Collisions In One Dimension

1 Purpose

- To investigate conservation of energy and momentum in one dimensional two-body collisions.
- Learning to make histograms Gaussian distribution

2 Theory

2.1 Conservation of Momentum

Let the mass of a particle be m and its velocity $\mathbf{v}$. The momentum $\mathbf{p}$ is defined by $\mathbf{p} = m\mathbf{v}$ and Newton's 2nd law may be written as $\mathbf{F} = d\mathbf{p}/dt$ where $\mathbf{F}$ is the net force on m. If $\mathbf{F} = 0$, then $\mathbf{p}$ is a constant, which is confirmed by Newton's 1st law.

For a system of particles we identify each particle with a subscript i so that $\mathbf{F}_i = d\mathbf{p}_i/dt$. If each side of this equation, where n is the number of particles, $\mathbf{F}$ is the net force on the system, and $\mathbf{P}$ is defined as the total momentum of the system, is summed over all the particles

$$\sum_{i=1}^{n} \mathbf{F}_i = \mathbf{F} = \frac{d}{dt} \sum_{i=1}^{n} \mathbf{p}_i = \frac{d}{dt}\mathbf{P}, \tag{1}$$

then the internal forces (the forces between the particles), can be excluded, as Newton's 3rd Law guarantees. We don't even have to know them. The terms $\mathbf{F}_i$ in the sum of the above equation can therefore be taken to refer only the forces outiside of the system acting on the system. A consequence of the equation therefore is that if any component of the sum of external forces applied to a system is zero, that component of the system's momentum $\mathbf{P}$ will be a constant. This is true even if some of the particles that make up the system make violent collisions with each other, because collisions involve only internal forces. One can say that the component of momentum is the same before the collisions as it is after the collisions, or that the component of momentum is conserved. If all components of $\mathbf{F} = 0$ then all components of the momentum are conserved.

Conservation of energy and conservation of momentum are different statements. Momentum may be conserved when energy is not and vice versa.

2.2 Some Definitions

In this lab we restrict ourselves to two body collisions. A collision is said to be elastic if the combined KE of the two objects is the same before and after the collision. If the KE is less after, then the collision was inelastic. If the two objects stick together the collision is called completely or perfectly inelastic, which means the maximum KE that can be lost (consistent with other conservation laws) will be lost.

2.3 Calculations for Two-Particle One-Dimensional Collisions

Consider a situation with two paticles where the first particle, with mass m_1 and initial velocity v_1, collides with a second particle of mass m_2 which is intially at rest. Denote the velocities after the collision with primes. As we are working only in one dimension, we will deal only with the components in that direction and vector notation is not needed.

For an elastic collision, conservation of energy and momentum give

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 {v_1'}^2 + \frac{1}{2}m_2 {v_2'}^2 \tag{2}$$

$$m_1 v_1 = m_1 v_1' + m_2 v_2' \ . \tag{3}$$

These equations can be solved (very easily if the right method is chosen!) for v_1' and v_2' to give

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad \text{and} \quad v_2' = \frac{2m_1}{m_1 + m_2} v_1 \ . \tag{4}$$

For perfectly inelastic collisions, energy is not conserved, and the corresponding conservation equation cannot be used. However, since the the particles stick together, there is only one unknown final velocity. Momentum conservation then gives $m_1 v_1 = (m_1 + m_2)v_1'$, which can be solved to give

$$v_1' = v_2' = \frac{m_1}{m_1 + m_2} v_1 \ . \tag{5}$$

3 Equipment Used

This experiment uses gliders, which float on an air track to approximate frictionless motion, as colliding objects. Photogate sensors are used to measure the velocity of the gliders. Stands are provided to set up the photogate sensors in the appropriate orientation. Index cards are mounted on the gliders to help the photogates calculate velocity. As was the case when you used the photogate in other experiments, you must provide Capstone with the size of the cards for the velocity measurements to be meaningful.

As collisions between the gliders' spring bumpers are nearly - but not completely - elastic, magnets will be provided. These can be mounted on the bumpers to provide repulsion without friction. In a completely inelastic collision velcro strips can be attached to the glider bumpers with paper clips so that colliding gliders stick together. Also, you will use inelastic needle with container.

Remember to be careful with the gliders, and not to move them on the air track unless the blower is on.

4 Experiment Procedures

4.1 General Procedures

Turn on the air blower. Set up the photogates so that the velocity of the gliders can be measured before and after collisions. Choose the position of sensors to make the best possible measurement of the velocities immediately before and after the collisions.

Plug the two photogate sensors into digital channels. Program Capstone for the first sensor by clicking **One Photogate (Single Flag)** from the dropdown menu. Click on the gear icon in the lower right corner of the hardware setup window and change the **Flag Width** to the width of the card on top of the glider. Follow the same procedure for the second sensor.

Create a **Table** by dragging its icon to the center screen. Under **Select Measurements** choose “Speed, Ch1 (m/s)” for one column and “Speed, Ch2 (m/s)” for the other so you that the velocities from both sensors are in the same table. **Always be aware of which photogate that the glider passes through.**

4.2 Elastic Collisions

4.2.1 Procedures

Perform collisions where one of the carts, of mass m_2 is initially at rest. Take data for $m_1 = m_2$, $m_1 < m_2$, and $m_1 > m_2$. Try a few velocities for m_1. Try collisions with and without magnets on the bumpers.

4.2.2 Analysis and questions

1. For what circumstances does the incident glider reverse its direction during the collision?

2. Explain how you chose the position of the sensors to make the best possible measurement.

3. Make estimates of the uncertainties involved in all of your measurements, including the length of the cards used to interrupt the photogate, the masses, and any other quantities you measure.

4. Compare your data to the equations for elastic collisions. Are momentum and energy conserved within the uncertainties of your measurement?

5. Do your results differ from the theory in the way you might expect if a bit of KE is lost in the collision?

6. If energy appears to be lost, where did the energy go? Explain any differences between results with spring bumpers and magnets on the bumpers.

4.3 Perfectly Inelastic Collisions

4.3.1 Procedures

To make the two gliders stick together during the collision use 4 paper clips to attach 5-cm pieces of Velcro to one bumper of each glider. Use the same combinations of masses as in the previous section. Finally, for one combination of masses, do the experiment 25 times. Repeat for the inelastic needle and wax container. You will insert an inelastic needle into the end of one glider and then insert the wax container into the colliding glider.

4.3.2 Analysis and questions

1. Compare your data to the equations for perfectly inelastic collisions. Are differences consistent within the uncertainties in your measurement?

2. Verify that KE is lost. Where does the energy go?

3. Explain any differences between results with velcro and inelastic needle with wax container.

4. This next analysis is lengthy, so do the rest of the experiment first and come back to this later, in lab if you have the time, otherwise after class.
   For the experiment you did 25 times:

   (a) Write a python program that histograms the values of v'/v_1 you obtained. Choose the bin size of the histograms so that you get multiple entries in at least a few bins. That is, if your bin size is too small then all bins will have either 0 or 1 entry, and if your bin size is too big all entries will be in one bin.

   (b) Have the program calculate the average and standard deviation of your values of v'/v_1.

   (c) What fraction of the measurements are outside the range average $\pm$ standard deviation and average $\pm$ two times the standard deviation?

5 Comment

When using the concepts of momentum and energy be aware that waves carry both of these quantities. When the gliders collide do you hear a sound? Could this contribute to the energy and momentum balance in your experiments. Probably not. For example the sound from a "quiet conversation" has an intensity of about 10^{-6} W/m^2. If it is assumed that this level of sound is produced by the collision for 0.1 s at a distance of 1 m from the collision, and that the sound radiates isotropically from the collision point, then the total energy in the sound wave would be 1.3×10^{-6} J. How does this compare with the kinetic energy of your gliders?