

Ballistic Pendulum

Caution

In this experiment a steel ball is projected horizontally across the room with sufficient speed to injure a person. Be sure the “line of fire” is clear before firing the ball, and be aware of other students who are preparing to do so.

1 Purpose

- To measure the speed of a projectile by a kinematic method and also by the use of a ballistic pendulum. The latter method illustrates the use of conservation of energy and momentum. The two results are compared for consistency within experimental uncertainty.
- To learn about modeling uncertainties with a Gaussian distribution.

2 Description

A spring gun fires a steel ball horizontally from a bench top. The steel ball is either allowed to fly free and strike the floor, or embeds itself in a pendulum. In the first case the initial velocity of the ball can be obtained by using basic kinematics. In the second case the initial velocity of the ball can be obtained by conservation laws derived from Newton’s 2nd law.

The ballistic pendulum was invented in 1742 to measure the speed of bullets. As you can see from this experiment it is not necessary to use a ballistic pendulum to measure the speed of a slowly moving object, but the ballistic pendulum does illustrate the use of several important conservation laws in physics.

3 Equipment

Ballistic pendulum apparatus: Ball, gun to fire ball, and a pendulum to catch it and rise to some height.

Measurement devices: 2 meter ruler, 30 cm ruler, blank paper, carbon paper, masking tape, scale.

4 Kinematic Measurement Of Speed

4.1 Theory

A ball fired horizontally with velocity v from a spring-loaded gun then falls a vertical distance d before hitting the floor a distance D from the point where it left the gun, as shown in Fig. 1.

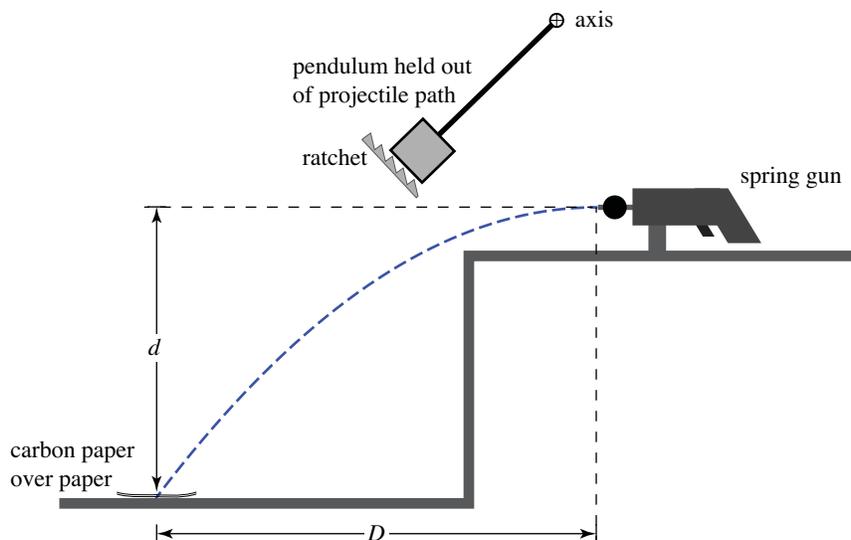


Figure 1: Arrangement for studying a projectile in free fall.

Ignoring air resistance there are no forces acting in the horizontal direction, so the horizontal distance traveled is related to the time of flight t by $D = vt$. The ball falls vertically with constant acceleration g so the distance fallen is related to the time of flight by $d = \frac{1}{2}gt^2$. Eliminating t between these two equations and solving for the initial velocity gives

$$v = D\sqrt{\frac{g}{2d}}, \quad (1)$$

where D and d can be measured and $g = 9.80$ m/s.

4.2 Methods

- Swing the pendulum up so the latching mechanism holds the pendulum out of the way.
- To shoot the ball: Put the ball, which has a hole in it, on the rod at the end of the gun. Push the ball into the gun against the action of the spring, pushing it far enough that the spring is latched. Aim the gun across a clear spot in the lab room. **Being sure no one is in the way**, depress the firing lever on top of the gun.
- To measure where the ball hits the ground: Tape paper to the ground, and put carbon paper, carbon side down, on top of the white paper. When the ball hits the carbon paper it will leave a mark on the paper. Note: firing the ball once or twice before taping the paper to the ground may help you determine where to put the paper.

4.3 Procedures, Analysis, and Questions

1. Measure d . State the sources of uncertainty and provide both the value of d and its uncertainty.

2. Make 25 measurements of where the ball hits the carbon paper, making sure to record both dimensions (you should decide on the most useful axis orientation to define).
3. List the sources of error in D and estimate their size.
4. Where you can, use the multiple measurements to estimate the size of some of the uncertainties. Are there any uncertainties not accounted for in this way? Provide the value of D and its overall uncertainty.
5. Determine v and its uncertainty.
6. The following analysis will take some time, so first complete the rest of this experiment then return to this part, in lab if you have the time, otherwise at home.
 - (a) Write a python program that creates two histograms for the two components of the 25 positions where the ball hit the carbon paper. Choose the bin size of the histograms so that you get multiple entries in at least a few bins. That is, if your bin size is too small then all bins will have either 0 or 1 entry, and if your bin size is too big all entries will be in one bin.
 - (b) Add a feature to your program that draws on top of the two histograms Gaussian distributions of the form $(25/\sigma\sqrt{2\pi})e^{-(x-x_0)^2/2\sigma^2}$. How will you determine the values of x_0 and σ to use for each of the histograms so that the curves are as close as possible to your data?
 - (c) What fraction of the measurements are outside the range $x_0 \pm \sigma$ and $x_0 \pm 2\sigma$? Does your data look Gaussian? Explain how you determined this, and why you think it is the case.
 - (d) Is σ the same for both dimensions? Should it be?

5 Ballistic Pendulum Measurement Of Speed I

5.1 Theory Assuming A Simple Pendulum

Here we assume that the pendulum is initially unlatched and hanging vertically at rest as shown in Fig. 2. For this section we will assume that the pendulum consists of a mass of small dimensions suspended by a massless cord. Let the mass of the ball be m and the mass of the pendulum be M . Let the horizontal speed of the ball before the collision be v and of the pendulum plus ball right after the collision be V . When the ball strikes the pendulum and sticks the collision is completely inelastic - energy is not conserved. Define the system as the pendulum mass plus the ball. The horizontal momentum of the system is conserved for a very short time from just before the collision to just after the collision, since for this very short period there are no external horizontal forces acting on the system. (When the pendulum has swung to the side a bit the cord of the pendulum exerts a horizontal force on the pendulum mass and horizontal momentum is no long conserved.) The horizontal linear momentum of the system just before the collision is mv and just after the collision is $(M + m)V$. Conservation of horizontal linear momentum gives $mv = (M + m)V$.

Right after the collision the pendulum mass and ball swing up against the force of gravity and eventually come to rest. Assuming no friction, energy is conserved during this part of

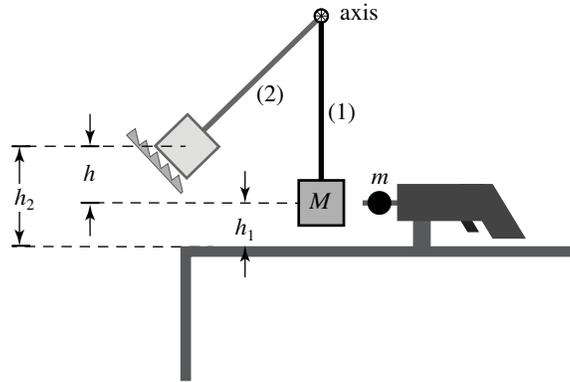


Figure 2: Ballistic pendulum at height h_1 before impact (1) and at height h_2 on reaching its point of maximum swing after impact (2).

the motion as the force of gravity is conservative and the force of the suspending cord does no work. Let the gravitational potential energy immediately after the collision be zero. At this time the total energy is then kinetic and equal to $\frac{1}{2}(M+m)V^2$. When the pendulum has stopped the kinetic energy is zero but the gravitational potential energy is $(M+m)gh$, where h is the vertical distance that the pendulum mass and ball have gone up. Equating the total energies at these two times gives $\frac{1}{2}(M+m)V^2 = (M+m)gh$. Eliminating V between these equations gives

$$v = \frac{M+m}{m} \sqrt{2gh}. \quad (2)$$

5.2 Procedures

- Pull the pendulum to the side, insert the ball into the gun, and compress and latch the gun spring. Release the pendulum so that it hangs vertically. Fire the gun. The pendulum will latch near the highest point of its swing.
- Make sure to record which notch the pendulum reached, as you will also need this information for the analysis in the next section.
- Measure the relevant heights and masses necessary to use Eq.(2) to determine v . The mass of the pendulum M is written on the pendulum.

5.3 Analysis and Questions

1. List the final measured values and uncertainties for all the quantities you measured directly.
2. If you used multiple measurements, how did they provide you estimates of uncertainties?
3. Calculate v and its uncertainty.

4. Derive an expression for the fraction of the kinetic energy lost in the collision. How does this compare to your experimental result?
5. Compare your determination of v with that obtained with the kinematic measurement.

Comments. There are a number of approximations that have been made in the analysis for this section. The pendulum used is not a simple pendulum which is a point mass suspended by a weightless string. The pendulum is actually a physical pendulum which has a distribution of mass. Unless the physical pendulum is struck by the ball at one specific spot, the center of percussion, the pendulum support will impart a short impulse during the collision. Horizontal momentum will not strictly be conserved. The kinetic energy immediately after the collision is not that of a moving point mass but of a solid body rotating about an axis. In calculating the change in gravitational potential energy of the pendulum plus ball, the center of mass should be used rather than the position of the ball. The analysis in the following section addresses these difficulties.

6 Ballistic Pendulum Measurement Of Speed Analysis II

6.1 Theory Assuming A Physical Pendulum

Assume the pendulum is a physical pendulum free to swing in a vertical plane about a horizontal axis. We use the previous definitions for M , m , V , v and introduce the following additional quantities:

- a : the vertical distance from the pendulum axis (pivot) to the initial trajectory of the center of the ball.
- b : the distance from the axis to the center of mass of the pendulum *with the ball in it*.
- ω : the angular velocity of the pendulum with ball immediately after the collision.
- I : the moment of inertia of the pendulum plus ball about the axis.
- H : the maximum vertical distance that the center of mass of the pendulum with ball rises after the collision.
- T : the period of oscillation of the pendulum with the ball in it.

As before, the system is the pendulum and the ball. We invoke conservation of angular momentum about the pendulum axis of the system for the time immediately before the collision to the time immediately after the collision (before the pendulum has swung upward). There are no external torques on the system for this period of time. Any impulse from the pendulum support during the collision contributes zero torque about the pendulum axis as there is no lever arm. The angular momentum of the pendulum does change as the pendulum swings upward due to the torque exerted by gravity about the pivot point, but there will be no torque due to gravity before the pendulum swings away from the vertical. The angular momentum of the system just before the collision is mva , and the angular momentum right after the collision is $I\omega$. Conservation of angular momentum gives $mva = I\omega$.

Let the gravitational potential energy of the system be zero right after the collision. Immediately after the collision the total energy of the system is kinetic and given by $\frac{1}{2}I\omega^2$. When the pendulum has risen to its maximum height the kinetic energy is zero and the potential energy is $(M + m)gH$. Conservation of energy gives $\frac{1}{2}I\omega^2 = (M + m)gH$.

The period of the physical pendulum is given by $T = 2\pi\sqrt{I/[(M + m)gb]}$. Combining the equations for conservation of angular momentum and energy, and the equation for the period, the initial velocity of the ball becomes

$$v = \left(\frac{M + m}{m}\right) \frac{gT}{\pi a} \sqrt{\frac{bH}{2}}. \quad (3)$$

The quantities needed, in addition to m , M , and g , are a , b , T , and H .

6.2 Procedures

- Measure any additional quantities you need in order to use Eq.(3). The distance b , from the pivot to the center of mass of pendulum plus ball, is 27.0 ± 0.1 cm. This has been measured for you by disassembling the pendulum and balancing it on an edge.
- With the ball in the pendulum set the pendulum swinging and measure the period T .

6.3 Analysis and Questions

1. List the additional quantities you measured, their values, and their uncertainties.
2. Use Eq.(3) to determine the value of v and its uncertainty.
3. Compare your result for v with those obtained with the previous two determinations. What do you think is the most important random error in this experiment? Which method do you think gives the best value? The worst?

7 Finishing Up

Please leave your bench clean and tidy, as you found it. Thank you.