Pendulum

Equipment Capstone, meter stick, large bench clamp with heavy rod, large triangle base stand, photogate stand, clamp for cross rod, 40 cm cross rod, pendulum mounting bar, set of weights with hooks, calipers, ruler, photogate, rotary motion sensor, rotary motion sensor three step pulley, rod and 1 brass weight from rotational accessory, scale, 1 meter string with loop at one end

Comments The monitor is difficult to read, but the relevant coefficient is the frequency a2. This is given in cycles per second, or Hz. When fitting a sine wave to your data, fit to just 3 or 4 cycles. The match then is quite good. If you try and fit to too many cycles, the match becomes very bad.

1 Purpose

To investigate the simple and physical pendulum. Also, determine how the period varies with length, mass, and amplitude.

2 Theory

2.1 Simple Pendulum

An ideal simple pendulum consists of a point mass suspended by a mass less non-extensible string. We assume the oscillations take place in a vertical plane. Let the length of the string be L, the value of the mass M, and the angle that string makes with the vertical θ. The acceleration and component of the gravitational force in the direction of motion are $ML\frac{d^2\theta}{dt^2}$ and $-Mg\sin(\theta)$ where t is the time. For small angles $\sin(\theta)$ may be approximated by θ. The relevant component of Newton's 2nd law becomes

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0. \tag{1}$$

The solution is $\theta_m \sin(\omega t + \phi)$ where θ_m is the amplitude, ϕ is the phase, and $\omega = \sqrt{\frac{g}{L}}$ the angular frequency of the motion. The period T of the motion is given

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}. \tag{2}$$

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The frequency and period of the pendulum do not depend on the mass.

2.2 The Physical Pendulum

Consider a rigid body pivoted about a fixed point or axis and constrained to move in a vertical plane. Let I be the moment of inertia about the fixed point, R the distance from the pivot point to the center of mass, M the mass, and θ the angle between the vertical and a line from the pivot point to the center of mass. The pivot point is assumed to be friction less and to exert no torque on the body. The torque τ on the body about the axis is $-MgR\sin(\theta) \cong -MgR\theta$, and the $\tau = I\alpha$ equation becomes

$$\frac{d^2\theta}{dt^2} + \frac{MgR}{I}\theta = 0. \tag{3}$$

Again the solution is $\theta_m \sin(\omega t + \phi)$, where θ_m is the amplitude, ϕ is the phase, and $\omega = \sqrt{\frac{MgL}{I}}$ is the angular frequency of the motion. The period T of the motion is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR}}. \tag{4}$$

The period does not depend on the total mass but does depend on how mass is distributed. **Exercise. Show that Eq. 4 reduces to Eq. 2 for a point mass.**

2.3 Correction To Period

The above analysis for the period T has made the approximation $\sin(\theta) = \theta$. The larger an angle, the worse this approximation will be. The following correction can be used. Let T_c be the corrected value for the period and let θ_m be the amplitude or the maximum value of θ. Then, where T is given by Eq. 2,

$$T_c = \frac{T}{1 - \frac{1}{4}\sin^2\left(\frac{\theta_m}{2}\right)}. \tag{5}$$

Equation 5 predicts that the period will increase when θ_m increases.

3 Simple Pendulum Experiment

3.1 Description

A pendulum is formed by a mass hooked into a loop on the end of a string. The upper end of the string is clamped, but the length of the string and the height of the string support are easily changed. A photogate on the bench is positioned with the legs up and arranged so that the mass swings through it. The period of the pendulum is measured by the photogate. The period is measured for various lengths of the string and different values of the mass.

3.2 Programming

Setup Capstone for the digital photogate sensor. Then you need to program photogate sensor for a pendulum setup. This can be done by clicking on **Timer Setup** icon. You need to check off the following measurements Period, Block- to Block Times, Speed, and Time in Gate. Also, the object width needs to be measured and set in Capstone parameter. Look at the following illustration for guidance.

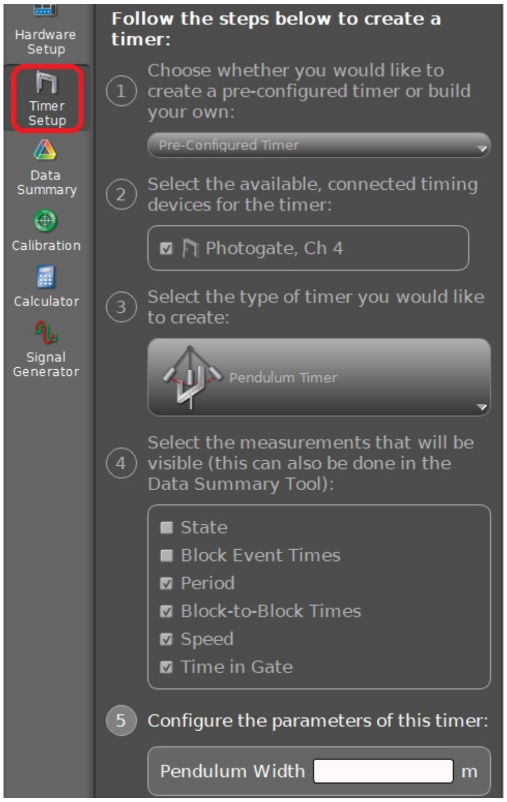

Figure 1:

3.3 Dependence On Mass

For this section you will vary the mass of the pendulum.

- Determine the center of mass of a 200 g weight by balancing it on a toothpick, ruler, or similar object.
- Set up the pendulum mounting bar onto the rods and clamp.
- Hang 200 g mass on the string.
- Adjust the length from the support to the center of mass to be about 80 cm.
- Setup a data table in Capstone and select **Block to Block times**

Measure the period for small amplitudes, using the block to block times from the table display. Note that the time difference between two successive time index numbers will be half a period. (The time that the mass spends in the photogate beam, which is the number in the lower right hand part of the table box, is not important, but you might note that this number will slowly increase due to friction.) Check your results against Eq. 2. Repeat for a mass of 50 g keeping the pendulum length constant. **Note: That the center of mass of the 50 g mass will lie at a slightly different distance, you should readjust the string length so that you put the center of mass at the same distance from the support as was done for the 200 g mass.** Compare and discuss your results.

3.4 Dependence On Length

Using a 200 g mass measure the period for a pendulum length for about 40 cm. Compare to the longer 200 g pendulum and discuss your results, using Eq. 2.

4 Thin Tube As A Physical Pendulum

4.1 Description

In this section one end of a thin tube from the rotational accessory is fastened to the pulley on a rotary motion sensor. This will be used as a physical pendulum. The tube is thin enough so that it can be considered as a thin rod. The motion is recorded by using a graph display. A sine curve is fitted for 3 or 4 cycles. Using the graph you can obtain frequency and hence the period. Later you will investigate the period as a function of maximum angular deflection θ_m.

4.2 Set Up

Using a cross rod clamp, mount the 40 cm rod on the vertical rectangular rod. Then mount the rotary motion sensor in the vertical direction on to the 40 cm rod. Then mount the three step pulley and thin tube on to the rotary motion sensor.

4.3 Center Of Mass

The tube extends on both sides of the pivot. Let the length of the shorter segment be a and the length of the longer segment be b. **Exercise. Show that the distance R_t of the center of mass from the pivot is given by**

$$R_t = \frac{b-a}{2}. \tag{6}$$

4.4 Moment Of Inertia

The moment of inertia is the sum of the moments of inertia of the two segments, each considered as as thin rod pivoted about one end. **<u>Exercise</u> Show that the moment of inertia of the tube I_t about the pivot is given by**

$$I_t = \frac{M_t}{3}\left(\frac{a^3+b^3}{a+b}\right), \tag{7}$$

where M_t is the mass of the tube. The moment of inertia of a thin tube of mass m and length d about an end is $\frac{1}{3}md^2$.

4.5 Programming

Plug in the rotary motion sensor and setup Capstone for the digital rotary motion sensor. Click on properties and in resolution choose 1440 divisions/rotation.

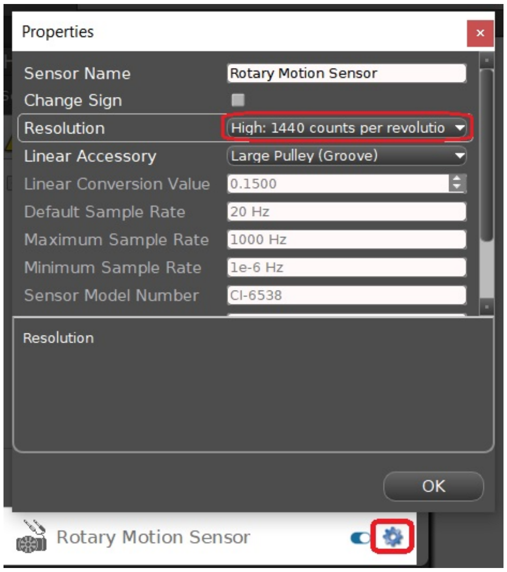

Figure 2:

Then setup a graph display window and choose angle.

4.6 Procedures

- With the pendulum at rest, click RECORD. (If you click REC with the pendulum going, the graph trace will not be centered on the horizontal axis.)
- Then pull the pendulum to one side about 0.2 rad and let it go for about 6 cycles before clicking STOP.
- Fit a sine curve to about 3 or 4 cycles of your data.

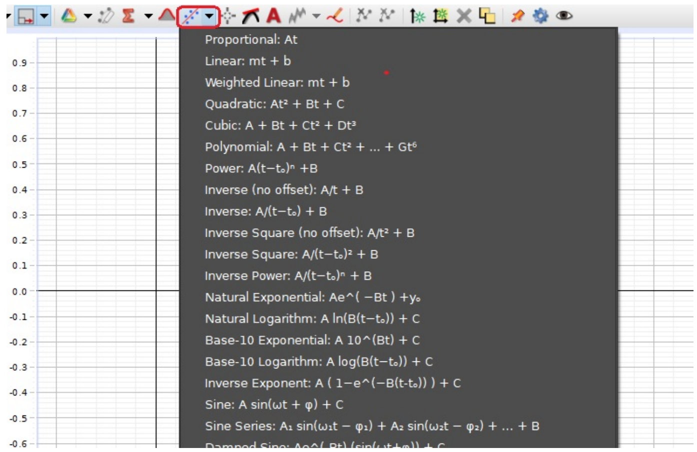

Figure 3:

Determine the frequency a2, and the period $\frac{1}{a2}$. Use Coordinates Tool located on top of the graph to determine the value of θ_m.

Compare your data to Eq. 4 and Eq. 5. Repeat for values of θ_m going up to about 1.0 rad in steps of about 0.2 rad. Is T_c a better fit to the data than T?

For this relatively light rod damping is readily apparent. For a modest deflection take data for about 1 minute. **If the damping is proportional to the angular velocity, the envelope of the amplitude is exponential. Does this seem to be the case? Measure the amplitudes at 10 s, 20 s, . . . , 60 s. Plot the natural logs of the amplitudes versus the times. Do you get a straight line?**

5 Tube Plus Cylinder As A Physical Pendulum

5.1 Description

Within this last section a rod (or thin tube) as a physical pendulum is made more complicated by putting a cylinder on the rod. After the period is measured and compared to theory.

5.2 Center Of Mass

The position of a center of mass with respect to the pivot can be calculated as if the tube and cylinder were point masses; located at their respective center of masses. Let M_c be the mass of the cylinder and R_c distance of the center of mass of the cylinder from the pivot. The distance R of the center of mass of the tube plus the cylinder from the pivot is given by

$$R = \frac{R_t M_t + R_c M_c}{M_t + M_c}. \tag{8}$$

5.3 Moment Of Inertia

If I_c is the moment of inertia of the cylinder, the total moment of inertia I is given by $I = I_t + I_c$. Let R_1 and R_2 be the inner and outer radii of the cylinder and let L be the height of the cylinder. The moment of inertia of the cylinder about the pivot is given by

$$I_c = \frac{1}{4}M_c(R_1^2 + R_2^2) + \frac{1}{12}M_cL^2 + M_cR_c^2. \quad (9)$$

The 1st two terms on the right hand side are the moment of inertia of the cylinder about an axis through the center of mass and perpendicular to the symmetry axis. The last term arises from the parallel axis theorem.

5.4 Procedures

Measure the diameter of the largest pulley, the masses of the rod and cylinder, and all other relevant dimensions. Slip the cylinder onto the rod and push it against the pulley. Measure the period of the pendulum for small amplitudes. **Compare your results to the prediction of Eq. 4. Answer the following questions.**

1. What is the value of I_c/I_t? Explain this number in light of the fact that the cylinder is quite a bit heavier than the tube.

2. Compare the period of this pendulum to that of the tube alone and discuss, noting that the moment of inertia has been increased with the addition of the cylinder.

5.5 For Your Edification Only

Move the cylinder to the end of the tube to lengthen the period. Click the add plot button and add a graph of angular velocity. Do the same for angular acceleration. Press REC and set the pendulum going for a few oscillations and then press STOP. Click the auto scale button. (The purpose of the above is to get a reasonable vertical axis scale for observation.) Press REC and set the pendulum going at a slightly lower amplitude so as to avoid re-scaling the vertical axis. Observe each of the 3 graph plots while you look at the pendulum motion. Get a feel for how the pendulum motion translate into graph plots. Note that the 3 quantities being observed are sinusoidal but that the phases are different.

6 Finishing Up

Please leave the bench as you found it. Thank you.