RC Circuits

Equipment SWS, RLC circuit board, 2 voltage sensors (no alligator clips), leads [35 in. (2), 12 in. (1)])

Reading Review operation of oscilloscope. Review operation of signal generator when used with power amplifier II.

1 Introduction

The 3 basic linear circuits elements are the resistor, the capacitor, and the inductor. The voltage across a resistor is proportional to the current through the resistor. The voltage across a capacitor is proportional to the time integral of the current into the capacitor. The voltage across an inductor is proportional to the time derivative of the current through the inductor. This lab is concerned with the characteristics of capacitors and circuits consisting of a resistor and a capacitor in series (RC circuits). The primary focus will be on the response of an RC circuit to a step voltage and a voltage square wave. (A step voltage is a constant voltage which instantaneously changes to a different constant voltage.)

2 Capacitors

A capacitor is a 2 terminal circuit element that stores energy in its electric field. In its simplest form a capacitor consists of 2 parallel plates separated by a small gap which is usually filled with a non-conducting dielectric material. In practice many capacitors are made by sandwiching a dielectric sheet such as mylar between 2 pieces of metal foil and rolling this assmebly up into a cylinder.

If a positive current I enters one plate of a capacitor and exits the other plate a positive charge Q builds up on the first plate and a negative charge -Q builds up on the second plate. A voltage V_C developes across the capacitor. The charge Q and voltage V_C are linearly related by the equation

$$Q = CV_C, \tag{1}$$

where C is a constant called the capacitance. Eq. 1 can also be written as

$$V_C = \frac{1}{C}\int I dt. \tag{2}$$

The units of C in S.I. units are farads (F). One farad is a huge capacitance and more usual units are the micro farad (μF) and the pico farad (pF). The constant C depends on the geometry of the capacitor and the dielectric properties of the insulator between the plates.

The most important specification for a capacitor of a given value is its maximum voltage rating.

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A series RC circuit with a voltage source V(t) connected across it is shown in Fig. 1. The voltage across the resistor and capacitor are designated by V_R and V_C, and the current around the loop by I. The signs are shown and are chosen in the conventional way. I is positive if it is in the direction of the arrow. Kirchoff's law, which says that the voltage changes around the loop are zero, may be written

$$V_R + V_C = V. \tag{3}$$

Letting $V_R = IR = \frac{d}{dt}Q$ and $V_C = \frac{1}{C}Q$ this may be written as

$$R\frac{d}{dt}Q + \frac{1}{C}Q = V. \tag{4}$$

The solution to the homogeneous equation (V(t)=0=a short circuit) is $Q = Q_0 e^{-\frac{t}{RC}}$, where Q_0 is the charge on the capacitor at t=0. This solution leads immediately to $V_C = V_0 e^{-\frac{t}{RC}}$ and $I = -I_0 e^{-\frac{t}{RC}}$, where $V_0 = Q_0/C$ and $I_0 = Q_0/(RC)$. The homogeneous solution decays exponentially with a time constant of RC.

The function $e^{-\frac{t}{RC}}$ describes the time dependence of the charge, the voltages, and the current in an RC circuit if a constant voltage V is applied across the terminals. It is only necessary to use physical intuition and put the appropriate constants into the solution. One guiding principle is that V_C does not change the instant the voltage across the RC circuit is changed, and that V_C will exponentially approach the applied constant voltage V. Another is that with the application of the external voltage the initial current will be (V-$V_{C,b}$)/R, where $V_{C,b}$ is the voltage on the capacitor just before the constant voltage V is applied to the circuit. These considerations apply whatever the previous history of the RC circuit. Consider an uncharged RC circuit with no voltage applied to it. If at t=0 a constant voltage V is put across the circuit V_C and V_R are given by

$$V_C = V(1 - e^{-\frac{t}{RC}}) \qquad and \qquad V_R = Ve^{-\frac{t}{RC}}. \tag{5}$$

This behavior is illustrated in Fig. 2. Remember that the current is proportional to V_R. Initially all the applied voltage appears across R and the current is a maximum. As the capacitor charges up V_C increases and approaches V, and V_R and I decrease. If we wait a time T/2 where T/2 is many time constants we will have $V_C = V$ and $V_R = 0$. If now V is set equal to 0 (this is equivalent to shorting the RC circuit) V_C and V_R will be given by

$$V_C = Ve^{-\frac{t}{RC}} \qquad and \qquad V_R = -Ve^{-\frac{t}{RC}}. \tag{6}$$

This is also illustrated in Fig. 2. Note the minus sign for V_R. (When V=0, $V_C = -V_R$.) The above response of an RC circuit to a constant voltage which is at first V and then 0 can be observed by applying a square wave to the circuit where one leg of the square wave has zero voltage and the other has voltage V. If T is the period of the square wave, it is also necessary that $T \gg RC$. QUESTION. If $T \gg RC$, what is the response of an RC circuit to a symmetric square wave where the voltage oscillates between +V and -V? Hint. At the instant the square wave "switches" the magnitude of the voltage across the resistor is 2V.

If a high frequency square wave, such that $T \ll RC$, is applied to an RC circuit, the voltages changes across the capacitor are minimal. The capacitor does not have time to charge much before the voltage is reversed. Most of the voltage changes occur across the resistor. If the square wave is not symmetric with respect to ground, the average voltage of the capacitor will be the time average voltage of the square wave. (You can think of a square wave offset with respect to ground as constant DC voltage plus a square wave that is symmetric with respect to ground.) Fig. 3 shows the voltages for a square wave that oscillates between the constant voltage V and 0 (ground), and Fig. 4 shows the voltages for a square wave that oscillates between +V and -V. In both these figures, the exponential dependences of V_C and V_R are approximated as straight lines.

4 Experiments

In this section the response of an RC circuit will be examined experimentally using the SWS signal generator and power amplifier II, 2 voltage sensors, and the oscilloscope display. Drag an analogue plug icon to channel C and choose power amplifier. Don't forget to click the AUTO button in the signal generator window that appears. Drag the oscilloscope display icon to the voltage terminal icon. The oscilloscope display will open with the top or green trace channel having the signal generator voltage (analog output) as the input. Drag an analogue plug to channel A and choose voltage sensor. Do this again for channel B. Program the middle scope channel (red trace) for Channel A and the bottom scope channel (blue trace) for channel B.

In the experiments below different resistors and capacitors on the RLC network board will be connected in series. To do this connect a short lead across the coil (or "short" the coil). Hook the circuit and voltage sensors up as shown in Fig. 5, paying attention to the polarities of the leads and terminals. The polarities are very important in this experiment. A positive voltage from a voltage sensor will correspond to a positive voltage as given by the convention of Fig. 1. The input square wave to the RC circuit will be in the top scope channel (green trace), V_C will be on the middle scope channel (red trace), and V_R will be on the bottom scope channel (blue trace). There are buttons on the far right of the scope display that add adjacent channels. In the experiments try adding the middle and bottom scope channels, as they should add up to the voltage on the top channel. Why?

You may wonder about the particular choice of frequencies used in the experiments. It turns out that the results are better if the signal generator does not operate at too high a frequency and the time constants and frequencies have been chosen accordingly.

Some remarks.

- When using the oscilloscope, use MON rather than REC.
- When you click STOP, the last traces are stored. The stored trace is better for examination that the "live" trace as it is steady.
- You can you the smart cursor on the stored trace.
- You can use many of the scope functions on the stored trace.
- You can print out the stored trace.
- Label your printed out traces immediately by V, V_C, and V_R as the printer is not a color printer.
- Use the same vertical sensivity for all 3 traces so that you can easily compare the trace values.
- The default voltage of 5 V for the signal genertor is satisfactory for all the experiments.
- For reliable triggering with a positive only square wave, make the trigger voltage somewhat positive.

4.1 $T \gg RC$, Positive Only Square Wave

Hook up an RC circuit with R=10 Ω and C=100 μF. What is the time constant and how does it compare to the period T? Examine V_C, V_R and V using a 50 Hz square wave for V that oscillates between 5 V and ground. Are the results what you expect? Use the smart cursor to measure the time constant of the exponentials and compare your measurements to RC (1/e = 0.37). This measurement can be made more accurate by an appropriate choice of sweep time. Note that V_C is fairly similar in form to the input square wave.

4.2 $T \gg RC$, Symmetric Square Wave

Use the same parameters as in section 4.1 except use a square wave that is symmetric with respect to ground. Compare your results to what you expect. Due to limitations in the equipment used, the peak voltages will probably be less than expected.

4.3 $T \ll RC$, Positive Only Square Wave

Apply a 100 Hz positive only square to an RC circuit with R=100 Ω and C=330 μF. What is the time constant and how does it compare to T? Compare your results to Fig. 3. Note that V_R is fairly similar in form to the input square wave. In Fig. 3 is approximating the exponentials by straight lines reasonable?

4.4 $T \ll RC$, Symmetric Square Wave

Use the same parameters as in Section 4.3 except use a square wave that is symmetric with respect to ground. Compare to Fig. 4. Is approximating the exponentials by straight lines reasonable?

4.5 $T \approx RC$, Symmetric Square Wave

Use the same RC components as in Section 4.3. Apply a symmetric square wave with a frequency of 30 Hz. Are the results what you expect? If you have time, try somewhat lower and higher frequencies.

5 Finishing Up

Please leave your bench in a state of maximum order. Thank you.

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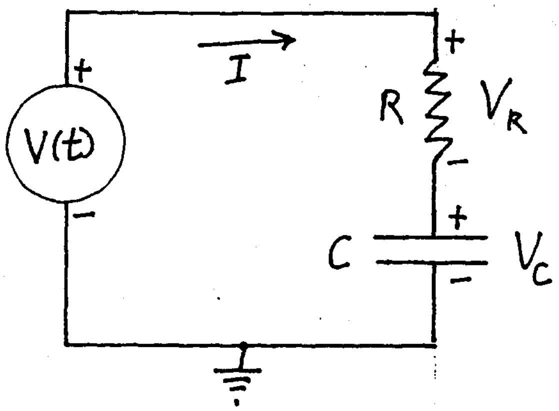

Fig. 1

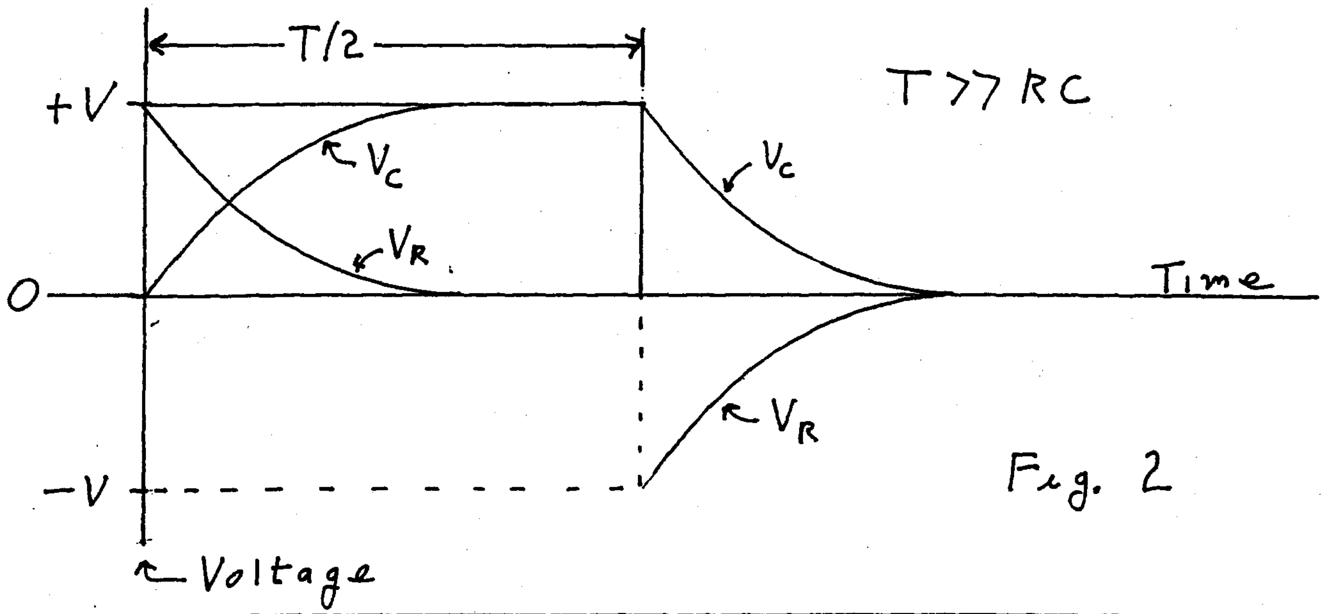

Fig. 2

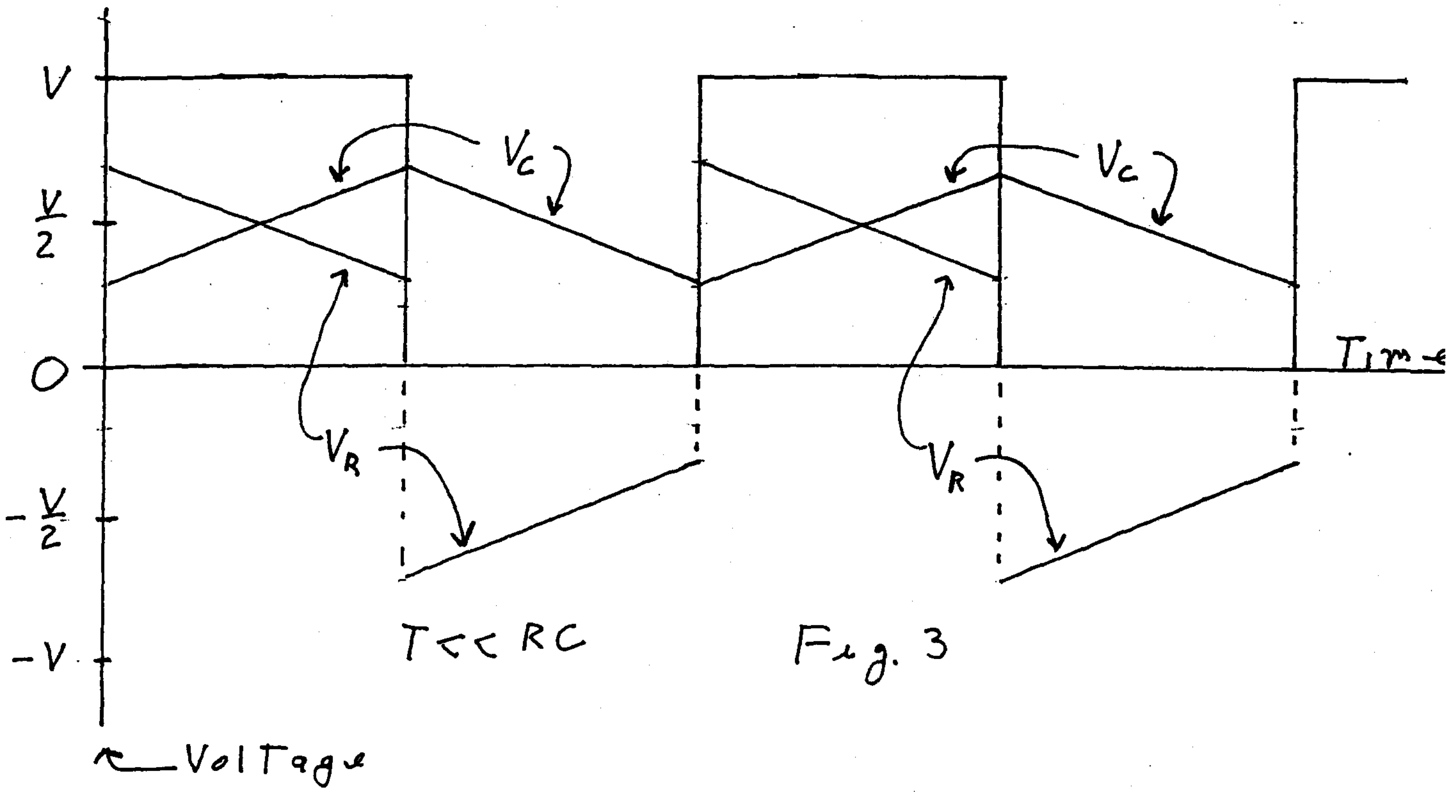

Fig. 3

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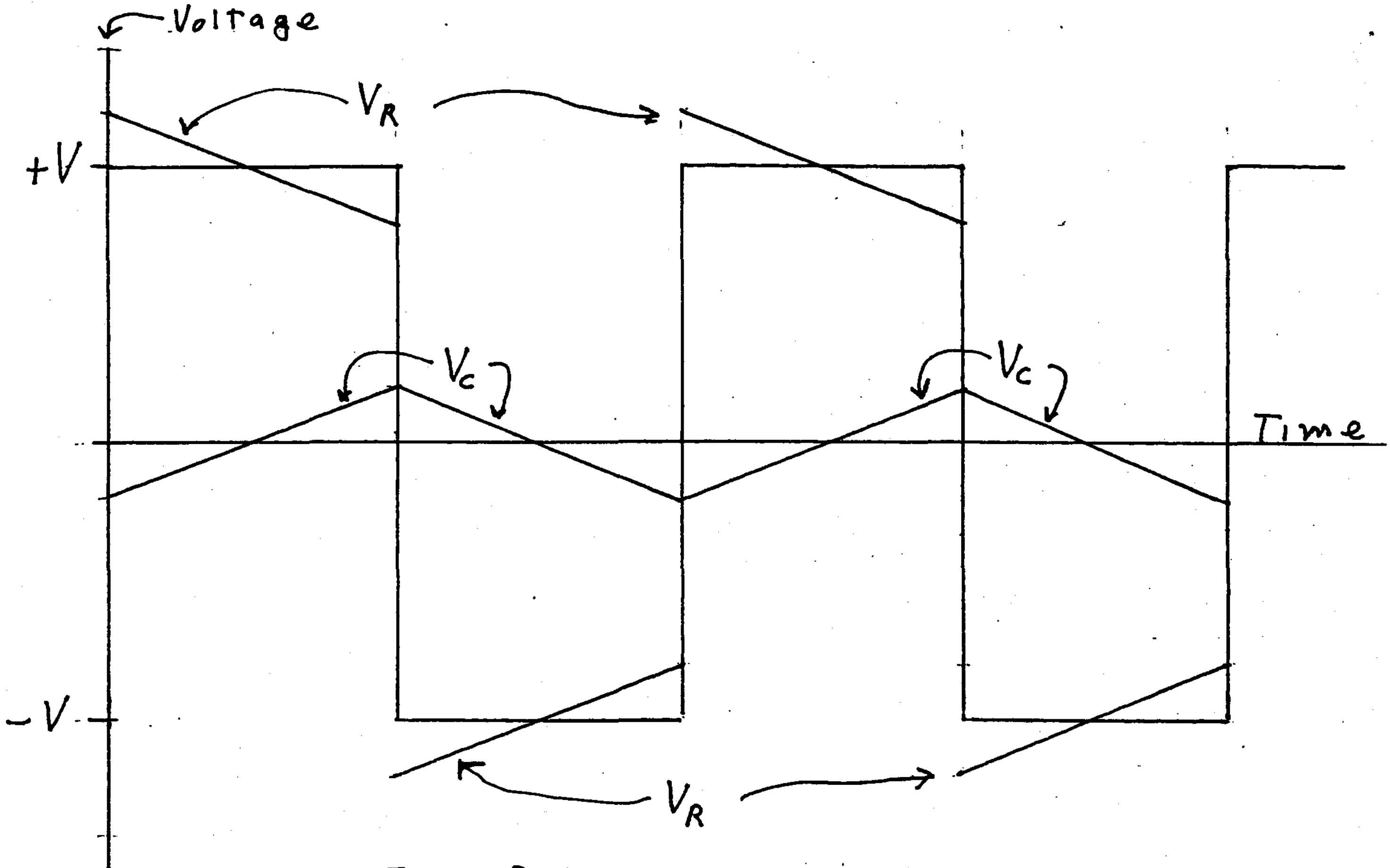

$T << RC$ Fig. 4

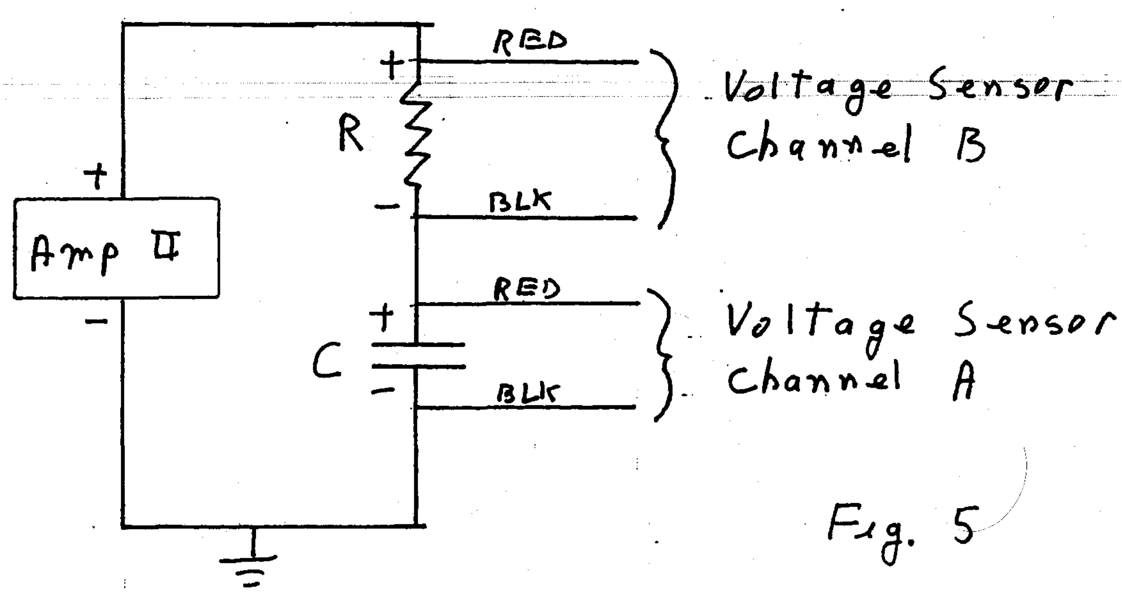

Fig. 5