RL Circuits Lab 10

Equipment SWS, RLC circuit board, 2 voltage sensors (no alligator clips), 2 leads (35 in)

Reading Review operation of oscilloscope, signal generator, and power amplifier II

1 Introduction

The 3 basic linear circuits are the resistor, the capacitor, and the inductor. This lab is concerned with the characteristics of inductors and circuits consisting of a resistor and an inductor in series (RL circuits). The primary focus will be on the response of an RL circuit to a step voltage and a voltage square wave.

2 Inductors

An inductor is a 2 terminal circuit element that stores energy in its magnetic field. Inductors are usually constructed by winding a coil with wire. To increase the magnetic field inductors used for low frequencies often have the inside of the coil filled with magnetic material. (At high frequencies such coils can be too lossy.) Inductors are the least perfect of the basic circuit elements due to the resistance of the wire they are made from. Often this resistance is not negligible, which will become apparent when the voltages and currents in an actual circuit are measured.

If a current I is flowing through an inductor, the voltage V_L across the inductor is proportional to the time rate of change of I, or $\frac{dI}{dt}$. We may write

$$V_L = L\frac{dI}{dt}, \tag{1}$$

where L is the inductance in henries (H). The inductance depends on the number of turns of the coil, the configuration of the coil, and the material that fills the coil. A henry is a large unit of inductance. More common units are the mH and the μH. A steady current through a perfect inductor (no resistance) will not produce a voltage across the inductor. The sign of the voltage across an inductor depends on the sign $\frac{dI}{dt}$ and not on the sign of the current. A positive current that is decreasing will produce a negative voltage across an inductor.

If an inductor has a resistance R_L the voltage across the inductor will be

$$V_L = L\frac{dI}{dt} + IR_L. \tag{2}$$

The most important specification for an inductor is its maximum current rating.

3 RL Circuits

A series RL circuit with a voltage source V(t) connected across it is shown in Fig. 1. The voltage across the resistor and inductor are designated by V_R and V_L, and the current around the loop by I. The signs are chosen in the conventional way. I is positive if it is in the direction of the arrow. Kirchoff's law, which says that the voltage changes around the loop are zero, may be written

$$V_L + V_R = V. \tag{3}$$

Assuming $R_L = 0$ and letting $V_L = L\frac{dI}{dt}$ and $V_R = IR$ Eq.(3) becomes

$$L\frac{dI}{dt} + RI = V. \tag{4}$$

The solution to the homogeneous equation (V(t)=0=a short circuit) is $I(t) = I_0 e^{-\frac{t}{L/R}}$, where I_0 is the current through the circuit at time t=0. This solution leads immediately to $V_R = I_0 R e^{-\frac{t}{L/R}}$ and $V_L = -I_0 R e^{-\frac{t}{L/R}}$. The homogeneous solution decays exponentially with a time constant of L/R.

Of interest is the response of an RL circuit to a constant voltage V applied across the circuit. The inhomogeneous solution is a constant. The function $e^{-\frac{t}{L/R}}$ describes the time dependence of the circuit. It is only necessary to use physical intuition and put the appropriate 2 constants into the solution. One guiding principle is that the current and V_R do not change the instant the voltage across an RL circuit is changed. V_R and I are continuous. The entire voltage change must appear across the inductor. Another is that after the constant voltage V is applied, the current in the circuit will exponential approach V/R if the inductor has no resistance, or V/(R+R_L) if the inductor has a resistance R_L. These considerations apply whatever the previous history of the RL circuit. Consider an RL circuit where V=0 and there is no current. We assume that $R_L = 0$. If at t=0 a constant voltage V is put across the circuit, V_L and V_R are given by, for t≥0,

$$V_L = Ve^{-\frac{t}{L/R}} \qquad and \qquad V_R = V(1 - e^{-\frac{t}{L/R}}). \tag{5}$$

This behavior is illustrated in Fig. 2. The current is not plotted, but remember that the current is proportional to V_R. Initially all of V appears across L because the current just before and after the application of V is zero. As the current exponentially builds up the voltage across the resistor increases and the voltage across the inductor decreases. If we wait a time T/2 where T/2 is many time constants we will have $V_R \cong V$ and $V_L \cong 0$. If now V is set equal to 0 (this is equivalent to shorting the circuit) V_L and V_R will be given by

$$V_L = -Ve^{-\frac{t}{L/R}} \qquad and \qquad V_R = Ve^{-\frac{t}{L/R}}. \tag{6}$$

This is also illustrated in Fig. 2. The above response of an RL circuit to a constant voltage which is at first V and then 0 can be observed by applying a square wave to the circuit where one leg of the square wave has zero voltage and the other has voltage V. If T is the period of the square wave, $T \gg L/R$. QUESTION. How would Fig. 2 be modified if the inductor had some resistance? QUESTION. If $T \gg L/R$, what is the response of an RL circuit to a symmetric square wave that oscillates between +V and -V (assume $R_L = 0$).

If a high frequency square wave, such that $T \ll L/R$, is applied to an RL circuit, the changes in current and V_R are minimal. There is not enough time for the current through the inductor to change much before the voltage is reversed. If the square wave is not symmetric with respect to ground the average V_R will be the average voltage of the square wave, assuming $R_L = 0$. Fig. 3 shows the voltages for a square wave that oscillates between the constant voltage V and 0 (ground), and Fig. 4 shows the voltages for a square wave that oscillates between +V and -V. In both these figures, the exponential dependences of V_L and V_R are approximated as straight lines and it has been assumed that $R_L = 0$.

4 Measuring R_L

In this section the DC value of R_L will be measured. In the right experiment setup window, drag the analog plug icon to channel C and choose power amplifier. Click the AUTO button in the signal generator window that appears. Program the signal generator for 2 V DC. Drag the digits display icon to the power amplifier icon below channel C. This digits display will give the current delivered by the power amplifier. Program the digits display for 2 decimal places. Connect the output of the power amplifier directly across the coil and click MON and then STOP. From your data calculate R_L, the resistance of the coil. Click the FILE menu button, NEW, and DON'T SAVE. Remove the wires from the coil.

During your experiments compare the value of R_L to the values of the resistances used in the RL circuits.

5 Experiments

In this section the response of an RL circuit will be examined experimentally using the SWS signal generator and power amplifier II, 2 voltage sensors, and the oscilloscope display. Drag an analog plug icon to channel C and choose power amplifier. Click the AUTO button in the signal generator window that appears. Drag the scope display icon to the voltage terminal icon. The oscilloscope display will open with the top or green trace channel having the signal generator voltage (analog output) as the input. Drag an analog plug icon to channel A and choose voltage sensor. Do this again for channel B. Program the middle scope channel (red trace) for channel A and the bottom scope channel (blue trace) for channel B.

In the experiment various resistors will be connected in series with an 8.2 mH inductor. Hook the circuit and voltage sensors up as shown in Fig. 5, paying attention to the polarities of the leads and terminals. The polarities are very important in this experiment. A positive voltage from a voltage sensor will correspond to a positive voltage as given by the convention of Fig. 1. The input square wave to the RL circuit will be in the top scope channel (green trace), V_L will be on the middle scope channel (red trace), and V_R will be on the bottom scope channel (blue trace). There are buttons on the far right of the scope display that add adjacent channels. In the experiments try adding the middle and bottom scope channels, as they should ad up to the voltage on the top channel. Why?

You may wonder about the particular choice of frequencies used in the experiments. It turns out that the results are better if the signal generator does not operate at too high a frequency and the time constants and frequencies have been chosen accordingly.

Some remarks.

- When using the oscilloscope, use MON rather than REC.
- When you click STOP, the last traces are stored. The stored traces are better for examination than the "live" traces as they are steady.
- You can use the smart cursor on the stored traces.
- You can use many of the scope functions on the stored traces.
- You can print out the stored traces.
- Label your printed out traces immediately by V, V_L, and V_R as the printers are not color printers.

- Use the same vertical sensitivity for all 3 traces so that you can easily compare the trace values.
- The default voltage amplitude of 5 V for the signal generator is NOT satisfactory for the experiments. Use 2 V. (The current rating for the inductor is 0.8 A.)
- For reliable triggering with a positive only square wave, make the trigger voltage positive but less than 2 V.
- Due to R_L, your graphs will differ somewhat from Figs. 2-4 of this write-up, which assumes $R_L = 0$.

5.1 T $\gg$ L/R, Positive Only Square Wave

Hook up an RL circuit with R=33 Ω. What is the time constant and how does it compare to the period of a 300 Hz square wave? Examine V_L, V_R, and V using a 300 Hz square wave for V that oscillates between 2 V and ground. How do the traces compare to Fig. 2? Is there an effect due to R_L? Use the smart cursor to measure the time constant of the exponentials (1/e=0.37) and compare your measurements to L/R and to L/(R+R_L). Which of these expressions do you think is the best one to use?

5.2 T $\gg$ L/R, Symmetric Square Wave

Use the same parameters as in section 5.1 except use a square wave that is symmetric with respect to ground. Compare your results to what you expect. Due to limitations in the equipment used and the non-zero value of R_L, the peak voltages will be less than calculated.

5.3 T $\ll$ L/R, Positive Only Square Wave

Apply a 3,000 Hz positive only square wave to an RL circuit with R=10 Ω. What is the time constant and how does it compare to T? Compare your results to Fig. 3. In Fig. 3 is approximating the exponentials by straight lines reasonable? Note the distortion in the square wave.

5.4 T $\ll$ L/R, Symmetric Square Wave

Use the same parameters as in section 5.3 except use a square wave that is symmetric with respect to ground. Compare to Fig. 4. The effect of R_L is quite small here as the average current is small.

5.5 T $\approx$ L/R, Symmetric Square Wave

Apply a 500 Hz square wave to an RL circuit with R=10 Ω. Are the results what you expect? If you have time, try somewhat lower and higher frequencies.

6 Finishing Up

Please leave the lab bench as you found it. Thank you.