Work-Energy

Caution: For this lab, force sensors are mounted near the floor on rods that stick out into the aisle. Please be careful.

1 Purpose

- To investigate the validity of the work-energy relationship.

- To perform a straight line fit to data.

2 Theory

Consider a point object of mass m acted upon by a net force $\mathbf{F} = \mathbf{F(r)}$. The position, velocity, and acceleration of the mass are given by $\mathbf{r}$, $\mathbf{v}$, and $\mathbf{a}$, and the time by t. The force $\mathbf{F}$ may be a function of $\mathbf{r}$. Newton's 2nd Law for the object is $\mathbf{F} = m\mathbf{a}$. The work W done on m as it moves from $\mathbf{r}_i$ to $\mathbf{r}_f$ is defined as the integral over the path of the component of the force in the direction of motion times the displacement of the object:

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = m \int_i^f \mathbf{a} \cdot d\mathbf{r} \ . \tag{1}$$

It is only the component of the force parallel or anti-parallel to the motion of m that contributes to the work. The right hand side can be transformed to

$$m \int_i^f \frac{d\mathbf{v}}{dt} \cdot d\mathbf{r} = m \int_i^f \mathbf{v} \cdot d\mathbf{v} = \frac{1}{2} m \mathbf{v} \cdot \mathbf{v} \Big|_i^f = \left(\frac{1}{2} m v^2 \right)_f - \left(\frac{1}{2} m v^2 \right)_i \ . \tag{2}$$

The quantity $\frac{1}{2}mv^2$ is defined as the kinetic energy, K, of the object. In words, when m moves from $\mathbf{r}_i$ to $\mathbf{r}_f$, the net work W done on m between those two points is equal to the change in kinetic energy of m between those two points. More succinctly,

$$W = \Delta K \ . \tag{3}$$

This is the most basic form of the work-energy theorem. To be used correctly, W must include all the work done on the object by all the forces during the entire path taken between the two points under consideration.

The work-energy theorem can be applied to a system of particles. The total work done on the system is equal to the total change in kinetic energy ΔK of the system. Care must be taken to include the work done by the forces between the constituents of the system, or the "internal" work. Unlike the case for forces and change in momentum, the total internal work need not cancel out to zero.

The motion in this experiment is one dimensional and the relevant forces are along the direction of motion, so that if one defines the one dimension as x, the work becomes $\int_{x_i}^{x_f} F dx$.

3 Equipment

Equipment list: Capstone, air track, 28-cm glider, set of masses with hooks, photogate/smart pulley, small bench clamp, approximately 40 cm metal rod, force sensor, 100 cm string with loops, 100 cm string with loops and 3 or 4 rubber bands, 130-cm string with loops, meter stick, and analog scale.

Gliders on a horizontal air track will be used as the bodies upon which force is exerted. Forces are exerted by attaching one end of a string to the glider, running it over a pulley, and attaching a hanging weight or stretched rubber bands to the other end of the string.

Velocity is measured using a photgate with smart pulley that is read out with Capstone. When the glider is exerted by stretched rubber bands, force is measured using a force sensor read out by Capstone. When a hanging weight is used, the force can be calculated using what you know about gravity.

4 Work done by gravity

4.1 Procedures

4.1.1 Overview

The work-energy theorem is tested here for a system of two masses in which one is falling due to gravity and the second is connected to it with a string. The theorem can be applied to the individual masses and to the system as a whole.

A string is attached to a glider on a horizontal air track. The string goes over a photogate with pulley and has a mass attached to its hanging end. The glider is released and the velocity of the glider is measured as a function of distance. The velocity of the glider is measured at various points and the change of K calculated for each mass (glider and hanging mass) and for both together. The work done on each mass and for the system is calculated.

4.1.2 Setting up photogate with pulley and programming Capstone

You will program the Capstone to measure linear position and speed from the pulley.

- Assemble the photogate with pulley on rod.

- Plug the photogate/smart pulley into the interface, and noting which channel it is plugged into, program Capstone for **photogate with pulley**.

- Check that both Linear Speed and Position in the **Timer Setup** window under **Tools** column has been checked off.

- Open a graph display by clicking on **Graph** icon in the **Displays** column and drag it to the center of the white screen.

- On the graph display move your mouse to the horizontal axis of the graph and click on **Select Measurements**. At this point click on the left side of the mouse and select **Position** from the submenu. Next, go to the vertical axis and click on **Select Measurements** and select **Linear Speed**. Your graph will have **Linear Speed** on the vertical axis and **Position** on the horizontal axis.

4.1.3 Doing the experiment

Level the air track and mount the photogate/smart pulley assembly in front of the air track. Hook the string to the glider and pass the string over the smart pulley. Adjust heights and positions so that the string doesn't rub on anything. Hang a 50g mass from the string. Pull the glider back until the 50g mass is close to the photogate/smart pulley, Click the Record button, and let the glider go. Click Stop just before the mass hits the floor or the glider hits the end of the track.

Once you have a data set, you should adjust the vertical and horizontal scales on the Graph Display so that you can clearly see your data. You can do this, for example, by selecting the data with a mouse and using **Scale axes to show all data** in the upper left of the **Graph** window. In the graph display you can use **Highlight range** icon to select certain parts of the curve or **Add a Coordinate Tool** to give exact data points of the curve.

4.1.4 Saving your data

Save your data to a text file. First, go to the **Displays** column and click on the **Table** icon. Set one table column as **Linear Speed** and the other as **Position**. In the top row of each column; select the data run you're interested in analyzing. Next, left click along the two top rows of one column. This will highlight all the data in that column. After, scroll down to where data begins in the column and right click on that column. Select **Copy values** Open an Excel sheet and paste the data into a column. Repeat for the other table column in Capstone. Select the base directory, my documents in most cases, and save your data with an appropriate name like WErun1.txt. Avoid spaces, keep your file name short (8 or fewer characters), and be sure to save your data as a **Text (Tab delimited)** file. You can do this under **save as**. You will analyze the data in this file later using Python. E-mail the data files you saved to yourself (and anyone else working with you).

4.2 Analysis and Questions

1. Describe how you leveled the air track.

2. Denote the hanging mass by M_1 and the glider mass by M_2. Show that the tension T in the string is $T = M_1 M_2 g/(M_1 + M_2)$.

3. Show that the work done on M_1 is $(M_1 g - T)(x_f - x_i)$. Explain the minus sign in front of T. Similarly, explain why the work done on M_2 is $T(x_f - x_i)$.

4. The work done on the system is $M_1 g(x_f - x_i)$. Explain clearly why T does not appear in this expression. You may want to consider Newton's 3rd law here.

5. Pick suitable x_i and x_f and determine the velocities at those points. Calculate the work done and the change in K for M_1, M_2, and for the system as a whole. Determine the uncertainties in each of these quantities, by propagating uncertainties from the measured quantities. Make sure to provide the uncertainties in the measured quantities for which you can estimate them, and state why you cannot for those you don't know. Are work and the change in K equal within errors for each of the masses and the whole system?

6. Should the graph of velocity versus displacement look like a straight line? Why or why not?

7. Create a data table and save your data to a text file. Write a Python program that fits velocity squared versus distance to a straight line. Plot your points and the straight line. What is the slope of your line? What should it be? Does the slope have units? If yes, make sure to include them.

5   Work done with a variable force

Here rubber bands are used to exert a varying force and perform varying work on an object.

### 5.1   Procedures

5.1.1   Overview

A glider of mass m is on a horizontal air track. A horizontal string attached to the glider goes over a smart pulley. The end of the string that goes over the smart pulley is attached to 3 or 4 rubber bands that are connected in series, and the last rubber band is attached to a force sensor which is clamped to a rod near the floor. The glider is pulled back to stretch the rubber bands and then let go. The force on the glider and linear speed of the glider are measured as a function of the position of the glider. Capstone integrates the force curve to give the work, and the linear speed curve can be examined to give the velocities at the chosen beginning and end points. The change in K is calculated and compared to W.

The force on the glider is not constant and as a consequence the acceleration is not constant. The tension T in the string will be the same on both sides of the pulley if the mass of the string and pulley can be neglected and if the pulley is frictionless. The magnitude of the force between the last rubber band and the force sensor will be the same as T if the rubber bands are massless.

5.1.2   Programming Capstone

Restart Capstone; program the Capstone to measure the force and velocity of the pulley as a function of the displacement x.

- Check that the (analog) force sensor and the (digital) photogate/smart pulley are plugged into the interface, and, noting which channels they are plugged into, program Capstone for these two sensors.

- Check that both Linear Speed and Position in the **Timer Setup** window under **Tools** column has been checked off.

- Drag the graph icon from the **Displays** column onto the Capstone page. Use add new **plot button** for the second graph.(The add a new plot option is located on the top bar above the plot) Program the graph so position is on the horizontal axis. Program the two veritcal axes for **Force** and **Linear Speed**. The following figure illustrates setting up the graph.

- Go to the bottom left center of the Capstone work page. Where it states Force Sensor next to "Recording Conditions", increase the periodic samples to 200 Hz.
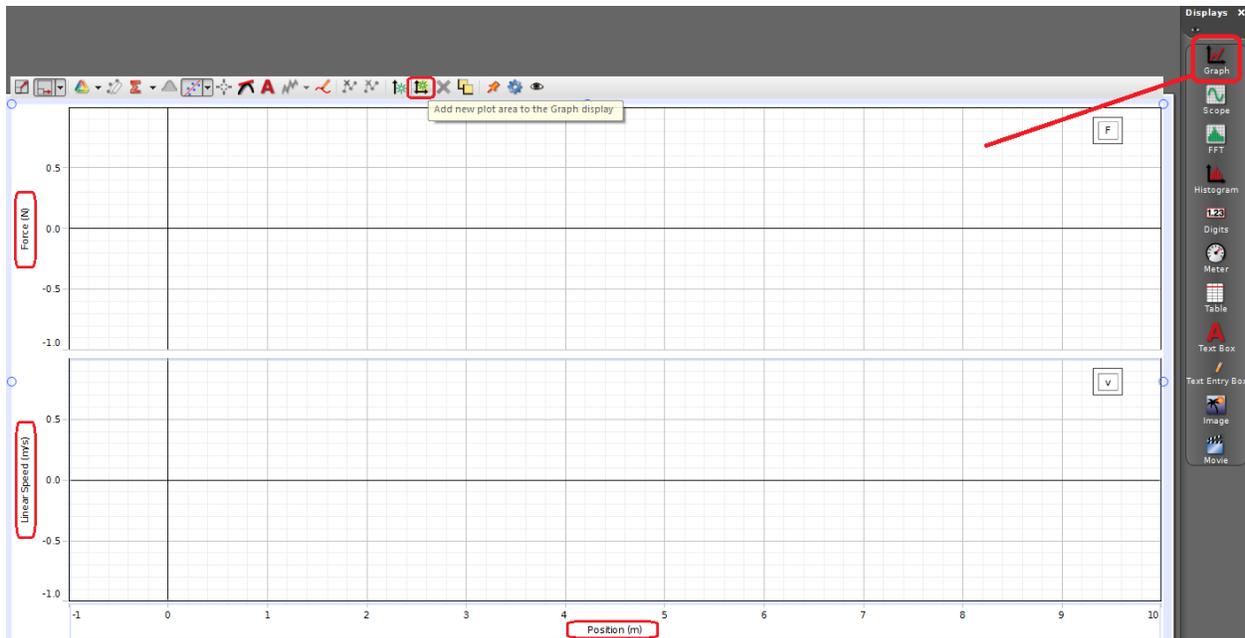
5.1.3   Setting up and calibrating the force sensor

The force sensor is calibrated by setting its values when there is no force on it and when there is a known force on it.

- On the side of the table along the bottom beam mount the force sensor, that the metal hook of the sensor is up in the vertical dircetion. The location of the force sensor will be in front of the air track.

- Leave the force sensor in the positon in which it will be used.

- You need to run the program to calibrate the Force Sensor. In Capstone, click the **Record** button.

- With nothing touching the hook on the force sensor, push the tare button located on the side of the sensor.

- Go to the **Tools** column and click on the **Calibration** icon.

- Select the force sensor and click next.

- In choosing the type of calibration you like select   **Two Standard (2 points)** and click next.

- In the box labeled Calibration first Point. Enter 0 in the box labeled **Standard Value** and click on **Set current value to standard value**.

- Take a 100 cm string with loops at both ends and attach one end to the hook of the force sensor. Pass the other end of the string over the pulley and down through the

hole in the bracket of the small bench clamp holding the photogate/smart pulley. Hang a 100 g mass from that end of the string.

- In the box labeled Calibration second Point. Enter the **weight** of the 100 g mass in the box labeled Standard Value. Click on **Set Current value to standard value**.

- Click Finish.

- Click **Stop**.

- Go to **Data Summary** in the Tools column and delete the data it recorded.

5.1.4 Taking Data

Remove the string used for calibrating the force sensor. Level the air track. Take the string with the rubber bands at one end and hook the last rubber band to the force sensor. Pass the string over the photgate/smart pulley and hook the end to the 28-cm glider. Pull the glider back to stretch the rubber bands but do not pull the rubber bands into the photogate/smart pulley. Click Record, and let go of the glider. Click Stop just after the string goes slack.

Once you have a data set, you should adjust the vertical and horizontal scales on the Graph Display so that you can clearly see your data. Note that you are not interested in any data taken after the string goes slack and the force falls to zero.

Click the **Scale axes to show data** button. Make sure the data are clearly visible in the plots, but only over the distance until the force falls to zero, as discussed above. Click the **Display area** button and select the area. Box in the part of the force curve you wish to integrate. Your beginning integration point x_i should exclude perhaps the first two data points. The end of your integration x_f should be just before the velocity curve levels off for the first time and before the force curve goes to zero for the first time. Record the value of the integral which is the work W done between the limits chosen. Use the **Add a Coordinate Tool** determine the velocities at the end point of your integration and calculate the change in K.
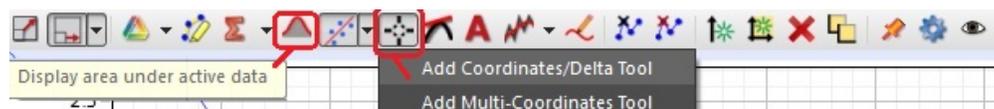
5.1.5 Saving your data

Save your data to a text file as described in the procedures of the "Work done by gravity" section. You will analyze the data in this file later using Python. E-mail the data files you saved to yourself (and anyone else working with you).

5.2 Analysis and Questions

1. Compare the change in K with the calculated work done. Are they equal within experimental uncertainties?

2. The trapezoidal rule states that an integral from a to b of a function $f(x)$ can be approximated by breaking the interval into N segments of length $(b-a)/N$, approximating the area under $f(x)$ for each segment as that of a trapezoid, and summing the areas of the segments. In equation form,

$$\int_a^b f(x)dx = \frac{b-a}{2N}\left[f(x_1) + 2f(x_2) + 2f(x_3) + ... + 2f(x_N) + f(x_{N+1})\right] \qquad (4)$$

where x_i is the left edge of segment i and x_{N+1} is the right edge ($= b$) of segment N. Show with pictures and a calculation why this equation correctly implements the verbal description.

3. After the lab, write a Python program to read the data you took and then repeat the analysis above: Integrate your force *vs.* distance data using the trapezoidal rule to obtain the the work W. Compare the work done to the change in the kinetic energy. Does your data support the work-energy theorem to within the experimental uncertainty of your measurements? Be sure to include accurate estimates of experimental uncertainties.

4. In previous labs, we saw that differentiating data increases the relative uncertainty of a quantity. Does integrating the data increase or decrease the uncertainties? Briefly explain.

6   Finishing Up

Please return the bench to the condition in which you found it. Do not save the experiment using the Capstone software. After mailing your data to each group member, move all your data files to the trash. Thank you.