

# RLC Circuits

Equipment: Capstone, 850 interface, RLC circuit board, 4 leads (91 cm), 3 voltage sensors, and Fluke multimeter.

Reading: Review AC circuits write-up

## 1 Introduction

Oscillating systems continually exchange one kind of energy for another. Familiar examples of oscillating systems are a mass on the end of a horizontal spring and a pendulum (conservation of energy experiment). In the former, kinetic energy and energy of the spring are traded back and forth. In the latter, kinetic energy and gravitational potential energy are exchanged. If these systems are excited and then left alone, the oscillations die down from friction. If these systems are driven by a sinusoidal driving force, the oscillations will be maintained and magnitude of the oscillations will depend on the frequency. If the frequency of the driving force is near the natural frequency of the system and friction is small, the oscillations will be quite large. Another example of an oscillating system is a resistor, capacitor, and inductor connected in series, which is often called a tuned circuit or an RLC circuit (**R=resistor, L-inductor, and C=capacitor**). **In this system, the magnetic energy of the inductor is exchanged with electrical energy of the capacitor.**

### 1.1 Undriven Systems

If friction or damping is not too large on an oscillating system is that is left alone, then the system will oscillate. If damping is zero, it will oscillate at an angular frequency  $\omega_0$  and amplitude of the oscillations will not change. For a mass  $\mathbf{m}$  on a spring  $\mathbf{k}$  that has no damping,  $\omega_0 = \sqrt{k/m}$ . For an RLC circuit with  $R = 0$ ,  $\omega_0 = 1/\sqrt{LC}$ . For a system with friction, oscillations decay, and when the friction is small, the frequency of oscillation is slightly less than  $\omega_0$ .

### 1.2 Driven Systems

If an oscillating system is sinusoidally driven at an angular frequency  $\omega$ , the system will oscillate at the same angular frequency. If the driving frequency is near  $\omega_0$ , the oscillations can be quite large. In this experiment an oscillating system that consists of a resistor, capacitor, and inductor connected in series will be investigated. The parameters that oscillate

are the charge on the capacitor, the current in the circuit and the voltages across all the components. Damping is furnished by the resistor.

## 2 Theory Of RLC Resonance

### 2.1 Qualitative Discussion

The circuit is shown in Fig. 1. The same current flows through all three elements.  $v_R$  will be in phase with the current. In a capacitor, current flows while voltage potential increases, and an inductor will resist any change in current. So,  $v_L$  will lead the current and  $v_R$  by  $\pi/2$ , and  $v_C$  will lag the current and  $v_R$  by  $\pi/2$ .  $v_L$  and  $v_C$  will be out of phase by  $\pi$ . At high enough and low enough frequencies, the impedance of the inductor will be much larger than the impedance of the capacitor or resistor, and the current will be small. **At the resonant frequency the reactance of the capacitor and inductor are equal and their impedances cancel out.** The current is a maximum. At resonance the sum of the voltages across the inductor and capacitor is zero, but  $v_C$  and  $v_L$  can be quite large if R is small.

### 2.2 Quantitative Discussion

The total impedance  $Z$  of the RLC combination is

$$Z = Z_R + Z_L + Z_C = R + j\omega L + \frac{1}{j\omega C} = [R^2 + (X_L - X_C)^2]^{\frac{1}{2}} e^{j\theta}, \quad \arctan \theta = \frac{X_L - X_C}{R}. \quad (1)$$

Where inductive reactance  $X_L = \omega L$ , capacitive reactance  $X_C = 1/\omega C$ , and  $j^2 = -1$ .

#### 2.2.1 Explanation of phasor voltages

In AC circuits that have inductors and capacitors, voltage and current are not in phase with one another. The period difference between voltage and current, defined as phase difference, is in degrees.

The undriven oscillating frequency with  $R = 0$  is given by  $\omega_o = 1/\sqrt{LC}$ . Let the voltage applied to the circuit be  $v(t) = V \cos(\omega t)$ . Then phasor voltage  $\bar{V} = V$ . The phasor current  $\bar{I}$  will be

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{V e^{-j\theta}}{[R^2 + (X_L - X_C)^2]^{\frac{1}{2}}} \quad (2)$$

Phasor voltage across the resistor  $\bar{V}_R$  is

$$\bar{V}_R = \bar{I} Z_R = \frac{R V e^{-j\theta}}{[R^2 + (X_L - X_C)^2]^{\frac{1}{2}}} \quad (3)$$

Phasor voltage across the inductor  $\bar{V}_L$  is

$$\bar{V}_L = \bar{I} Z_L = \frac{V X_L e^{j(-\theta + \frac{\pi}{2})}}{[R^2 + (X_L - X_C)^2]^{\frac{1}{2}}} \quad (4)$$

Phasor voltage across the capacitor  $\bar{V}_C$  is

$$\bar{V}_C = \bar{I}Z_C = \frac{V X_C e^{j(-\theta - \frac{\pi}{2})}}{[R^2 + (X_L - X_C)^2]^{\frac{1}{2}}} \quad (5)$$

From these equations, the following conclusions can be drawn:

1. The current is a maximum when  $X_L = X_C$ . This occurs at the resonant frequency which is given by  $f_0 = \frac{\omega_0}{2\pi}$ , where  $\omega_0 = 1/\sqrt{LC}$ .
2. At resonance, the magnitude of the voltage across the capacitor is equal to the magnitude of the voltage across the inductor.
3. At resonance, the magnitude of the voltage across the capacitor and the magnitude of the voltage across the inductor are equal to Q times the source voltage, where  $Q = (\omega_0 L)/R$ . Q is the “quality factor” of the circuit. Q is large if R is small.
4. The larger Q is the narrower the resonance.
5. The phase difference between capacitor voltage and inductor voltage is always  $\pi$ .
6. For  $f \gg f_0$  the current is small and determined by  $X_L$ .
7. For  $f \ll f_0$  the current is small and determined by  $X_C$ .

### 3 The Q or Quality Factor

Generally, the less friction in an oscillating system, the better the system. A dimensionless parameter is defined as Q (for quality factor). A large Q represents less resistance and a better system response. For a series RLC circuit,  $Q = \omega_0 L/R$ , which is the inductive reactance at resonance divided by the resistance. The Q of an oscillating system can be defined as

$$Q = 2\pi \frac{\text{the energy stored in the circuit}}{\text{the energy lost per cycle}} \quad (6)$$

#### 3.1 Q and the Undriven Series RLC Circuit

If R is low enough and the series RLC circuit is excited and the left alone, the current will oscillate and decay exponentially as  $e^{-\omega_0 t/2Q}$ . If R is large enough, the current will not oscillate but decay monotonically. The circuit is said to be over-damped. Critical damping separates these two behaviors and is given by the resistance  $R_C = 2\omega_0 L$  or  $Q = 1/2$ .

#### 3.2 Q and the Driven Series RLC circuit

If the series RLC circuit is driven by a constant sinusoidal voltage source, the current as a function of frequency will exhibit resonant behavior with the maximum current at a frequency  $\omega_0 = 2\pi f_0$ . See Fig. 2. The width of the resonance curve decreases as Q increases. If  $Q \gg 1$ ,

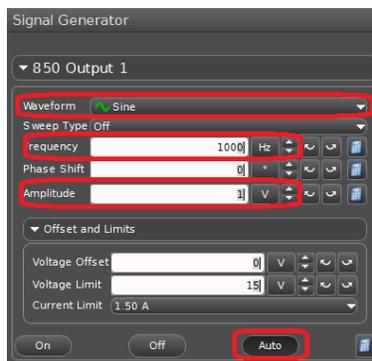
the half power points (where the current is  $1/\sqrt{2}$  of the maximum value) occur at the two frequencies  $f_0 = \pm f_0/2Q$ . At the half power points, the current and voltage are  $\pm 45$  degrees out of phase. The “bandwidth”  $B$  of the circuit is  $f_0/Q$ . **In this experiment the  $Q$  is not nearly high enough to use these approximations.**

## 4 Experiment

The experiment will use a 10 ohm resistor, 8.2 mH inductor (no iron) and a  $100 \mu F$  capacitor on the circuit board. **Calculate  $\omega_0$  and  $Q$ .** The  $R$  you’re using in calculating  $Q$  should include the resistance of the inductor. **Because the inductor has some resistance, both magnitude and phase of the voltage across the inductor will not completely agree with the calculations presented.** This will become more evident as the applied frequency is made lower.

### 4.1 Setting up the experiment

- To measure the ohmic resistance of the coil (inductor), connect the Fluke multimeter across the coil, and switch the knob to  $\Omega$ .
- Set up the circuit, using the voltage sensors, as shown in Fig. 3.
- The Fluke should be connected across the 400 mA terminals in series to all three circuit components to measure the AC current. **(Set the fluke meter to read AC mA)**
- Connect the voltage sensors to 850 Analog inputs. Start the Capstone software and set up three voltage sensors on the 850 interface.
- In the tools column click on the signal generator icon.
- The output of the 850 interface will be controlled by the 850 output 1 signal generator. Remember you need to click on Auto!



The items in red will be the functions you will be using and adjusting when you run the experiment.

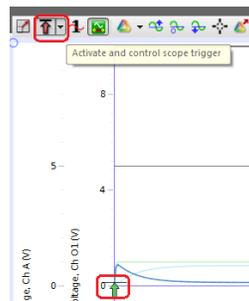
- Open the scope display by dragging the scope icon to the center of the screen.
- Click on the orange tack to prevent overlapping.
- Next, click on **Add a new y-axis** twice. It's located on top bar of the scope display. You should now have 3 y-axes.
- For each axis, click **Select Measurements** and choose one to be **Output Voltage Ch A (V)**, the second to be **Voltage Ch B (V)**, and the third to be **Voltage Ch C (V)**.

#### Remarks on running Capstone:

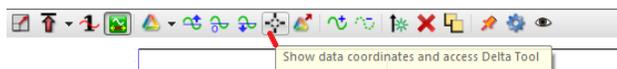
- When using the oscilloscope, use Fast Monitor Mode rather than Continuous Mode.



- You need to use the trigger function on the scope to obtain a stable signal. Click on the trigger icon and on the side of the scope move the trigger arrow up.



- For reliable triggering with a positive only square wave, make the trigger voltage positive but less than 1 V.
- When you click STOP, the last traces are stored. The stored traces are better for examination than the “live” traces as they are steady.
- You can use the Show data coordinates and access delta tool, and many of the other scope functions, on the stored traces.



- You can print out the stored traces, but label your printed out traces immediately as the printers are not color printers.

- **Be aware of vertical scales for all 3 traces so that you can easily compare the trace values.**

### Comment about voltage

Unfortunately for this experiment, the impedance of the RLC circuit and the voltage across the circuit will change with frequency. At a given frequency, the various phase relationships between the components, as well as the relative magnitude of the voltage, will be properly displayed. However, the relative magnitudes of the voltages for two different frequencies will not be correct unless the voltage across the RLC circuit is adjusted to be the same for the two frequencies.

- Use a sine wave function. Adjust the output of the function generator to some reasonable value like 2 V at resonance and then sweep the frequency from well below the resonant frequency to well above the resonant frequency.
- Do this slowly several times. On the scope, carefully scrutinize the relative magnitudes and phases of the voltages across the R, L, and C for the various frequencies and discuss what you see.
- Now do a more careful investigation by choosing a set of discrete frequencies separated by about 30 Hz (exact value up to you). These frequencies should start well below resonance and end well above resonance.
- At each frequency, check the voltage output of the signal generator is the *same* value, and measure the amplitudes of the voltages  $v_R$ ,  $v_L$ , and  $v_C$ . **Do they all add up to the voltage of the 850 output? Why or why not?**
- Use a coordinate tool to measure the phases of  $v_L$  and  $v_C$  with respect to  $v_R$ . With Python, plot the magnitude of the current versus the frequency and determine the frequency at which the maximum current occurs. **Does this agree with you calculation of  $\omega_0 = 2\pi f_0$ ?** Plot the magnitude and phases of  $v_L$  and  $v_C$  versus frequency. Discuss your results.
- Insert the iron rod into the inductor and determine the resonant frequency. **What is the value of the inductance with the iron rod inserted?**

## 5 Question

This experiment has been concerned with the series RLC circuit driven by a sinusoidal voltage source.

1. Qualitatively discuss the parallel RLC circuit driven by a sinusoidal current source. At resonance is the voltage maximum or minimum?
2. At frequencies way above and below resonance is the voltage high or low?

## 6 Finishing Up

Please leave the bench as you found it. Thank you.

RLC Resonance

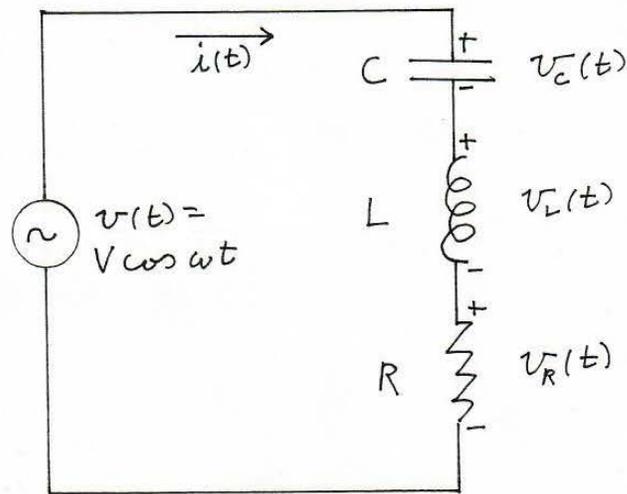


Fig. 1

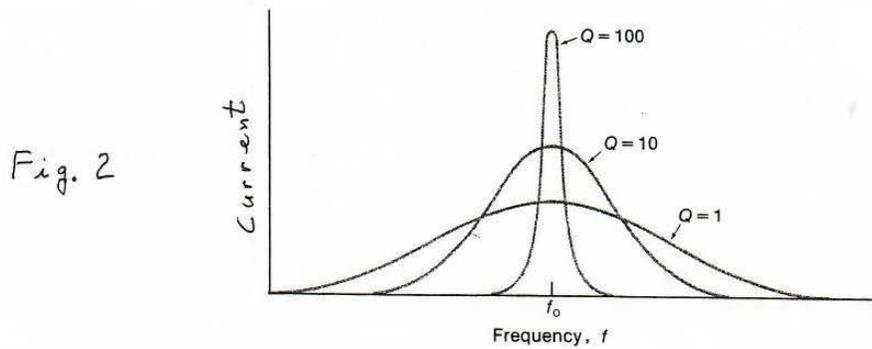


Fig. 2

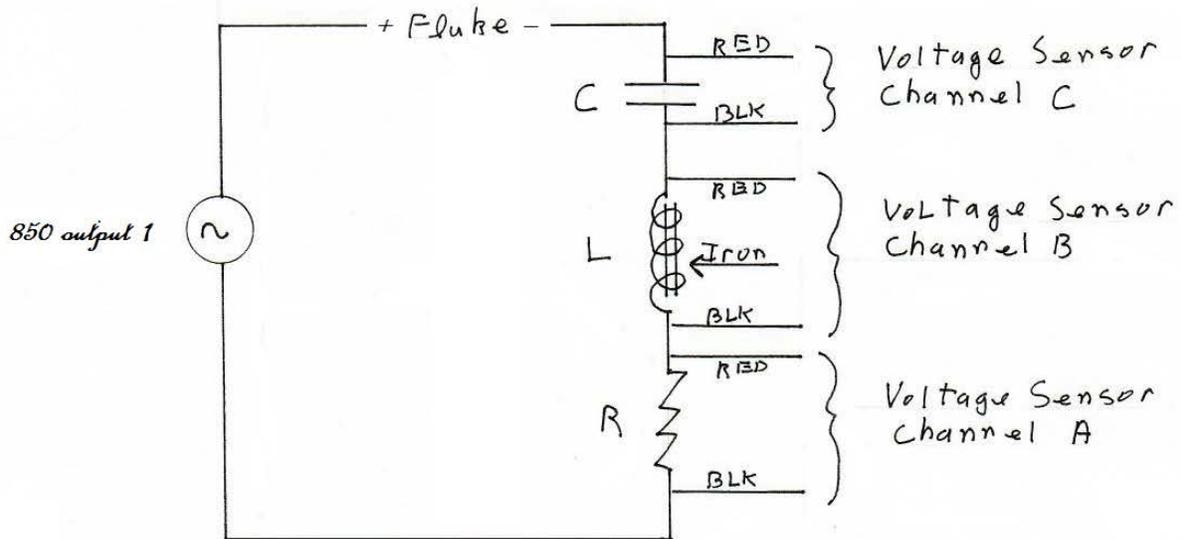


Fig. 3