Rutherford Scattering

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Goals

The goal of this experiment is to verify the Rutherford formula for nuclear scattering. You will measure the distribution of alpha particles scattered by a gold foil as a function of scattering angle, and compare your results with the theory developed around 1913 by E. Rutherford and his students H. W. Gieger and E. Marsden.

Background

A great deal of what is known about the atomic nucleus has been deduced from the analysis of scattering experiments done with charged particles and neutrons. The first such experiments were performed with alpha particles from naturally radioactive substances, the counting of the scattered particles being done by visual observation of the scintillations (faint flashes of light) from a zinc sulfide phosphor. The data from the first experiments by Geiger and Mardsen formed the basis for the Rutherford model of the atom. The crucial fact from these first experiments was that, although most α-particles (He nuclei) passed through a thin foil undeflected, there were a large number of large angle scatterings. Subsequent, more precise experiments verified Rutherford's quantitative predictions in detail.

This type of scattering experiment has also played an important role in the exploration of the moon. One of the unmanned instrument packages observed the large angle scattering of α-particles off the surface of the moon. Since the energy loss of the scattered particle depends on the mass of the scatterer, it was possible to determine many of the elements on the Moon's surface.

Scattering data is used to determine an experimental value for the differential scattering cross-section, $\frac{d\sigma}{d\Omega}(\theta)$, which is defined by the following:

$$\frac{d\sigma}{d\Omega}(\theta)\,\Delta\Omega \equiv \frac{\text{number of particles scattered per unit time into a solid angle } \Delta\Omega(\theta)}{(\text{number of scatterers}) \times (\text{incident flux})}\,, \tag{1}$$

where "incident flux" is the number of particles incident per unit area and time.

Rutherford showed that a positively charged point nucleus has a differential scattering cross-section for charged particles equal to

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{Z^2 z^2 e^4}{16E^2}\frac{1}{\sin^4(\theta/2)}\,, \tag{2}$$

where Ze is the nuclear charge, ze is the charge of the scattered particle, E is the energy of the scattered particle, and θ is, as above, the scattering angle.

Apparatus

Geometry

The geometry used in this experiment, originated by Chadwick (who discovered the neutron), is shown in Figure 1. The apparatus we use was designed for student laboratories at MIT. The values of some of the parameters used in this experiment are:

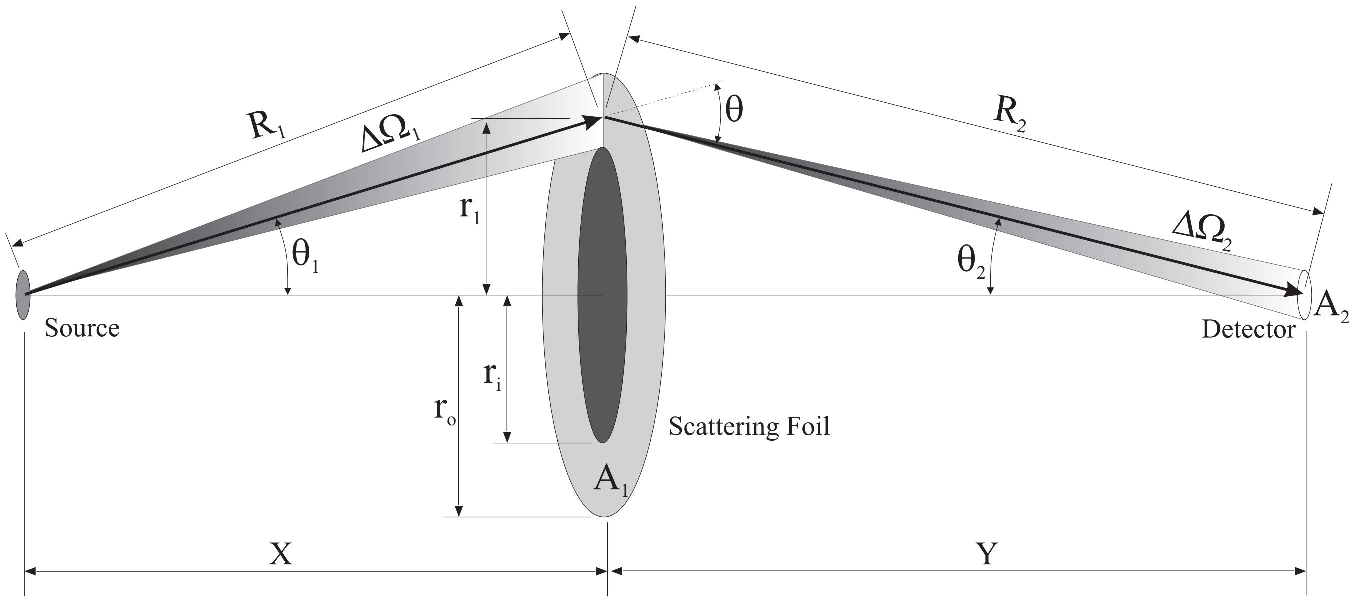

Figure 1: Experimental Geometry

$r_s = 0.32 \pm 0.01$ cm	radius of the source	$\rightarrow$	**Do Not Attempt to Measure**
$\mathrm{X} = 7.22 \pm 0.01$ cm	source to plane of scattering foil	$\rightarrow$	**or Check These Numbers**
$r_i = 2.30$ cm	radius of inside of scattering foil.		
$r_o = 2.70$ cm	radius of outside of scattering foil.		
$r_d = 0.48$ cm	radius of detector.		

The symbols $\Delta\Omega_1$ and $\Delta\Omega_2$ are defined as follows:

$\Delta\Omega_1 \equiv$ average solid angle subtended by the gold annulus (scattering foil) at the source.

$\Delta\Omega_2 \equiv$ average solid angle subtended by the detector at the scatterer ($\Delta\Omega_2 = \Delta\Omega$ in the definition of $d\sigma/d\Omega$).

To a good approximation (i.e., for this experiment,) we may calculate angles and distances for an "average" scattering center located at $r_1 = (r_i + r_o)/2$. We may also use the approximate solid angles:

$$\Delta\Omega_1 = \frac{A_1 \cos(\theta_1)}{R_1^2}, \qquad \Delta\Omega_2 = \frac{A_2 \cos(\theta_2)}{R_2^2}.$$

Other experimental parameters are defined as follows:

N_0 = total number of particles emitted by the source per unit solid angle per unit time

A_1 = area of the gold annulus (scattering foil)

A_2 = area of the detector

ρ = density of the scattering material

t = thickness of the scattering foil

N_A = Avogadro's number $= 6 \times 10^{23}$

A_g = Atomic weight of scattering material (gold) = 197.2 amu

$n = \left(\frac{\rho N_A}{A_g}\right) A_1 t$ = total number of scattering centers

$N_1 = \Delta\Omega_1 N_0$ = total number of particles reaching A_1 per unit time

N_2 = total number of particles reaching A_2 per unit time(expected rate)

The rate of detected particles N_2 can then be expressed in terms of N_0. From Eq. (1)

$$N_2 = n\frac{d\sigma}{d\Omega}(\theta)\Delta\Omega_2 \times (\text{incident flux}). \tag{3}$$

Since

$$(\text{incident flux}) = \frac{N_1}{A_1 \cos\theta_1} = \frac{N_0 \Delta\Omega_1}{A_1 \cos\theta_1} = N_0/R_1^2\,, \tag{4}$$

we have

$$N_2 = \frac{nN_0}{R_1^2}\frac{d\sigma}{d\Omega}(\theta)\Delta\Omega_2 = \frac{nN_0 A_2 \cos(\theta_2)}{R_1^2 R_2^2}\frac{d\sigma}{d\Omega}(\theta)\,. \tag{5}$$

The scattering angle in this experiment is varied by adjusting the distance Y between the plane of the scattering foil and the detector. We eliminate R_2, from Eq. (5) by making the substitution $R_2 = r_1/\sin\theta_2$, and we substitute Rutherford's formula (Eq. 2) to get

$$\begin{aligned} N_2 &= N_0 \times \left(\frac{nA_2}{R_1^2 r_1^2}\frac{Z^2 z^2 e^4}{16E^2}\right) \times \left(\frac{\cos\theta_2 \sin^2\theta_2}{\sin^4(\theta/2)}\right) \\ &= N_0 \times G \times f(Y), \end{aligned} \tag{6}$$

where $\theta = \theta_1 + \theta_2$ and $Y = R_2 \cos\theta_2 = r_1 \cot\theta_2$.

Advance Preparation

1. Plot the function $f(Y)$ verses Y from $Y = 1$ cm to $Y = 20$ cm.

2. Assuming a scattering foil thickness of 2×10^{-4} cm, calculate a preliminary value for the constant G. What must N_0 be to give enough points, each with a statistical precision of 10%, to check the angular distribution in an hour's counting?

Special Precautions for this Experiment

The source used in this experiment (^{241}Am) is dangerous to a degree which requires licensing. It is very important to understand the dangers involved. The following general information relevant to the ^{241}Am source is essential:

a) In general, the maximum allowable amount of α-emitting material *inside* the human body is lower by orders of magnitude than that of γ, β-emitters. For ^{241}Am, the maximum allowable amount in the kidney, for example, is only 0.01 microcuries.

b) Many α-emitters have chemical properties which cause them to be retained by the human body for a long time. For example, radium is a "bone-seeker" because of its chemical similarity to calcium. Americium has an effective biological half-life of 10 to 20 years depending on the organ involved.

c) As long as the α-emitting material is not ingested, the α particles do not constitute a hazard since they have a range of only about 4 cm in air. Also, skin is not significantly damaged by α particles. Of course, most α-emitters also emit γ-radiation, which may constitute a problem. In the case of ^{241}Am, however the gamma rays have an energy of only 60 kev and are emitted only in 30% of the decays. The danger associated with this radiation is minimal especially if the source is inside the brass tube.

In summary, the source is dangerous only if it is damaged such that pieces of it are scattered around the laboratory and eventually inhaled and ingested. The source can be easily damaged if touched because it has to be very thin for the α-particles to get out.

For your own edification, you should measure the range of ^{241}Am α-particles in air by monitoring the flux from the source as a function of air pressure (as read on the gauge).

Note: many α-sources, but not Americium, have the problem of spontaneously spreading around. This is because the chemical bonds are sufficiently weak so that the recoil resulting from the α-emission frees microscopic pieces of the source. Even more diabolical are radium sources, which emit radon *gas*.

In light of these hazards, the following precautions must be observed:

1. The instructor **must** be present whenever the apparatus is opened.

2. **Do Not Touch the Source with Anything!!**
   As described above, the source is specially protected to prevent radioactive contamination, but the radioactive material could be spread by damaging its surface. There is a **lethal** amount of radioisotope in the source. Therefore, **DO NOT ATTEMPT TO MEASURE THE DISTANCE FROM THE SOURCE TO THE SCATTERING FOIL OR THE SOURCE DIAMETER!**

Also, when venting the chamber, admit air *slowly* to avoid breaking the fragile gold foil.

Source

The amount of ^{241}Am used in this experiment is 100 microcuries. ^{241}Am emits α-particles of several energies, all very close to 5.5 MeV. The radioisotope is uniformly distributed through a gold foil 10^{-4} cm thick, and this is over-coated with 10^{-4} cm of gold and 0.5×10^{-4} cm of palladium to keep the radioactive material from being spread about. Since 5 MeV α-particles lose about 0.5 MeV in 10^{-4} cm of gold, the average energy of the α's coming from this source is 4.4 MeV. The distribution of the Americium through a thick gold foil (10^{-4} cm thick) results in an energy spread of ± 0.25 MeV in the particles coming from the source. To determine the energy of the α's when they are scattered, note that the average rate of energy loss in the scattering foil for α-particles of this energy is 0.3 MeV per milligram/cm^2.

The source strength may be measured by removing the central plug and counting the particles coming directly from the source. (How does the count rate depend on the distance between the source and the detector?)

Scattering Foil

A sheet of gold about 10^{-4} cm thick will be supplied for the scatterer. It should be used double (fold it along a diagonal). The actual thickness can be determined by weighing it – it is advisable to support it on a piece of paper about the same size when you place it on the balance. The foil can be attached to the brass foil holder with silicone grease.

Detector/Pre-amp

This experiment makes use of a solid-state detector (Si surface barrier detector) sensitive to charged particles (and high-energy photons). Each particle detected generates a low-level pulse of charge; the charge is amplified by a charge-sensitive pre-amp.

Electronics

Overview:

The typical electronics associated with nuclear radiation detectors has the basic function of (a) transforming low level pulses issuing from the detector, through amplification and pulse shaping, into a pulse more suitable for measurement and analysis; (b) analyzing the pulse height distribution; and (c) counting the pulses. The requirements for different detectors are sufficiently similar so that standard "NIM" modules in different combinations can be used. A block diagram of the arrangement of components is shown in Figure 2:

1. Linear Amplifier
   This unit amplifies and shapes the pulses from the detector. A linear relationship between the input pulse amplitude and the output amplitude exists as long as the gain is set lower than what is needed to produce an output of $\approx$ 12 Volts. The gain should be sufficient to spread out the pulse height distribution over the operating range of the discriminator (0-10 volts).

2 Discriminator
   This unit allows the selection of pulses, from a *positive* pulse height distribution, which exceed a certain height (set by the E_1 control). Thus, for example, low amplitude noise pulses can be eliminated without

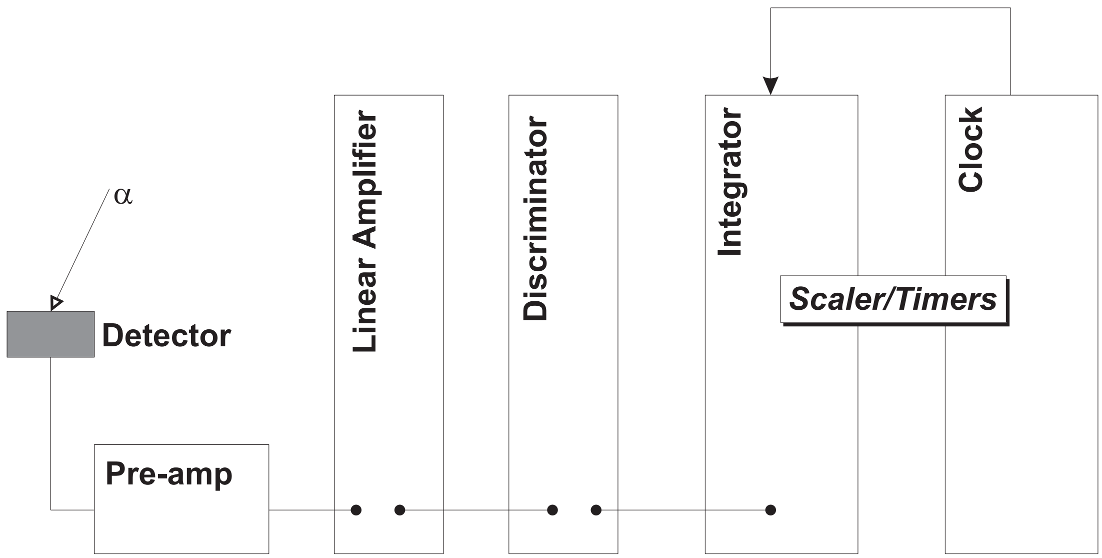

Figure 2: Schematic diagram of the electronics.

affecting the higher amplitude pulses due to α-particles. The output pulses from this unit (E_1 output) all have the same height (i.e., square pulses) and are suitable for driving the scaler.

3 Scaler/Timer
An electronic counter, capable of counting pulses with a rate as high as 10^7 per second. In this experiment, one Scaler/Timer is used as a clock, while the other is used as an integrator. That is, the integrator counts pulses for a fixed period of time determined by the clock settings.

Adjusting The Electronics:

1. **Check the operation of the detector.**

   Observe the output of the amplifier with an oscilloscope. The central plug should be removed during the test to get a high count rate. Note that the polarity switch should be on "inverted" to get positive output pulses. Trigger the oscilloscope *internally* so that the pulse height distribution can be observed. The gain of the amplifier can now be adjusted according to the considerations discussed above.

2. **Check the operation of the discriminator.**

   By triggering the oscilloscope with the output of the discriminator, it is possible to see from the display what part of the pulse height distribution is being selected.

   A good discriminator setting is one for which about 20% of the pulses due to α-particles are rejected. The reason for using a setting for which not only noise pulses but also α-particle pulses are rejected is the following. In the experiment there will be, in addition to α-particles scattered from the gold foil, some α-particles scattered from the walls, which are expected to have, on average, lower energies, as considerable penetration of the wall may occur before and after a nuclear scattering. Also, for light nuclei (copper and zinc) the non-negligible recoil energy results in a reduction of energy of the wall-scattered α's. (Calculate this recoil energy for a 180 deg scattering.) The wall-scattered α's with their lower energies will give smaller pulses, which can be partly eliminated by the discriminator.

Miscellaneous

Poisson Distribution If the expected number of counts in a given counting period is N, then the probability of observing n counts is given by the Poisson distribution:

$$P_N(n) = \frac{N^n e^{-N}}{n!} \tag{7}$$

If the number of counts is not too small, then the expected rate will have a 68% chance (i.e., 1-σ) of falling in the range $n \pm \sqrt{n}$. The percentage error is thus reduced by increasing the number of counts.

The Poisson distribution can be verified for the source used in this experiment by taking many counts with the geometry fixed. This is also a useful check on the functioning of the apparatus: a) a distribution much broader than a Poisson distribution may result from an electrical problem such as a bad connection, b) a systematic drift of the count rate, due to some other problem, may be discovered.

Required Count Rate Measurements Three types of count rate measurements are needed: (a) N_2 as a function of Y; (b) N_0; (c) wall-scattering count rate as a function of Y.

Energy Loss in the Scattering Foil Multiple Scattering of an α-particle with electrons give rise to an energy loss of about 1/2 MeV per 10^{-4} cm of gold. Since the efficiency of the detector depends on the energy of the α's, it is important to have the energy distributions approximately the same in the determination of N_0 and N_2. This can be done by measuring N_0 with the central opening covered with the same thickness of foil as used in the scattering experiment.

In comparing the theory to the experimental results, the energy loss in the scattering foil should be taken into account.

Questions

Is your data also consistent with a theory that predicts a differential cross-section

$$\frac{d\sigma}{d\Omega}(\theta) \sim \frac{1}{\sin^2(\theta/2)}? \tag{8}$$

Can you determine the value of Z from your data?

What upper limit on the nuclear radius do your data imply?

References

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