The Mössbauer Effect

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Goals

The main goal of this experiment is to exploit the Mössbauer effect to measure the linewidth of the $^{57}$Fe 14.4 keV gamma ray. This measurement is one of the most precise in physics: the relative width of the line, $\Delta \nu / \nu$ is only about one part in $10^{13}$!

Background

The photons emitted by nuclei making a transition from an excited state to the ground state are not monoenergetic, but have a distribution of energies because of the energy width of the excited state. The natural width of the energy distribution of photons, $\Gamma$, is related to the mean lifetime, $\tau$, of the excited state by

$$\Gamma = \hbar / \tau, \quad (1)$$

consistent with the “time-energy” uncertainty principle. The width and thus the lifetime can, in principle, be determined from a measurement of the resonance fluorescence, the absorption and reemission of a photon emitted by a nucleus of the same type.

If the excited state has energy centered at $E_0$ with width $\Gamma$, the cross section for the absorption of photons has a sharp maximum at the excitation energy $E_0$ and drops to half the maximum at $E_0 \pm \Gamma / 2$. The integral of the cross section over energy is proportional to $\Gamma$, so a measurement of the absorption cross section versus energy can be used to determine $\Gamma$ and therefore the lifetime $\tau$.

Resonance fluorescence is observed for atomic transitions, but generally not for nuclear transitions because of the nuclear recoil. This recoil is negligible for atomic transitions because of their low energy ($\sim$1 eV), whereas for nuclear transitions of energy of the order of 1 MeV, the recoil shifts the energy of the emitted photon completely off resonance. A nucleus or atom emitting a photon of energy $E$ will recoil with momentum approximately equal to $E/c$. The energy of the photon will be reduced by the nuclear recoil energy $E_r = p^2 / 2M \approx E^2 / 2Mc^2$. If the energy of the excited state is $E_0$, the energy of the emitted photon will then be $E_0 - E_r$ with $E_r \approx E_0^2 / 2Mc^2$. Similarly, if the nucleus is to absorb a photon and make a transition of energy $E_0$, the photon must have energy $E_0 + E_r$. The center of the distribution of energies of the emitted photons is therefore displaced from that of the distribution of absorbed photons by $2E_r = E_0^2 / 2Mc^2$. (See Figure 1.)

If the width of the excited state is less than this amount, no photons will be absorbed by another nucleus of the same kind, and resonance fluorescence cannot take place. For a typical mean lifetime of $10^{-8}$ sec, the width is about $\Gamma = h / \tau = 10^{-7}$ eV. For a typical atom of atomic number $A = 100$, the nuclear recoil energy is about $10^{-11}$ eV for $E_0 = 1$ eV and about $10^{-1}$ eV for $E_0 = 1$ MeV. For atomic transitions, therefore, the recoil energy is negligible compared with the natural width $\Gamma$, but for most nuclear transitions, it is much greater than $\Gamma$, and there is, in general, no overlap of the emission and absorption lines.

However, the above discussion refers to the case of the emission and absorption of a gamma ray by a free nucleus. In the case of a nucleus in a crystal lattice, the situation is somewhat different, as was shown by R. Mössbauer in 1958. For atoms bound in a crystal lattice, a nucleus does not recoil individually, but the momentum of the nuclear gamma ray is shared by the whole crystal, in the form of vibrational energy, or phonons. (This can be understood if we consider that the binding energies of the atoms in a lattice site are of the order of 10 eV, whereas the recoil energies are always less than 1 eV.) However, since phonons are a quantized process, if the recoil energy of the transition is less than the lowest energy phonon that can be excited for a given crystal, then all the energy of the transition is, in fact, taken by the gamma ray, leading to the so-called recoilless emission or absorption of the photon; in such a case, resonance fluorescence can, in fact, occur.
The Mössbauer Effect

Figure 1: (a) Nuclear energy level has a half width $\Gamma$. (b) Shift in energy of emitted photon due to recoil of nucleus. (c) Shift in energy of absorptive transition because of the need of the absorbing nucleus to recoil. If the shifted levels (b) and (c) overlap, resonance fluorescence can occur.

Consider now a crystalline source of nuclear gamma rays, for which the Mössbauer conditions for recoilless emission and absorption are met, and resonance fluorescence occurs. We now observe that the resonance fluorescence can be destroyed if we simply move the source at a small velocity relative to an absorber, since the overlap of the emission and absorption lines varies with the velocity of the source, as a result of the Doppler shift. That is, if the emitting nucleus is moving in the laboratory with velocity $v$ in the direction of the gamma ray, which has energy $E_\gamma = h\nu_0$ in the rest frame of the nucleus, then the laboratory energy of the gamma ray $E'_\gamma$ is given through a Lorentz transformation as

$$E'_\gamma = \frac{1}{\sqrt{1 - \beta^2}} (E_\gamma - v p_\gamma) = E_\gamma \frac{1 - \beta}{\sqrt{1 - \beta^2}},$$

where $\beta = v/c$. For $\beta \ll 1$ we obtain to first order

$$\Delta E = E'_\gamma - E_\gamma = \beta E_\gamma,$$

or

$$\frac{\Delta E}{E} = \beta = v/c,$$

which, when written as $\Delta \nu/\nu = v/c$, is the first order Doppler shift of a wave emitted (absorbed) by a moving observer. The degree of overlap between the emission and absorption lines, and hence the amount of radiation absorbed, will thus depend on the value of $v$.

In this experiment, we will consider the 14.4 keV gamma ray of $^{57}$Fe, which follows the decay, by electron capture, of $^{57}$Co, as shown in Figure 2. The Mössbauer conditions for recoilless emission and absorption are
met for this material (due to the particular details of the crystal lattice) and so there is considerable overlap between the emission and absorption lines. By varying the velocity of the source, the condition of resonance fluorescence will be destroyed. The 14.4 keV line has a lifetime \( t \approx 10^{-7} \) sec, and hence \( \Delta \nu/\nu = 4.5 \times 10^{-13} \). Thus, velocities of order \( v = c(\Delta \nu/\nu) = 1.4 \) mm/sec will be sufficient to destroy the resonance fluorescence. Such velocities are easy to achieve and control in the laboratory, and we measure the transmission of the 14.4 keV gamma ray through an Fe absorber (either stainless steel or enriched \(^{57}\text{Fe}\)) as a function of the velocity of the source.

**Apparatus**

The apparatus used in this experiment is shown in Figure 3. A \(^{57}\text{Co}\) source is mounted on a mechanical actuator (Pasco “Mechanical Wave Driver”, model SF-9324), and a thin absorber of either stainless steel or enriched \(^{57}\text{Fe}\) is placed between the source and the detector. Counting electronics consist of a pre-amp, amplifier, and multichannel buffer. The actuator is driven at constant velocity by means of a sawtooth waveform from a computer controlled voltage source (Pasco), after appropriate amplification and bias. Note that the sign of the displacement of the actuator is dependent upon the polarity of the connection to the Pasco voltage source.

The Pasco voltage supply is controlled by the Science Workshop (SWS) software on your PC. SWS allows you to set the amplitude and frequency, as well as the shape, of the output AC waveform. There are four choices for the sawtooth waveform: two completely positive and two that go between \( \pm \) the amplitude. The two waveforms of each type are inverted over time with respect to each other. Table 1 in the appendix gives the displacement of the actuator from its center position as a function of the applied voltage. Take care, then, when calculating velocities that you are considering the total displacement of the actuator.

**Detector/Pre-amp:**

The detector in this experiment is a thin crystal of sodium iodide doped with thallium, NaI(Tl). The use of NaI as a \( \gamma \) ray detector was pioneered by Hartmut Kallman who was a faculty member in the NYU physics department. A \( \gamma \) ray photon produces a pulse of light (scintillation) in the NaI whose amplitude is
proportional to the $\gamma$ rays energy. The light is detected by a photomultiplier that views the NaI crystal. The output of the photomultiplier is an electronic pulse consisting of roughly a million electrons.

**Electronics:**

The typical electronics associated with nuclear radiation detectors has the basic function of (a) transforming low level pulses issuing from the detector, through amplification and pulse shaping, into a pulse more suitable for measurement and analysis; (b) analyzing the pulse height distribution; and (c) counting the pulses. The requirements for different detectors are sufficiently similar so that standard ‘NIM’ modules in different combinations can be used.

1. **Linear Amplifier**

   Electrical pulses from the photomultiplier are passed to a linear amplifier, so called because a linear relationship exists between the input and output signals as long as the gain is set low enough such that the strongest output pulses are no larger than $\sim 12$ V. The amplifier amplifies and shapes input pulses. There are two outputs from the amplifier, one labeled UNI and the other labeled BI, where the UNI output is the modulus of the amplified input signal. Power to the amplifier is switched using the Tennelec switch on the right hand side of the equipment rack.

2. **Multichannel Buffer (MCB)**

   Pulses from the amplifier are passed to an ORTEC MCB card in the PC at your lab bench. The ORTEC Maestro software records and counts pulses processed by the card, binning them by voltage. The pulse counts are displayed as a histogram over voltage. Thus, the MCB combined with the recording software emulate the functionality of a multichannel analyzer. Note that pulses should be passed to the MCB from the UNI output of the amplifier in order to be correctly interpreted. Maestro has a built-in discriminator, which in this case is a software implementation where pulses outside of the discriminator range are simply not counted by the program.

   Spectra are stored by Maestro in a memory buffer, which can also be written to file. There are “Start” and “Stop” functions that begin and end data collection periods, and a “Clear” function that clears the buffer. Once a spectrum is recorded, it can be integrated over energies inside of a so-called “Region of Interest” (ROI). To mark a ROI, place the vertical line cursor at one of the desired boundaries of the ROI, select ROI... “Mark” from the menu bar, then move the cursor around with the arrow keys. A bin that is swept over by the vertical cursor is highlighted in red and thus included in the new ROI (this can be undone by using “Unmark” from the ROI menu). Right-click on an ROI and select “Peak Info” to display the approximate center and spread of the highlighted peak, as well as the integral over the ROI.

   Note that two values for the integral are displayed: one labeled “Gross” and one labeled “Net” (the latter of which comes with a confidence interval). The “Net” integral is supposed to take into account noise in the data which is subtracted from the “Gross” data, and then give an estimate of the confidence interval of the “noise-free” data. However, the manner in which Maestro models noise and estimates the confidence interval is unknown. It is recommended that both values are recorded for each data point and compared later.

**Procedure:**

1. **Check the operation of the detector.**

   Turn on the AC power on the power supply for the photomultiplier. Check that the power supply is set for positive voltage output. Allow the power supply a few minutes to warm up, then turn on the high voltage power. Slowly increase the voltage to $+600$ V.

   Observe the output of the amplifier with an oscilloscope. Place the source directly in front of the detector during the test to get a high count rate. Note that the polarity switch on the amplifier may need to be inverted to get positive output pulses. Trigger the oscilloscope internally so that the pulse height distribution can be observed.
The gain of the amplifier can now be adjusted according to the considerations discussed above.

2. **Check the operation of the discriminator.**

   To maximize the change in the counting rate due to the Mössbauer effect, it is necessary to select only the 14.4 keV transition and to reject the large background of higher energy gamma rays also emitted by the $^{57}$Co source. Acquire a spectrum by collecting 3 minutes worth of pulses. You can preset the data acquisition interval in Maestro. Identify the peaks. What is the origin of the peak at the lowest energies (in the lowest channels)? See Figure 5 in the Appendix and note that the detector in this experiment is unable to resolve the two lowest energy peaks shown in this figure.

   Now set a region of interest (ROI) that includes the 14.4 keV peak. You will be monitoring the number of $\gamma$ rays in this ROI as a function of the velocity of the cobalt source.

3. **Calibration of the actuator movement.**

   Table 1 of the appendix shows the result of the calibration of the displacement of the source as a function of the voltage of the sawtooth. Note that there is a linear relationship between the frequency of the sawtooth, the amplitude of the sawtooth, and the velocity of the actuator, which you can use with the data of Table 1 to produce a desired velocity of the actuator. This linear relationship has been verified up to frequencies of 6 Hz. Turn on the voltage power supply for the sawtooth generator. Practice using the amplitude and sweep time of the sawtooth to generate a velocity of 1 mm/sec for the source. It was recommended by past students that you use only the positive sawtooth waveforms (two reverse the velocity, switch from the positive sawtooth with the rising edge to the one with the falling edge).

4. **Check for the Mössbauer effect.**

   Using the stainless steel absorber, check that the apparatus is working properly by comparing the count rate measured with the source stationary to that obtained with the source moving at high velocity ($\sim$ 2 mm/sec). The count rate obviously varies as the source to detector distance changes, and so the ‘stationary’ count rate should be measured not with the actuator in its zero-voltage equilibrium position, but with the source moving very slowly such that its average position is the same as when it is moving rapidly.

**Questions**

Using the stainless steel absorber, determine the absorption line profile, and thus the lifetime of the $3/2−$ excited state in $^{57}$Fe, by measuring the count rate as a function of source velocity. Keep in mind that the absorption line is not necessarily symmetric about zero velocity, and so data must be taken for both positive and negative velocities. In your report, be sure to provide an estimate of the experimental uncertainty in your determination of the lifetime, and discuss the sources of error and how they might be minimized or eliminated.

Now replace the stainless steel foil with the enriched $^{57}$Fe absorber, and again measure the absorption line profile. You should now be able to observe the ‘Zeeman effect’ for the nucleus, due to the strong magnetic fields present at the site of the Fe nucleus; as a result of the magnetic field, both the $3/2−$ and the $1/2−$ states are split, and consequently the 14.4 keV line has six hyperfine structure components. See the appendix (Figure 3) for an energy level diagram and its associated spectrum.

**References**


Appendix

Figure 4: Low energy photon spectrum of $^{57}$Co Acquired with a High-Resolution Si(Li) detector.

<table>
<thead>
<tr>
<th>Voltage [V]</th>
<th>Displacement [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.1651</td>
</tr>
<tr>
<td>0.35</td>
<td>0.2032</td>
</tr>
<tr>
<td>0.40</td>
<td>0.2286</td>
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<tr>
<td>0.45</td>
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<tr>
<td>0.50</td>
<td>0.3048</td>
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<tr>
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<td>0.3048</td>
</tr>
<tr>
<td>0.60</td>
<td>0.4064</td>
</tr>
<tr>
<td>0.65</td>
<td>0.4572</td>
</tr>
<tr>
<td>0.70</td>
<td>0.508</td>
</tr>
</tbody>
</table>

Table 1: Calibration of the vibrator: Table of the displacement $Z$ versus the applied voltage $V$.

Figure 5: Graph of the above table
The Mössbauer Effect

\[ M_1 \]

\[ \frac{3}{2} \]

\[ \frac{1}{2} \]

\[ \frac{-1}{2} \]

\[ \frac{-3}{2} \]

\[ E_0 = 14,400 \text{ eV} \]

\[ g_n H_{\text{int}} = 3.2 \times 10^{-7} \text{ eV} \]

\[ g_n H_{\text{int}} = 1.9 \times 10^{-7} \text{ eV} \]

\[ P_6 - P_1 = (3g_n^* + g_n^0) \beta_n H_{\text{int}} = 10.66 \text{ mm/s} \]

\[ P_5 - P_2 = (g_n^* + g_n^0) \beta_n H_{\text{int}} = 6.17 \text{ mm/s} \]

\[ P_4 - P_3 = (-g_n^* + g_n^0) \beta_n H_{\text{int}} = 1.68 \text{ mm/s} \]

1 mm/s = \[ \frac{h(1 \text{ mm/s})}{c} = 14.4 \text{ keV} / 3 \times 10^{11} = 4.8 \times 10^{-8} \text{ eV} \]

Figure 3. Iron Metal $^{57}$Fe hyperfine splitting and Mössbauer spectrum