# Motion $1 + 2$

## 1 Introduction

This experiment serves multiple purposes:

- By measuring position, velocity and acceleration of an object you move, it is designed to provide some intuitive feel of the meaning of these kinematic quantities.

- It serves to demonstrate that one way of doing a rough check of your measurement device is to measure something of which you have a rough idea of the true value. Also, measuring something that is precisely known provides a calibration of the device.

- This laboratory will give you some practice in propagation of errors and statistical analysis of random uncertainties.

- Numerical differentiation will be performed.

- Your analysis will involve plotting with Python.

- This lab will introduce you to the Capstone software system and PASCO hardware that will be used in a number of experiments conducted this semester.

## 2 Theory

Velocity $v$ is defined as $v = dx/dt$ and acceleration $a$ as $a = dv/dt = d^2x/dt^2$. Integrating the last definition twice for a situation where $a$ is constant gives

$$v = v_0 + at \quad \text{and} \tag{1}$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \quad , \tag{2}$$

where the constants $x_0$ and $v_0$ are the position and velocity when $t = 0$. When these are plotted, time is usually chosen as the horizontal axis, with position and velocity as the vertical axes. The first equation is then a straight line and the second a parabola.

## 3 Equipment Used

**Equipment List:** An air track, 2 gliders, 2 wooden blocks, index cards, a meter stick, a motion sensor, and hardware for mounting the motion sensor on the lab bench.

**Air Tracks & Gliders:** In this experiment you will be using an air track, a long straight metal beam of triangular cross section with many small holes in it. A blower pumps air through the holes so that a glider placed on top of the air track floats on a cushion of air with little friction. The blower sits on the floor and has a switch on the top. The blowers are noisy and run hot. Run them only when you need them!

*Please handle the gliders carefully.* They are expensive, and if dropped or abused, may be damaged. Do not move the gliders on the air track without the blower on, as this damages both the glider and air track. A motion sensor, which works by detecting reflected sonic (sound) pulses from an index card attached to the glider, can be used to detect the glider's position. Fix the index card on top of the glider and orient it perpendicular to the track to reflect the sonic pulses.

**The Motion Sensor:** The motion sensor works by measuring the time from when a sound pulse leaves the sensor to when it is reflected back to the sensor. Like all waves, sound can be reflected (producing an echo). In this experiment, the motion sensor emits a short sound pulse, say at time $t_1$, that travels out in a cone at a speed $v_s$ towards the glider. The pulse is reflected by the index card on the glider and returns to the motion detector where its arrival time $t_{1a}$ is detected. The time of flight $\Delta t_1 = t_{1a} - t_1$ is related to the distance between the position detector and the position of the index card on the glider by the relation

$$x_1 = \frac{1}{2} v_s \Delta t_1 \tag{3}$$

Obtaining the correct distance $x_1$ thus depends on knowing the speed of sound in air. Sound is a pressure wave whose speed depends on the properties of the gas in which it propagates. For a given gas the speed of sound is proportional to $\sqrt{T}$, where $T$ is the absolute temperature. For air at 20°C ($= 293$ K) the speed is 344 m/s.

In order to measure the position of the glider as a function of time, the motion sensor emits a train of equally-spaced sound pulses at regular intervals, that is, at times $t_1, t_2, t_3, ....$ The number of pulses emitted per unit time is called the sample rate, and can be varied from 1 to 250 Hz. The default sample rate is 20 Hz, which means that 20 pulses per second are emitted. The pulses are ultrasonic (pressure variations that are too fast to be heard) but you can hear a click when each pulse is emitted. The motion sensor detects the reflected pulses at times $t_{1a}, t_{2a}, t_{3a}, ...$ and measures the time of flight $\Delta t_i = t_{ia} - t_i$ between the emission and reflection of a given pulse. The Capstone digital display can be programmed to show the time of flight of the pulses. It can also show the distance $x_i = v_s \Delta t_i / 2$ of the glider from the motion sensor, as well as the glider's velocity and acceleration. Note, however, that only the time-of-flight $\Delta t_i$ is measured; the other quantities are calculated by the Capstone software. You will use these derived quantities in the lab, to get a sense of what is going on. However, you will not rely solely on these derived quantities when you analyze the data for your report. In the last part of the experiment, you will determine the velocities and accelerations for yourself from the position vs time data that are directly measured.

The motion sensor has two plugs. The yellow plug is used to deliver electrical pulses to the sensor that in turn generates the sound pulses; it is inserted into Channel 1 of the Capstone interface. The black plug is used to send back from the sensor the signal generated by the echo; it is inserted into channel 2 of the interface. The maximum distance that can be measured depends on the sample rate. All of the motion sensors have a switch that allows you to choose between a pulse emitted in a narrow or somewhat broader cone. Choose the switch position that gives the best results.

**Software:** Use of the software will be practiced in step-by-step instructions below.

# 4   Overview of the lab procedures

In Part 1 of this lab you will set up Capstone for the motion sensor and digital display. You will use the motion sensor to examine the movement of a notebook that you move manually; you will also learn how to use the Capstone software. As this is the first lab using Capstone, the instructions will be more detailed than they will be in future labs. The instructions will also become less detailed further on in this write up. Please remember that the procedures used with respect to the motion sensor window, digits display set up window, and input menu, apply, with very little variation, to other sensors and window displays to be used in other experiments in this course.

In Part 2 of this lab, you will be presented with a curve of **position vs time** and a curve of **speed vs time**. You will analyze and answer all the questions of both graphs in your lab reports.

In Part 3 you will study the relationships between position $x$, velocity $v$, acceleration $a$, and time $t$. In general, $x$, $v$, and $a$ are vectors but here, where we consider motion in only one dimension, so they can be treated as scalars. For the setup of this part (a glider accelerating due to gravity) we make an assumption that holds fairly well, that $a$ is a constant. This assumption is particular to this experiment; in general, $a$ may be a function of position and/or velocity and/or time.

# 5   Part 1: Programming Capstone

To program Capstone for the motion sensor, open Capstone by double clicking on it from the desktop. When the program opens go to the sidebar on the left, choose the tab "Hardware Setup." The Pasco 850 interface should come up in the hardware window setup. Typically the next step would be to select the input on the interface that the sensor is plugged into. For this experiment we're using a motion sensor. Click on the hardware port that the sensor is plugged in and scroll down to **Motion Sensor II**. In the hardware setup window you should see the motion sensor active. Look at the figure in the next page.
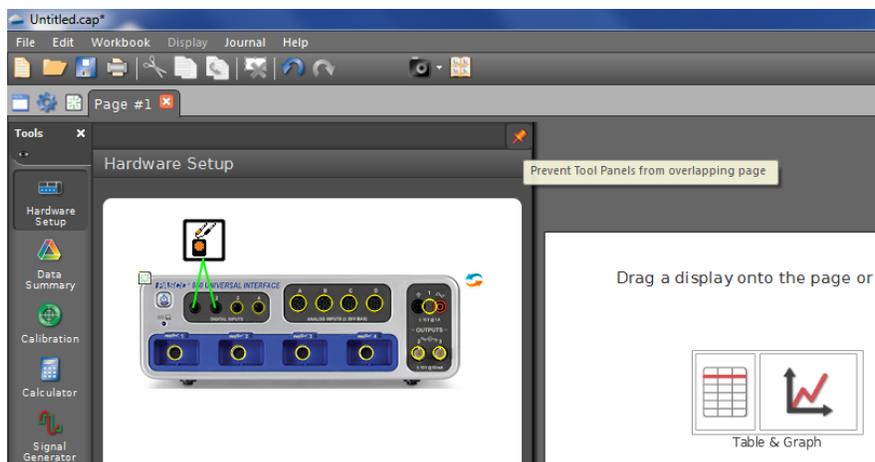
In the tools menu located on the left click on **Data Summary.** Under the Sensor Data Summary go to the motion sensor and click on the small eye. and select **Ping Echo Time**. You should see Position, Velocity, Acceleration, and Ping Echo Time all checked off. Look at following illustration for guidance.
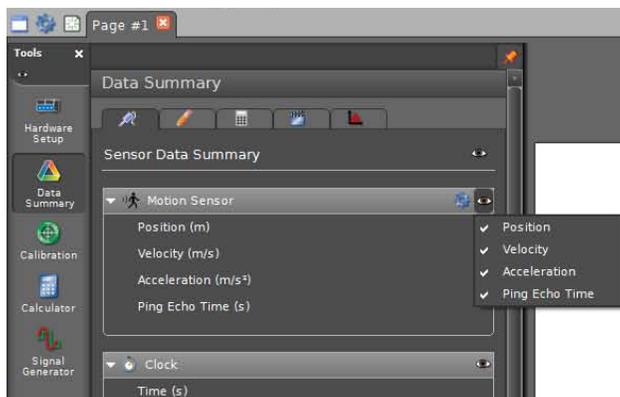


At the bottom of the experiment setup window the default sample rate is 20 Hz. Changing the sampling rate adjusts the number points being measured per unit time. If you increase the sampling rate the number points recorded increase per unit time. Vary the sampling rate as you please.

You can click on the orange tack to reduce the overlapping of the pages.



Next step is to open a digits display. Click and hold on **Digits** box, which is located in the **Displays** menu on the right side of the screen. Drag the digits box onto the screen next to the hardware setup window. Click on **Select Measurement** and select **Position.** (The m in the parentheses means the units are meters.)
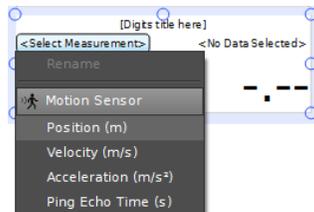
Note: You can change the unit of measurement in the digits box by clicking on the little (m) in parentheses.

Now try the following.

- Move the digits display window by clicking and dragging the center of the digits box.

- See if you can enlarge the digits display window by dragging an edge or a corner. You should be able to enlarge the display and this will be useful.

- Next try adding a Velocity Digits display. In order to add different digits displays, click and drag the digits box to the center window and click on select measurement as you have done before. Repeat for Acceleration and Ping Echo Time. When you're done getting a feel for the software close all the displays except Position by right clicking on the digits display box and selecting Delete.

What are all the units of measurements and dimensions for position, velocity, acceleration, and time? What is the speed of sound? Can you think of why it might vary from day to day?

## 5.1   Measuring Position

Mount the motion sensor so that it is on one side of the lab bench and about 30 cm above the bench. Then sit across the lab bench from this sensor. Place a meter stick on the bench with one end in the plane of the sensor grill and hold a notebook in your hands. Have your partner click the **Record** button in the lower left side of the eperiment set up window and examine the digits display and meter stick while you move the notebook in the acoustic beam of the sensor. Click the STOP button when finished. Then click on **Data Summary**. Run#1 appears in the digits display and in the Sensor Data Summary box. Keep in mind the digits display shows the last value measured. To change to the previous data you need to click on where it says run in the upper right corner of the digits display box. To increase or decrease the number of digits to the right click on one of the top two left corner buttons of the digits display.
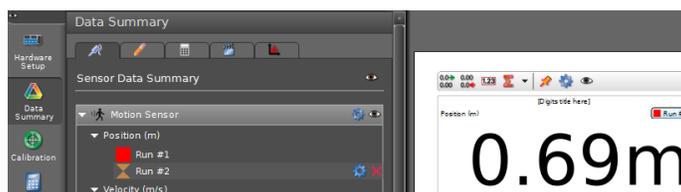
Do you think the number of digits to the right of the decimal point is sufficient? If not, increase the number to a reasonable value (look at the top left corner of the digits display box and click on the little green arrows by the zeros ) What would be a sufficient number of digits to the right and why? Take another run and see how close you can get the notebook to the sensor and obtain reliable values.

Try deleting Run # 1 by clicking on it in the sensor data summary window and clicking on the red X. Don't let too many runs accumulate.

## 5.2 Checking Capstone

It is a good idea to see if equipment you are using is doing what it is supposed to do. For example, if I had just constructed (or am considering buying) a fancy new length-measuring device, I might as well measure a meter stick length to see if I get reasonable results. Here we will check that Capstone is making reasonable measurements of the position of your notebook.

Set up a stationary stable reflector about 0.7 m in front of the motion sensor. Open digits displays, for **Position** and for the **Ping Echo Time.** Measure the position and the Ping Echo Time to 3 significant figures. Use the speed of sound and the ping echo time to calculate the position. Compare your calculated value to the value given in one of the digits displays for position in Capstone. How do these values compare to what you get with a meter stick? What the motion sensor actually measures is the round trip pulse time. Capstone then calculates the distance. Is it doing this calculation correctly? Repeat this procedure for a reflector 0.6 m from the sensor.

## 5.3 Measuring Velocity

Add Velocity to the digits display and take another run, seeing what happens as you move your notebook in various ways. What does a minus sign mean in the display? What does no sign mean?

## 5.4 Measuring Max and Min velocity

Carry out an experiment to see how fast you can move your notebook toward and away from the sensor. Hint: open the digits display set up window and examine it. Do you think the default sample rate of 20 Hz is appropriate for this experiment? You can change it if you want. To do so, go to the bottom center of the Capstone interface. You can adjust the sample rate of each individual sensor. Remember that if you increase the sample rate the maximum distance that the motion sensor will detect is reduced.

## 5.5 Questions and Analysis for Part 1

This section lists questions and analyses regarding Part 1 that should be addressed in your laboratory report. Some of these are also discussed in context above, but are repeated here. All the questions are required to be answered in your report.

1. In the first part of this procedure you performed a calibration of the motion sensor. The calibration sets the conversion factor from time to distance, which is just the speed of sound. Why might the speed of sound vary from day to day or lab bench to lab bench?

2. Why is 1 m a good calibration distance? Would shorter or longer be better?

3. The maximum measurable distance depends on sample rate. Assuming that the system cannot deal with a second pulse emitted before the first reflected one is detected, estimate the maximum sample rate for a 1m distance.

4. Determine the uncertainty in your calibration.

5. How did you select the number of digits to display?

6. You checked the Capstone measurement by comparing distances of 0.6 and 0.7 meters with the value calculated by Capstone.

   (a) List the sources of error in your measurement of 0.6 and 0.7 m and estimate the uncertainty in these measurements.

   (b) List the sources of error in the Capstone measurement of 0.6 and 0.7 m and estimate the uncertainty in these measurements.

   (c) Are the two measurements consistent within these errors?

7. Calculate the speed of sound using your distance measurements and the Capstone times. Is the result reasonable?

# 6 Part 2: Graphs and Questions

First examine the **position vs velocity** curve and answer the following questions.
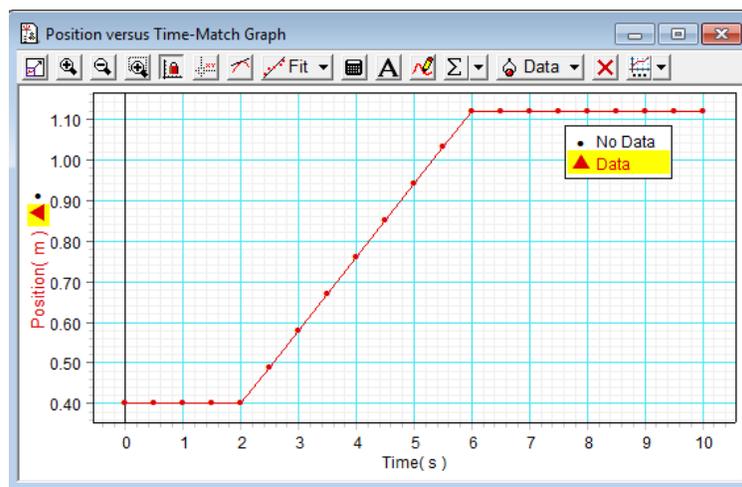


1. How close should your notebook be to the motion sensor at the beginning?

2. How far away should you move it?

3. How long should your motion last?
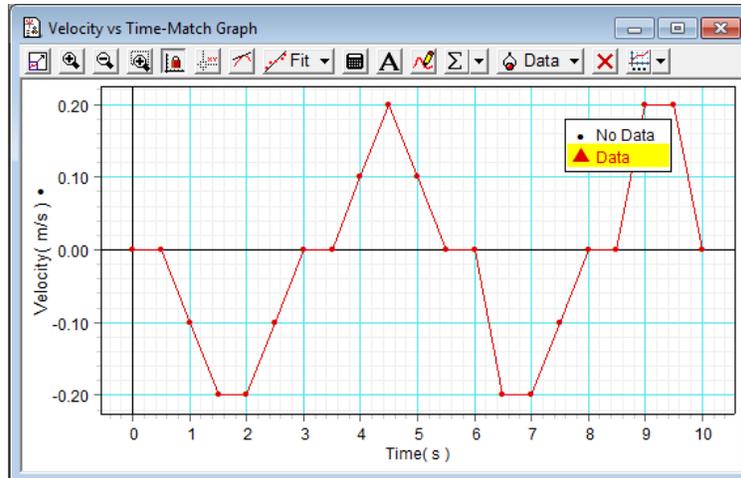
Examine the **velcoity vs postion** curve and answer the following questions.

1. In the graph, what is the slope of the line of best fit for the middle section of your plot?

2. What is the description of your motion? (Example: Starting at a position of 0.1 m, constant speed of 0.5 m/s for 2 seconds followed by no motion for 3 seconds, etc.)

1. In what direction should your notebook initially move and how far should in move it that direction before changing directions?

2. How far should your notebook move in the opposite direction before changing directions again?

3. At what time (or times) should your notebook be farthest from its starting point? How far should it have moved at that time (or times)?

4. How far should your notebook have moved from its initial position at the end of the run (10 seconds)?

5. With respect to the motion of the notebook, does it matter how far you are from the motion sensor when you take the data, assuming that you are always within the operating range of the sensor? Discuss.

# 7  Part 3: Motion of a Glider on an Inclined Air Track

## 7.1  Position, Velocity, Constant Acceleration

In this part, the motion sensor is used to measure the position of a glider on a tilted air track as a function of time. If there were no friction, Newton's 2nd law predicts that the acceleration of the glider would be $g \sin\theta$, where $g$ is the acceleration of gravity and $\theta$ is the angle between the air track and the horizontal. Note that the acceleration does not depend on the glider mass. Capstone calculates the velocity and acceleration from the position vs time data.

Level the air track by turning on the blower, putting a glider on the track, and adjusting the single screw at one end of the air track so that the glider does not move. Measure the distance between the single supporting screw at one end of the air track and the line formed by the two supporting screws at the other end of the air track. Measure the thickness of the two blocks of wood with a meter stick and then raise the end of the air track that is closer to the sensor (the closer end must be raised because the power law fit used below assumes that distance gets bigger with time). Calculate $\theta$.

Close Capstone and always click on **Discard**. Restart and program Capstone for the digital motion sensor. You may use the default sample rate and default speed of sound.
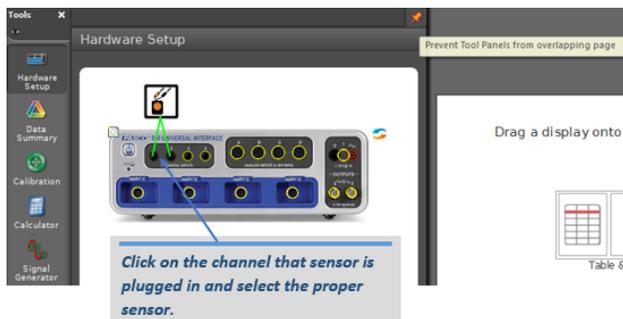


Program Capstone for the graph display, choosing position, velocity, and acceleration. The three quantities should be on three different graphs. To program Capstone for graph display, drag and drop the graph icon from the displays box to the center of the white screen of Capstone. Click on the **Select Measurement** tab and select position. Note: Reminder use the overlapping button which is the orange tack on the hardware setup window if the screens are overlapping.

Use **add a new plot** button to add the additonal graphs. The next graph will be velocity and repeat for acceleration. You should have three graph plots. On the top of the next page is an illustration on how to set up the graphs. Place the motion sensor near the spring bumper at the high end of the air track so that the grill looks down the track. You might have to adjust the position of the motion sensor on the rod. Position the card on the glider so that the pulses emitted from the motion sensor reflect off the card back to the motion sensor.    The motion sensor does not work for less than 0.2 m, so place the glider on the air track 0.2 m from the sensor and start your gliders from that point. Hold the glider
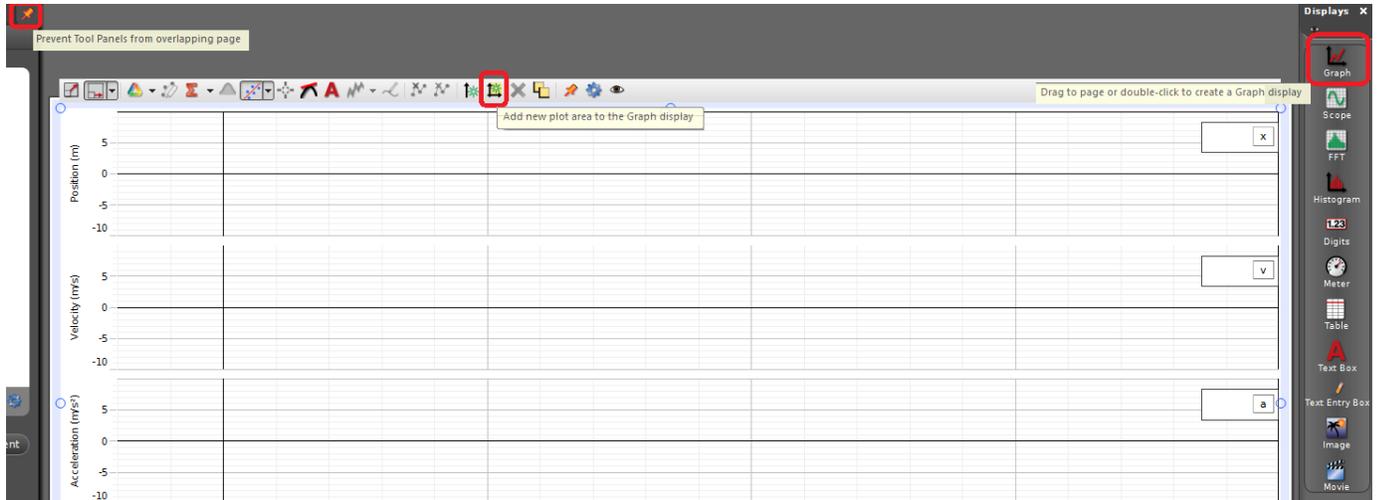
in place and turn on the air blower. Next click **Record** and release the glider at the same time. The record button is located on the bottom left of the Capstone screen. Click Stop just before the glider hits the end of the air track. Make a number of runs, making small adjustments in the orientation of the sensor until you get the best data. You will probably find that if the sensor points slightly up the data will be best.

### 7.1.1 First Set of Questions and Analyses to be submitted for Part 3

1. For your best run, click the scale to fit button (located on the top left of the graph window) and examine the three graphs in the display and discuss them. Are they what you expect and what is predicted by the theory? By inspection, are the graphs related, and if so, how?

2. Is it reasonable that the velocity curve is noisier than the position curve, and the acceleration curve noisier than the velocity curve? Explain.

3. Click the statistics button and then the scale to fit button. For the position plot choose fit and then power fit. For the velocity plot choose fit and then linear fit. In the statistics window, $y$ refers to the vertical axis and $x$ to the horizontal axis. If your data points are noisy near the ends, draw boxes to exclude the noisy data but include as much good data as possible. Draw the boxes so that they all begin and end at the same times. Your boxes should be lined up vertically.

4. For the velocity plot the acceleration is given by the slope m. Why?

5. In the acceleration plot the average acceleration for the data in the box is given by the mean of y. Compare this to m. Compare these values to the theoretical one.

6. For the position plot, the notation n(Power) means that the function is raised to a certain exponent. What is the n(power)? What does the theory say it should be?

7. Once you have data that you are satisfied with, you should save it to an Excel file. To do so, you first need to make a table of your data by dragging the Table icon over to the center of the white data window. Next, click on the top left corner button of the table. It is **insert empty column to the right** icon. You should have three data tables. Under select measurements pick position, velocity, and acceleration. There are two options to export data. First, you can save all the data by selecting the Export Data button under the File menu. Second, you can export selected data by using the columns in the data table. (Note: you can switch runs by clicking on the Run # on the top of the column in the data table and then select the data run you want.) In the data table left click on the position column. This selects all the data in that column. Next, right click on the data of the position column and select copy values. After, open the Notepad program and pasta the data. Also, you can use Excel. Save the data as `position.txt`. Repeat this for the velocity and acceleration data. When you are done, you should have three files: `position.txt`, `veloctiy.txt`, `acceleration.txt`. E-mail the data files to yourself and to your lab mates so that you have multiple copies for later use.

### 7.1.2 Additional Analysis for Part 3

We know exactly how the position $x$ data were obtained, namely by measuring the time-of-flight of sound pulses and converting them to distances using Eq. 3. The velocity and acceleration data were then simply output by the Capstone software. Evidently, they were obtained by numerically differentiating the position vs time data, but exactly how is not clear—the Capstone software is thus a kind of "black box" whose inner workings we cannot see. In general, you should be suspicious of data obtained by processing that you do not understand. Therefore we are going to take a closer look.

1. Write a Python program to read the position, velocity, and acceleration data from the text files using the `loadtxt` function of the Python NumPy library (see the Python manual). The program should then plot these three quantities as a function of time.

2. Consider a situation (like the one in this experiment) where position is measured at fixed time intervals $\Delta t$. Show that the velocity at time $i$ can be approximated in terms of positions at neighboring times as

$$v_i = \frac{x_{i+1} - x_{i-1}}{2\Delta t} \tag{4}$$

Show that the acceleration can be approximated from three points using

$$a_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{(\Delta t)^2} \tag{5}$$

Hints: Make sure to think about whether you are determining each quantity's value at a time $t_i$ of a measurement or at a time half way between two measurements. Why are these equations only approximate for $\Delta t > 0$?

3. Use the provisional uncertainty rules to find how the uncertainty in velocity and acceleration depend on the uncertainty in position (ignore any uncertainty in time).

4. Modify your program to calculate the velocity and acceleration and their uncertainties as a function of time by using the previous two equations. Graphically compare your results to the velocities and accelerations given by the Capstone software. You should find that the velocities and accelerations you obtain are noisier than those given by the Capstone software, especially the acceleration data. In fact, a lot of noise has been filtered out of the Capstone data using some algorithm that we can only hope is valid. If you want to know more about how this is done, consult your instructor.

5. Note that no matter how the data is processed, the velocity and acceleration data are noisier than the original position data. Why? HINT: When two quantities, say $a$ and $b$ are known to a finite number of digits (for example, 8), what happens to the number of significant digits of $c = a - b$ if $a$ and $b$ are very near each other?

6. Based on your responses above, is it better to use a slow or fast sample rate in this experiment? What are the advantages and disadvantages of each of them?

## 8   Finishing Up

Please leave your lab bench as you found it. Thank you.