Electromagnetic Induction

Equipment DataStudio, RLC circuit board, box with 2 coils and iron rod, magnet, 2 voltage sensors (no alligator clips), 2 leads (35 in.), bubble wrap to catch dropped magnet, Fluke multimeter

1 Introduction

The phenomenon of electromagnetic induction was discovered by Joseph Henry in New York in 1830 and by Michael Faraday in England in 1831. Faraday’s name is commonly associated with the phenomenon (Faraday induction), although in this century Henry’s contribution has been recognized by assigning his name as the unit for inductance. In the 1820’s it was known that an electric current produces a magnetic field, and Henry and Faraday were trying to reverse the process and produce a current with a magnetic field. The result is elusive as a steady magnetic field will not cause a current to flow in a circuit. It is a time changing magnetic field that “induces” a current to flow. The changing magnetic field produces an electric field which is not conservative. The line integral of the electric field around a loop or circuit is not zero and is called an electromotive force or EMF. The EMF can drive a current in a circuit. The unit of EMF is the volt (V).

In this experiment an emf will be induced in one coil by

- Moving a permanent magnet into and out of the coil,
- Moving a second coil with a current in it near the first coil, and
- Changing the current in the second coil.

Some of your results will be qualitative. You should describe and explain what you observe. Bear in mind that the strength of the induced emf depends on the rate of change of the magnetic field. When an emf is induced in a coil the current that results will depend on the resistance of the circuit which the coil is part of. A voltage will also appear across the coil which can be measured by standard means. Because the voltage across the coil used in this experiment will be measured by a high impedance (resistance) device, the amount of current that flows will be small.

2 Theory

The flux $\Phi$ of the magnetic field is defined in a similar way to the flux of the electric field. See Fig. 1. A magnetic field $\vec{B}$ passes through a differential area $d\vec{A}$. The differential element of magnetic flux associated with $d\vec{A}$ is $d\Phi = \vec{B} \cdot d\vec{A}$. It is only the component of $\vec{B}$ that is perpendicular to the surface area that contributes to the flux. For a finite area the magnetic flux is $\Phi = \int \vec{B} \cdot d\vec{A}$. The line integral of the electric field around the boundary of the area is called the EMF. The positive directions of $d\vec{A}$ and the line integral must be consistent with the right hand rule where the thumb points in the direction of $d\vec{A}$. Faraday’s law says that for a single circuit around the boundary of the area the EMF is given by $\text{EMF} = -d\Phi/dt$. The non-conservative electric field giving rise to the emf exists whether there is a wire around the boundary or not. But if there is a wire a current will flow if the circuit is “complete.” The current will be limited by the resistance of the circuit. If the loop of wire has two ends
a voltage of $V = +d\Phi/dt$ will appear between the two ends of the wire. This voltage is due to a conservative electric field that opposes the non-conservative electric field producing the EMF. If the wire has N complete turns it is called a coil. The Voltage across the N turn coil is $V = +Nd\Phi/dt$. The magnetic field is said to “link” the coil. It has been assumed that the cross section of the coil is small and that the same magnetic flux goes through each turn of the coil.

The changing magnetic flux through the coil can be produced by any means you can think of. This includes relative motion between a magnet and the coil, relative motion between the coil and a separate circuit with steady currents, changing currents in a separate circuit (mutual inductance), and a changing current in the coil itself (self inductance).

3 EMF Induced By A Permanent Magnet

In this section an EMF is induced in a coil by moving a magnet in and out of the coil. The emf will be is detected by measuring the voltage across the coil. The voltage across the coil as a function of time is shown on the graph display.

Plug in the voltage sensor into one of the analog plugs. In the experiment set-up window click on the 750 interface analog channel that has the voltage sensor. After choose the voltage sensor from the drop down menu. Open a graph by dragging its icon from the Displays window to the voltage sensor icon in the Data window. For a start, make the range of the vertical axis on the graph display -0.1 to +0.1 V and make the maximum value of the horizontal axis 20 s. (Double click on the graph display and click on the axis setting tab to change the axes scales). In the experiment set up window adjust the sampling rate to 100 Hz.

Connect a voltage sensor directly across the coil on the RLC circuit board and see that this sensor is plugged into the analog channel of the interface. In what follows, keep track of which end of the rod magnet you are using. The magnetization of the magnet is along the axis of the rod. Align the axis of the rod along the axis of the coil. Click Start.
Keeping the axes of the coil and magnet colinear, thrust the magnet through the center of the coil so that one end of the magnet rests on the lab bench. Wait a second, keeping the magnet motionless, and then pull the magnet out of the coil. Click stop. You should see two peaks of induced voltage in the graph. Note the polarities of the peaks. Try slower motions and faster motions. Turn the magnet around and repeat your motions, noting the voltage polarities. Try slower motions and faster motions. Try not moving the magnet at all. Turn the axis of the magnet perpendicular to the axis of the coil and bring the magnet down onto the coil. Is there an induced voltage?

Try the above experiments by keeping the magnet stationary and moving the coil.

Lift the RLC circuit board above the bench and place bubble wrap under the coil. Align the axes of coil and magnet, hold the magnet about one inch above the coil, and drop the magnet through the coil onto the bubble wrap. Which voltage peak is higher? Why?

4 EMF Induced By Moving A Coil With A Steady Current

At this point you will restart DataStudio, so close DataStudio and restart the program. In other words X out of the program and double click on the DataStudio icon in the desktop window. The experiments of section 3 will be repeated with a current carrying coil instead of a magnet. A coil with a current has a magnetic field similar to that of a bar magnet. If this coil is moved with respect to another coil the changing magnetic field will induce an EMF. Plug the Power Amplifier into the 750 interface. In the experiment set up window click on the analog channel that the power amplifier is plugged in and select power amplifier from the drop down menu. Program the signal generator for 3 V DC. Connect the power amplifier to one of the coils from the wooden box. Plug in the voltage sensor into one of the analog plugs. In the DataStudio experiment set-up window click on the 750 interface analog channel that has the voltage sensor. After choose the voltage sensor from the drop down menu. Open a graph by dragging its icon from the Displays window to the voltage sensor icon in the Data window. The induced voltages will be smaller so in the graph display, so change the vertical axes to read from -0.01 to +0.01 V. Double click the voltage sensor icon and change the sensitivity to medium (10X). The other parameters can stay the same as in section 3. Click Start and quickly move the 2nd coil in a coaxial fashion with the coil on the RLC circuit board. The recorder trace will be noisy, but an effect should be observable. Try reversing the current to the coil by switching the leads. (In theory, the coil could be turned over, but the connectors to the coil then make it difficult to move the 2 coils together.)

Comment There is a great deal of similarity between moving a magnet and moving a coil with a current. The magnet also has currents, but the currents are not produced by conduction electrons but by electron orbits and spins in the magnetized material from which the magnet is made.

5 EMF induced by Changing The Current In A Coil

In the previous experiment a coil 1 with a steady current was moved with respect to a coil 2 in such a way that the magnetic flux changed through coil 2, inducing an EMF and voltage in coil 2. Now assume the two coils are fixed in position and that there is a time varying current through coil 1 that produces a time varying magnetic flux through coil 2. A time varying EMF will be produced in coil 2. If there are no magnetic materials around the magnetic field produced by coil 1 will be proportional to the current $i_1$ in that coil. Likewise the magnetic
flux through coil 2 produced by coil 1 will also be proportional to \(i_1\). It is customary to write the magnitude of this EMF in coil 2 as \(M \frac{di_1}{dt}\). The quantity \(M\) is called the mutual inductance between the two coils and depends on the geometry of the coils and the number of turns of each coil. \(M\) does not depend on the current. It is possible to orient the two coils so that \(M\) is zero. In this experiment the EMF induced in one coil by a changing current in another coil will be investigated. If the coils are moved with respect to each other the mutual inductance \(M\) will change as will the induced voltage.

As in the previous experiment you should have a voltage sensor icon and a PWR amplifier icon in the Data window. Remove the graph display and drag the scope icon to the power amplifier voltage icon to open the oscilloscope display. Make sure in the power amplifier measurements tab that voltage is checked. This will allow the power amplifier voltage icon to appear in the Data window. The Channel 1 of the scope will record the voltage output of the power amplifier. Program the 2nd trace of the scope for the voltage sensor channel. Program the signal generator for a 1 V sinusoidal wave with a frequency of 1000 Hz. Connect one of the coils from the box to the power amplifier. Lay the wired coil on top of the circuit board coil in a coaxial fashion. Click Start and observe the power amplifier voltage and the induced voltage on the 2 scope traces. How are the 2 scope traces related? What happens if you reverse the two leads from the power amplifier to the coil? What happens if you move the coils apart? What is happening to the mutual inductance when you do this?

Lay the wired coil on top of the circuit board coil and do not move the coils. Now see what happens when you use frequencies of 500, 1000, 2000, and 4000 Hz. The voltage across the wired coil should not change (do not change the voltage of the signal generator). You should also find that the induced voltage across the circuit board coil does not change very much! As an increase in frequency will result in a more rapidly changing magnetic flux in the circuit board coil, this implies that the current in the wired coil goes down as the frequency goes up. This will be examined in the next section.

Iron is a magnetic material. If a modest magnetic field is applied to iron the iron will magnetize so as to greatly increase the magnetic field.

Return to a frequency of 1000 Hz. Insert the iron rod from the box partway into the wired coil. What happens to the induced voltage in the circuit board coil? Insert the iron rod fully so that it goes through both coils. What happens to the induced voltage? When the iron is inserted partway into the wired coil the current in the wired coil is reduced, reducing the flux in the circuit board coil. When the iron is completely inserted, the iron produces more than enough extra magnetic flux in the circuit board coil so as to make up for the decrease of current in the wired coil. The voltage increases. Note. In the above analysis the EMF produced in coil 2 by the current in coil 2 has been neglected. This is an excellent assumption as the only thing connected across coil 2 is a voltage sensor which has a very high resistance. The current in coil 2 is negligible.

Comment Two coils magnetically coupled by iron are called a transformer. A sinusoidal voltage is applied to the primary coil and the sinusoidal output voltage appears at the secondary coil. If the secondary coil has more turns than the primary coil the voltage is stepped up. If the secondary coil has fewer turns the voltage is stepped down. Neglecting losses power is conserved: The input voltage times the input current is equal to the output voltage times the output current. The existence of transformers is why the power distribution system is AC, not DC. Tesla had it right and Edison had it wrong.
6 Self Inductance

Current in a coil produces a magnetic flux in the same coil. This magnetic flux, if it varies with time, produces a self EMF in the coil. As the magnetic flux in the coil is proportional to the current \(i\) in the coil, the EMF in the coil can be written \(\text{EMF} = -L\frac{di}{dt}\). \(L\) is called the self inductance of the coil and depends on the coil geometry and the number of turns of wire but not on the current. If a time varying voltage is applied to a coil the electric field drives a time varying current through the coil. This current creates an induced counter electric field that limits the current. The EMF across the coil is \(-L\frac{di}{dt}\) and the voltage across the coil is \(+L\frac{di}{dt}\). This is most easily explored by applying a sinusoidal voltage \(v = v(t) = V_P \cos \omega t\) across the coil. \(V_P\) is the peak value of the sinusoidal voltage. In what follows it is assumed that the coil has no ohmic resistance. This is an assumption that for some parameters is not completely valid. We have \(v = V_P \cos \omega t = L\frac{di}{dt}\). Integrating this expression the current becomes \(i = \frac{V_P}{\omega L} \sin \omega t\). The voltage across the coil is said to lead the current through the coil by \(\pi/2\) rad. If \(V_P\) is kept constant the current decreases as \(1/\omega\) as the frequency increases. The quantity \(\omega L\) is called the inductive reactance \(X_L\). \(X_L\) is somewhat akin to resistance and has the units of ohms. In the same way that resistance determines the amount of current in a resistor, the inductive reactance determines the amount of current in an inductor. There is no energy dissipation in a coil which has no resistance. Energy is continually exchanged between the magnetic field of the coil and the voltage source and a averages to zero over one cycle. There is of course energy dissipation in a resistor. For a sinusoidal voltage, in a resistor the current and voltage are in phase. In an inductor the voltage leads the current by \(\pi/2\) rad.

Plug the voltage sensors into channels A and B. In the DataStudio experimental set-up window program channel A and B for the voltage sensors. Setup channel C for the power amplifier and make sure that voltage is checked in the power amplifier measurements tab. Drag the scope icon to the power amplifier voltage icon. The output voltage of the power amplifier should be the input to channel 1 of the scope. Choose voltage sensor A for channel 2 of the scope and voltage sensor B for channel 3 of the scope. Hook up the circuit shown in Fig. 2. The 10 ohm resistor is included in the circuit for current measurement. Start with a frequency of 1000 Hz and a peak voltage of 1.5 V across the coil. Measure the voltage across the resistor and calculate the current. Also note the phase difference between the current.
through and voltage across the coil. Repeat these measurements for frequencies of 500, 2000, and 4000 Hz, for each case adjusting the voltage across the coil to be 1.5 V. In light of the theory given above, discuss your results.

In our theory it was assumed that the coil had no resistance. Use the Fluke multimeter to measure the resistance of the coil. Calculate the inductive reactance of coil at 1000 Hz and compare it to the resistance of the coil. The resistance of the coil is not enough to drastically change our principle results.

The derivative of the current through an ideal coil is proportional to the voltage across the coil. The voltage across a resistor is proportional to the current through the resistor. At a frequency of 1000 Hz look at the waveforms on the scope for the square wave and triangular wave. Do the shapes of the voltage across the coil and resistor make sense? The shapes of the waveforms will not be strictly what you expect, but they will be close. For example, note that the applied square wave has a very finite rise time.

Returning to the 1000 Hz sine wave, thrust an iron rod into the coil. What happens to the current. Why?

**Problem** The power into a circuit element is the product of the voltage across the element and the current through the element. Assuming a voltage \( v(t) = V_P \cos(\omega t) \) across an inductor \( L \), integrate the power over one cycle and show that the net energy into the inductor is zero.

### 7 Finishing Up

Please leave the bench as you found it. Thank you.