Oscillations of a string

Equipment Capstone, mechanical wave driver, meter stick, digital scale, elastic cord, pulley on rod, 2 rods 30 cm or more, 1 large table clamp, 1 small table clamp, dual rod clamp, set of weights, sheet of cardboard, weight hanger, leads [180 cm (2)], stroboscope

1 Introduction

A taut string can be made to vibrate. If the string is plucked and then left alone, the oscillations of the string are "free oscillations." If the string is subjected to a time dependent external driving force the oscillations are "forced." The coupling between the string and the driving force can vary continuously from weak to strong. In this experiment the forced oscillations of a string will be studied. The displacement of the non-fixed end will be sinusoidally varied in the direction transverse to the string.

2 Remarks

If a system is displaced from its equilibrium configuration and let go, it will vibrate. The system will vibrate in one or more of its normal modes, eigenmodes, or modes for short. In a given mode all parts of the system vibrate sinusoidally at the same frequency and pass through their equilibrium positions at the same time, i.e., have the same phase. Each mode has a distinct frequency or eigenfrequency. There are as many normal modes as there are degrees of freedom of the system. A continuous system has an infinite number of modes. Any oscillations of a system can be described by a linear combination of its normal modes.

If a system has boundaries, the normal modes are affected by the boundary conditions. For example, a string fixed at both ends has one set of modes, whereas a string fixed at one end with the other end free to slide on a frictionless transverse rod, has a different set of normal modes. Another example is the air inside a pipe, e.g., a wind instrument; the modes are different if one end of the pipe is closed or open.

If a system is driven or forced at a particular frequency, the system will oscillate at that frequency. The response of the system does depend on the normal mode structure of the system and also on exactly how and where the driving force is applied.

3 Theory: Forced Oscillations

We consider a string fixed at both ends and assume it vibrates in the $x + y$ plane. See Fig. 1. The length along the string is designated by x, with the string fixed at $x = 0$ and $x = L$. The transverse displacement of the string is called $y = y(x, t)$. The string is assumed to have a uniform constant tension T and a uniform mass per unit length of ρ.

The normal mode vibrations of a string are often called standing waves. Standing waves are produced on a string if two sinusoidal waves of the same frequency and amplitude are traveling on the string in opposite directions. The periodic wave on the string will have the same recurring pattern. The distance from one peak to the next peak is the length of one wavelength, λ. There will be points in the standing wave where the string does not vibrate, making $y = 0$. These points are called nodes (destructive interference) and are separated by a distance of $\lambda/2$. The 2 fixed end points of a string are nodes. For the lowest frequency or 1st mode, there are no other nodes. Mode 2 has one extra node, mode 3 has 2 extra nodes, and so forth. The first 4 modes are shown in Fig. 2. The solid line shows the string at an instant in time when the amplitude is maximum and all points of the string are instantaneously at rest. A quarter of a period later the string is flat but all points of the string except the nodes are moving. A half period later the string is shown by the dotted line. The envelope of the motion, what you actually see in the lab is due to the finite time resolution of your senses, which consists of both the dotted and solid lines. The antinodes are those points along the string which have maximum amplitude. The antinodes are situated midway between the nodes. Antinodes are where constructive interference occurs.

Consider a taut string fixed at the end $x = 0$, so that $y(0, t) = 0$. The mechanical vibrator will displace the cord in the vertical direction creating transverse waves. The waves will have an amplitude of A (height of the displacement), frequency f (number of waves in one second), and angular frequency $\omega = 2\pi f$. The wave pattern will have a fixed speed v and a displacement length of one wavelength, which corresponds to a time of one period T. This means $v = \lambda/T$ and since $f = 1/T$ then,

$$v = \lambda f \tag{1}$$

Keep in mind the wave speed is affected by the material properties the wave is traveling in. The vibrator will displace the particles of the cord from one end. The other end at $x = L$ is the location of the pulley. For a maximum mode to exist the incident and reflected waves need to be in phase. This means the time for a standing wave to form has a period T equal to $2L/v$. Combining $2L/v$ with $v = f\lambda$ gives,

$$L = \frac{1}{2}\lambda \tag{2}$$

Keep in mind $f = 1/T$. The total length of the first maximum mode has a length equal to one λ. Standing wave patterns of higher order modes, n, will occur at periods of $2T, 3T, 4T$, etc. This means the maximum amplitude of higher order wavelengths will occur at,

$$\lambda_n^{max} = \frac{2L}{n}, \quad where \quad n = 1, 2, 3, \dots \quad . \tag{3}$$

If you take into account the speed of a periodic wave $v = f\lambda$ and substitute λ in Eq.(3). Then the speed a wave travels on a string is determined by the linear mass density of the string under force tension T_f, $v = \sqrt{\frac{T_f}{\rho}}$. This will allow you to determine the frquency for when the maximum amplitude will occur for each mode. The frequencies for maxima amplitude are,

$$f_n^{max} = \frac{n}{2L}\sqrt{\frac{T_f}{\rho}}, \quad where \quad n = 1, 2, 3, \dots \quad . \tag{4}$$

A minimum response of the string is obtained by driving at a frequency such that the driving point is an anti-node, or maximum, of the standing wave. **You will observe this in section 5.2**.

The response goes through a series of minima and maxima, the amplitude of a minimum response is just the driving amplitude of the mechanical vibrator.

4 Operation of the experiment

The string will be driven in the vertical direction and the displacement will be in a vertical plane. In the following experimental sections you may find that at and near resonance the string has horizontal as well as vertical oscillations. Look for this by looking at the string from *both* the side and the top. When the string vibrates in this manner each element of the string will be moving in a circle about the equilibrium position of the string. Use the stroboscope to examine the motion. **You should take the maximum amplitude of this motion as an indication of resonance.**

A possible explanation for this behavior is the following: the damping of the string is due not only to air resistance but also the stretching and contraction of the cord; in the circular modes the string does not expand and contract, therefore has less damping, and is more easily excited.

4.1 Preliminary Measurements

These are the measurements necessary in order to calculate the normal mode frequencies from Eq.(4): Remove the cord from the apparatus, leaving the small loop in one end for the weight hanger. Find the mass of the cord and measure its unstretched length. Calculate the unstretched mass per unit length ρ_0 of the cord.

Measure L, the distance between the center of the fixed rod and the top of the pulley. Put the cord back on the apparatus and, with no tension on the cord, measure the length of the cord from the top of the pulley to the small loop in the cord. Hang a mass for a total mass of M =200 g from the cord (this includes the mass of the weight hanger) and measure how much the cord stretches. From these measurements you can calculate the mass density ρ of the cord when it has been stretched by any mass M. We assume the cord obeys Hook's law, $F = -k\Delta \mathrm{X}$.

4.2 Set up of the experiment

See Fig. 3. Leave the 200 g on the end of the cord. Keep the vibrator rod 5 cm from the fixed rod. Put the cord in the notch at the top of the vibrating rod and move the vibrator so that when the cord is looked at from above, the cord is straight. Move the fixed rod up or down so that the angle that the cord makes between the fixed and vibrating rods is at about 15 deg with the horizontal. Having made these adjustments, you need to relieve any longitudinal stress on the vibrator rod. To do this, lift the cord out of the rod's notch and then put it back.

5 Experiments

5.1 Resonance

In this experiment, you will be adjusting the frequencies between 3 and 120 Hz to find the lowest 3 resonances. In determining the frequency of a resonance, a judgment has to be made as to how fine a frequency interval is worth while measuring. Consider 0.5 Hz adjustment as a starting point. Too fine an interval requires too many points and takes too long. Too coarse an interval does not give an accurate value of the resonant frequency. Resonant frequencies occur when the normal modes of oscillations have maximum amplitude. To assist you in finding the experimental value of resonance, you should first calculate the expected frequency.

You are using the **Output 1** on the interface. Make sure the banana leads are plugged into the mechanical vibrator. Click on the signal generator icon in Capstone. It's located in the tools column. Then click on **850 Output1** and adjust the function generator output for an amplitude of 5 volts. You will be using the default setting of **Sine** for your waveform. Make sure the mechanical vibrator is unlocked and click **On**. At the 3 lowest resonances, does the wave pattern look like normal modes of a string? Explain. How close are the resonant frequencies to those predicted by Eq.(4)? How close is the driving point to a node at resonance? Which side of the driving point is the node on (between the vibrating rod and the pulley, or between the vibrating rod and the fixed rod)?

5.2 Anti-Resonance

Find the 3 lowest anti-resonances. These are the frequencies at which the response of the string is a minimum. What happens to the initial pulse that is created from the mechanical vibrator? What happens to the reflected pulse from the pulley? Is the driving point (the vibrator rod) at an anti-node?

5.3 Dependence of f_1 on T

Put a mass of 100 g on the cord **(measure any stretch of the cord)**. To assist you in finding the experimental value of resonance, you should first calculate the expected frequency. Supply a 5 V sine wave to the vibrator and vary the frequency until mode 1 is at a maximum. Record this frequency. **Repeat for two other masses that your teaching assistant will select.** Calculate the frequencies predicted from Eq.(4). Make a table of your experimental and theoretical results. How well to they agree? What are the discrepancies?

5.4 Dependence of f_n on Mode

Place 200 g on the end of the cord. Find the experimental normal mode frequencies for modes 1-4. Are these frequencies related in the way you expect from Eq.(4)? Explain your results.

6 Finishing Up

Please leave the bench as you found it. Thank you.

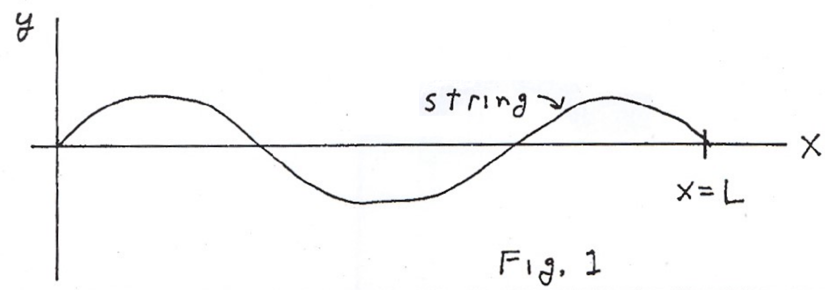

Fig. 1

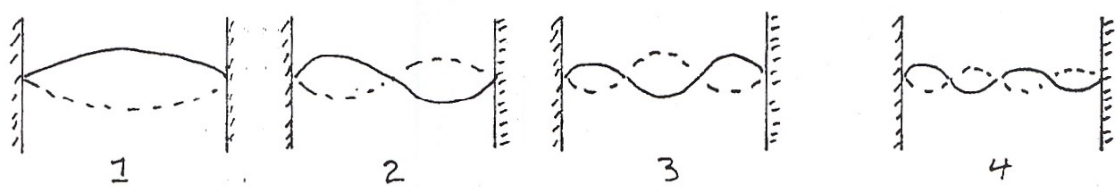

Fig. 2

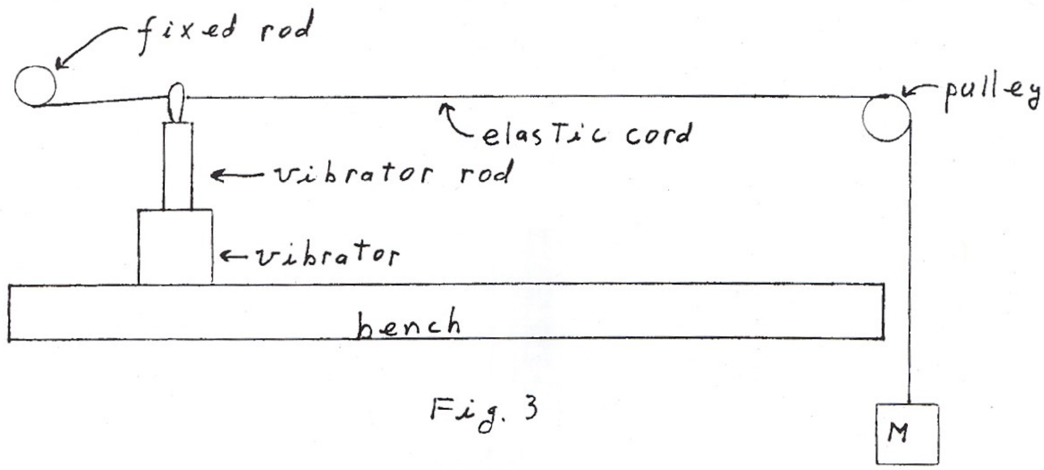

Fig. 3