Measurements and Uncertainties

1 Purpose

This laboratory introduces some basic laboratory and measurement techniques. It also provides some practice in error analysis.

2 General Directions for Keeping a Laboratory Notebook and for Laboratory Reports

2.1 Laboratory Notebook

Keep a record of everything you do in your laboratory notebook. For this course, this should include a record of how you carried out the step-by-step instructions in the experiments. At the end of the laboratory session, hand in to your instructor the carbon copy of your laboratory notebook.

Note that you do not need to include the step-by-step instructions themselves, as these are already printed in the lab description. So for example, if the instructions say, "Measure the height of the cart," your notebook entry may be something like:

"I used a ruler to measure the distance from the table top to the top of the cart. I leaned the ruler against the vertical post to make sure that the ruler was vertical. The uncertainty in this measurement comes from how well I can read the divisions, which is about 1 mm, and from the deviation of the ruler from vertical, which I estimate to contribute up to another 2 mm. The measured distance was 20.5 ± 0.3 cm."

Your notebook entry should start with your name and the date you are making the entry. It should be detailed enough so that you would be able, 6 months from now, to reconstruct what you did in the lab and what results you obtained.

The reported results of a measurement are only useful to the extent that they correctly describe what was done and what was found. Accurately reporting that you made a mistake, or that you got the "wrong" value and don't know why is much more important than getting the "right" answer. Get used to this now, as it is at the heart of what makes science possible.

When starting a new set of measurements, it is often useful to check that you are doing things roughly right by performing some simple calculations to make sure that you are getting reasonable values. Include any such calculations in your notebook.

2.2 Laboratory Report

A laboratory report including the items below is to be handed in the week following each lab.

The report should contain:

- A brief summary of the purpose of the lab (that is what did you intend to measure and why).
- A brief description of the equipment used.
- A brief description of the measurements made and results obtained. Note that the details are already in your lab notebook so don't need to be repeated here.
- The analyses and responses to questions listed in the laboratory description.
- Your conclusions based on the results you obtained. This is the place to make sense of everything you have done. For example, do the various measurements and expectations from theory agree or not? If they don't agree, do you have any ideas as to what the reason might be?

3 Theory

3.1 Measurement tools and techniques

Various tools are used for measurement of macroscopic lengths. The precision of a ruler is limited by the fineness of division thickness and separations that one can see. Clever ideas, many quite old, allow improvement over what can be done with a ruler.

Calipers use vernier scales to improve measurement precision. In a vernier scale, two rulers are positioned next to each other. The spacing in one is slightly different than the spacing in the other, so that two divisions on the two rulers will line up when they are separated by a multiple of the spacing difference. For example, consider a fixed scale with 10/100 cm between divisions and a moving scale with 9/100 cm between divisions, orginally positioned so that their zeroes coincide and numbers increase to the right. If the moving scale zero point is slid to the right a distance of 5.23 cm, its zero point will line up 0.03 cm to the right of the fixed scale 5.2 division, its first division will lie 0.02 cm to the right of the fixed scale 5.3 division, and its second will lie 0.01 cm to the right of the fixed scale 5.4 division. But its third division will lie directly across from the fixed scale 5.5 division. Seeing that the third division lines up thus allows determining the distance moved as 5.23 cm. Vernier scales come in a large variety of division numbers and spacings.

Micrometers use screw action to reduce distances. The screw thread angle is such such that a large turn of a screw results in only a small displacement, allowing precise measurements of the small displacement. Adding a vernier scale to the screw movement allows even more precise measurements.

Another method to measure small quantities is to make a bulk measurement and then to divide by the number of objects in the sample. For example, you can obtain the mass of an iron atom by weighing 1 mole of iron and dividing by Avogadros number. Note that you have to take into account whether the objects you are measuring are completely identical (are iron atoms all of the same isotope?) or vary from one to another.

Various electronic devices allow easy readout of calipers and other more precise measurements than those made by a micrometer, but these will not be discussed here.

For determination of weight (or mass) we use one mechanical and one electronic device in this lab. The mechanical device employs the ancient principle of balancing weights against each other. Note the implication here: for all measurements we must arbitrarily define some standard unit, and all other measurements are in terms of this unit. The electronic scale provides an alternative measurement used for comparison purposes in this laboratory exercise.

3.2 Propagation of errors and comparison of experimental results

Some of the quantities to be determined in this lab will be derived from others which you directly measure. It will be up to you to correctly propagate the errors from the measured quantities to determine the uncertainty in the derived ones. Once you have determined the uncertainties, you can then determine whether different measurements are consistent.

3.3 Conversion of units

Remember that measurements must always be converted to a single consistent system of units. The general convention is to use the SI system unless there is a specific reason to use another system.

4 Paper thickness experiments

4.1 Procedures

- Always remember to check that your measurement devices are properly zeroed. For example, check that the caliper reads zero for the ends pushed together.
- Using calipers, measure the thickness of a single sheet of paper. Record the measured value, remembering to include uncertainties in the quoted number.
- Using calipers, measure the thickness of a pile of 100 sheets of paper. From the measurement, determine the thickness of a single sheet of paper. Remember to include uncertainties in the quoted number.
- Using a micrometer, measure the thickness of a single sheet of paper. Remember to include uncertainties in the quoted number.
- Using a micrometer, measure the thickness of a pile of 100 sheets of paper. From the measurement, determine the thickness of a single sheet of paper. Remember to include uncertainties in the quoted number.
- Design and conduct an experiment to determine how uniform the thickness of a sheet of paper really is.
- Write on the paper. Perform a measurement that either determines the thickness of the ink layer or sets an “upper limit” to its thickness. An upper limit is a value such that the thickness must be less than this value (with a certainty determined by the measurement).

4.2 Questions and analysis

1. You have already seen that if $q = xy$ then $\delta q/|q_{best}| \sim \delta x/|x_{best}| + \delta y/|y_{best}|$. Use this to show that if $q = kx$, where k is a constant, then $\delta q \sim |k|\delta x$. This is actually an equality, but we will not show that here.

2. Use the above result to show for the measurement using a pile of 100 sheets how the uncertainty on the thickness of a single sheet of paper depends on the uncertainty in the thickness of the pile.

3. Describe how you determined the uncertainties for each of the 4 measurements you made of the thickness of a sheet of paper.

4. Explain in words how given two measurements and their associated uncertainties one determines whether they are consistent.

5. Are the measurements of paper thickness made with the different methods consistent within the uncertainties in the measurements?

6. Provide a short description of the experiment you did to determine the uniformity of the thickness of a sheet of paper and the results you obtained.

7. Describe the measurement you did to determine the thickness of the ink layer and give your results (either the measured value or the upper limit).

5 Weight of a sheet of paper

5.1 Procedures

- Use the adjustment screw-weights to zero the beam balance.

- Using the beam balance, weigh 100 sheets of paper. Determine the weight of a single sheet.

- Using the digital scale, weigh a single sheet of paper.

- Design and conduct an experiment to weigh your signature or to set an upper limit on it.

5.2 Questions and analysis

1. Are your different measurements of the weight of a sheet of paper consistent within errors? Explain your criteria for consistency.

2. Describe the experiment you conducted to weigh your signature and give your results.

6 Testing the volume of a cylinder formula

6.1 Procedures

Measure the dimensions of a can of soda in order to determine its volume.

6.2 Questions and analysis

1. Use the provisional uncertainty rules to show how the uncertainty in volume can be determined from the uncertainty in the measurements which you have made.

2. Write and hand in a python program that calculates the volume of the can from the dimensions you measured, and the uncertainty in the volume from the uncertainty in the measurements.

3. Determine whether your answer is consistent with the volume printed on the label. If not, why do you think this may be the case?