Compton Effect

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1 Introduction

One of the most important experiments in the early days of quantum mechanics (1923) studied the interaction between light and matter; it determined the change in energy and momentum of light (x-rays photons) when they scatter off electrons. Photons carry momentum. So light (i.e. photons) can exert a force on an object, such as particle, mirror, etc. If the object's momentum changes its energy changes. By conservation of energy there must be an equal and opposite change of energy of the photon. This is known as an inelastic collision—as the photon energy changes.

As the energy of a photon is proportional to its frequency ($E = hf$) its frequency must also change during the collision. Equivalently, the wavelength of the photon must change as $\lambda = c/f$. As the photon transfers energy to the electron its frequency decreases (there is a "red shift") on scattering and thus its wavelength increases.

The figure below shows the basic physics of Compton scattering:

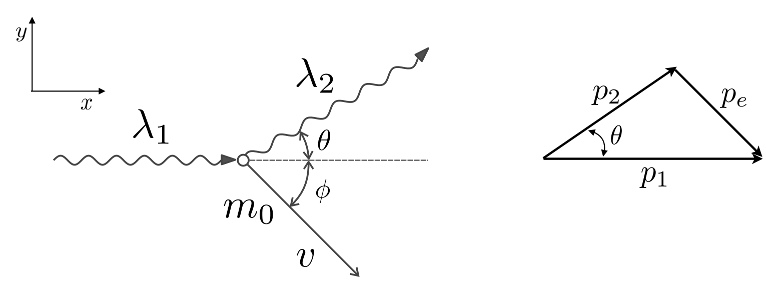

Figure 1: Compton effect. Left: An x-ray photon incident on a particle (an electron in this experiment) is scattered through an angle θ. The initial wavelength of the x-ray is λ_1 and the scattered x-rays wavelength is λ_2 ($\lambda_2 \geq \lambda_1$). The particle, initially at rest, acquires a velocity v in the scattering process. Right: Conservation of momentum in scattering process.

The goal of this experiment is observe the Compton effect and to measure the change in wavelength of x-rays on inelastic scattering from electrons in a solid. Since in the x-ray setup we can only count the number of x-rays per unit time incident on the detector, we need to convert a change in wavelength into a change in x-ray counting rate. This will be done using filters whose attenuation depends on x-ray wavelength. By measuring the change in the count rate we will be able to infer the change in the average x-ray wavelength.

2 Theory

2.1 Compton Effect

To determine the change in wavelength we need to consider the conversation of energy and momentum. We assume that an x-ray photon scatters from a particle with mass m_0, initially at rest. The conservation of energy is:

$$p_1 c + m_0 c^2 = p_2 c + E_e, \tag{1}$$

where the relativistic form of the kinetic energy is used (**Why is the relativistic form used? What is the maximum possible velocity of the electron in terms of the speed of light, i.e. what is** v/c**?**). The magnitude of the photon's momentum is $p = h/\lambda$. So prior to scattering the photon has momentum $p_1 = h/\lambda_1$ and after the scattering event: $p_2 = h/\lambda_2$.

The conservation of momentum (see the right hand side of Fig. 1) is:

$$\vec{p}_e = \vec{p}_1 - \vec{p}_2. \tag{2}$$

As shown in Fig. 1, θ is the angle by which the x-ray scatters and ϕ is the angle the particle moves in after the collision.

After some algebra (see below) one finds:

$$\lambda_2 - \lambda_1 = \frac{h}{m_0 c}(1 - \cos\theta). \tag{3}$$

The constant $h/(m_0 c) = 2.43$ pm is called the Compton wavelength λ_C. This is the wavelength of a photon having an energy equal to the rest energy of the particle (an electron in this experiment). $2\lambda_C$ is the maximum change in wavelength of the photon in the scattering process and corresponds to the photon backscattering ($\theta = 180°$) and transferring the maximum amount of energy and momentum to the particle.

Deriving the Compton formula (Eqn. 3). First take the square of both sides of Eqn. 2. This makes this vector equation an equation for scalars (i.e. just numbers) which are far easier to work with:

$$(\vec{p})_e^2 = (\vec{p_1} - \vec{p_2})^2 \quad (4)$$

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta. \quad (5)$$

Here and throughout the lab notes we use the usual convention that letters without a vector sign, e.g. p_1, represent the magnitude of the vector, e.g. $p_1 = |\vec{p_1}|$. Then take the square of (Eqn. 1):

$$(p_1c - p_2c + m_0c^2)^2 = E_e^2 = (m_0c^2)^2 + (p_ec)^2 \quad (6)$$

Now eliminate p_e in this equation (Eqn. 6) by substituting p_e from Eqn. 5 into Eqn. 6 and divide both sides of the resulting equation by c^2:

$$(p_1 - p_2 + m_0c)^2 = (m_0c)^2 + p_1^2 + p_2^2 - 2p_1p_2\cos\theta \quad (7)$$

Several terms cancel to give:

$$\frac{m_0c}{p_2} - \frac{m_0c}{p_1} = 1 - \cos\theta. \quad (8)$$

Finally, substitute $p_1 = h/\lambda_1$ and $p_2 = h/\lambda_2$ and multiply by $h/(m_0c)$ to give Eqn. 3!

In this experiment x-rays are scattered off an aluminum crystal. As aluminum is a metal its valence electrons (the outermost electrons of Al atoms) behave as nearly free electrons. We thus expect x-rays to scattering inelastically off the valence electrons in the Al.

3 Experiment

In this experiment the attenuation of unscattered x-ray radiation passing through a copper foil is compared to that of radiation scattered from aluminum. The transmission T_{Cu} of the copper foil depends on the wavelength of the x-rays (see Fig. 2). Therefore, a shift in the wavelength of the x-radiation due to Compton scattering induces a change in the transmission and thus x-ray counting rate. In other words, the average x-ray transmission through the Cu foil is used to determine the average x-ray wavelength both before and after scattering from the aluminum. The average change in x-ray wavelength as a function of scattering angle is compared to theoretical expectation for Compton scattering (Eqn. 3).

The transmission of the copper foil depends on wavelength as follows:

$$T_{\mathrm{Cu}} = \exp\left[-a\left(\frac{\lambda}{\lambda_0}\right)^n\right] \quad (9)$$

with $a = 7.6$ and $n = 2.75$ and $\lambda_0 = 0.1$ nm.

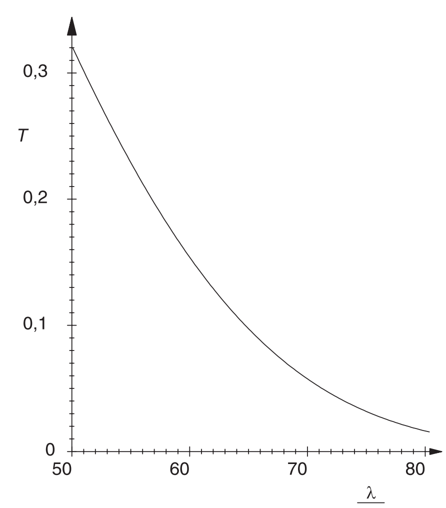

Figure 2: Transmission of a copper foil (d = 0.07 mm) in the wavelength range from 50 to 80 pm. Figure from Leybold physics leaflet P6.3.7.1.

Four different counting rates are measured to determine the wavelength shift:

1. R_0: The counting rate when the x-rays are scattered from the Al without being attenuated.
2. R_1: The counting rate when the x-rays are attenuated by a Cu filter *before* scattering from the Al.
3. R_2: The counting rate when the x-rays are attenuated by a Cu filter *after* scattering from the Al.
4. R_B: The background x-ray counting rate; the count rate the detector measures when the x-ray tube is *off*.

With these measurements one can then compute the average λ_1 from the transmission of the x-rays when they are attenuated before scattering from the Al:

$$T_1 = \frac{R_1 - R_B}{R_0 - R_B}. \tag{10}$$

One can also compute the average λ_2 from the transmission of the x-rays when they are attenuated after scattering from the Al:

$$T_2 = \frac{R_2 - R_B}{R_0 - R_B}. \tag{11}$$

(**Why is it necessary to include the background x-ray counting rates in the above expressions?**) One then can rearrange Eqn. 9 to determine the average wavelength before and after scattering in terms of the measured x-ray transmission rate:

$$\lambda = \lambda_0 \left(-\frac{\ln(T)}{a} \right)^{1/n}. \tag{12}$$

3.1 Setup

The main components of the x-ray experiment are:

- X-ray tube (left side of instrument)
- X-ray optics: slits to collimate and filter the x-rays
- Goniometer: computer controlled sample positioner
- X-ray detector
- Samples: single crystals of NaCl, LiF and Al

Figure. 3 has a schematic of the x-ray setup. A source of x-rays is collimated with slit 1. The x-rays strike the sample 2 and then pass through another collimating slit before begin detected an x-ray counter 3. A more detailed view of the setup is shown in Fig. 4.

Caution: X-rays are also not good for the human body, as they can ionize matter and lead to cell damage. In this experiment the x-rays are contained in the apparatus using lead lined glass and other materials that absorb the x-rays. Of course, care must be taken to ensure that the glass window is closed during the measurements, and that you are not in close proximity to the unit when it is running. The unit should also be off when you are not taking data.

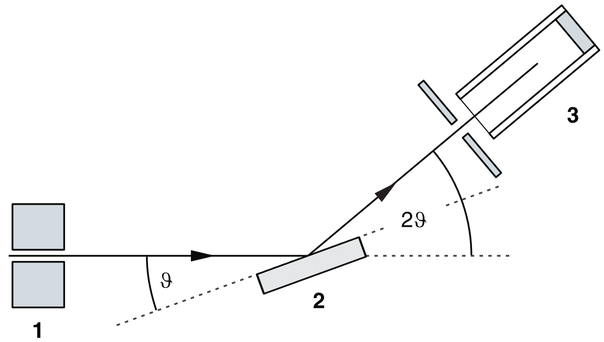

Figure 3: Diagram showing the basic components of the x-ray experiment. 1. Collimator, 2. Sample and 3. Detector slit and x-ray counter. In this experiment the incident and scattered angle are not necessarily equal.

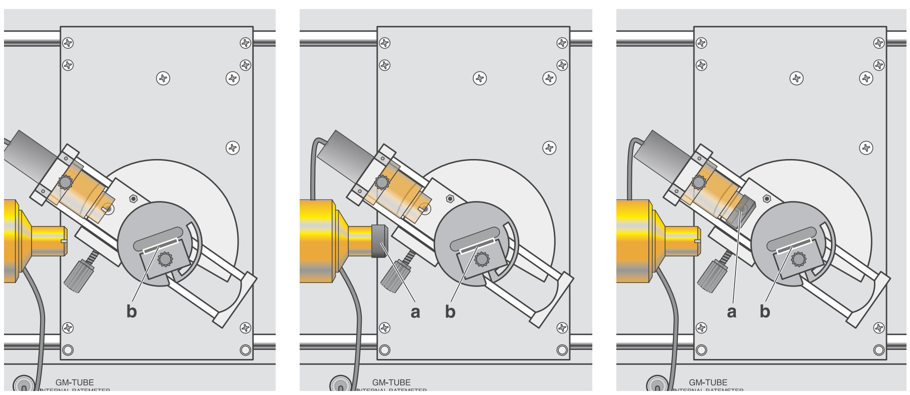

Figure 4: Experimental setup for Compton scattering. **Left**: without Copper filter, **Center**: with Cu filter (a) in front of Al scattering body (b), **Right**: with Cu filter behind the Al scattering body (b). Figure from Leybold leaflet P6.3.3.7.

3.2 Setup

Set up the experiment as shown in Fig. 4.

Carry out the following steps:

- Remove the collimator and mount the zirconium filter supplied with the x-ray apparatus on the radiation inlet aperture side of the collimator.
- Mount the collimator (in the correct orientation!) together with the zirconium filter.
- If necessary, mount the goniometer and end window counter (see the Instruction Sheet of the x-ray apparatus on the lab website).
- Set a distance between the collimator and target to about 9 cm and a distance between the target and the detector also to approximately 6 cm.
- Mount the aluminum scattering body from the Compton accessory x-ray as the target.

The Al should be pressed firmly against a ridge in the sample holder to be at proper height.

- Turn on the x-ray apparatus!
- Press the TARGET key and manually set the target angle to 20° by using the ADJUST knob.
- Press the SENSOR key and manually set the sensor angle to 120° by using the ADJUST knob.
- Gradually and carefully increase the sensor angle (steps of 5°) to 140° watching to be sure the GM tube does not touch the inner left side of the instrument—which would damage the sensor.

3.3 Carrying out the experiment

- Set the tube high voltage $V_T = 30$ kV and the emission current $I = 1.00$ mA.
- Set the angular step width $\Delta\theta = 0.0$; you will not be varying the scattering angles in the next series of steps, only counting x-rays at a fixed scattering angle.

a) Without the copper filter

- Set the measuring time per angular step to $t = 60$ s.
- Start the measurement with the SCAN key and display the mean counting rate R after the measuring time elapses by pressing REPLAY. Record the result as counting rate R_0.

b) With the copper filter in front of the Al scatterer

- Place the copper filter on the collimator.
- Increase the measuring time per angular step to $t = 300$ s. (You can also use $t = 600$ s to get better statistics.)
- Start the measurement with the SCAN key and display the mean counting rate R after the measuring time elapses by pressing REPLAY. Record the result as counting rate R_1.

c) With copper filter behind the Al scatterer

- Mount the copper filter on the sensor seat.
- Start the measurement with the SCAN key and display the mean counting rate R after the measuring time elapses by pressing REPLAY. Record the result as counting rate R_2.

d) Background counts

- Set the emission current $I = 0$.
- Start the measurement with the SCAN key and display the mean counting rate R after the measuring time elapses by pressing REPLAY. Record the result as counting rate R_B.

3.4 Results and further studies

From the data you have taken compute T_1 and T_2. Use the results to find the average wavelength shift on scattering from the Al body. Compare you results with the theoretical expectations. Do you they agree? If not, what are possible reasons for the discrepancy?

In addition, vary the scattering angles (at least two additional angles) and repeat the above experiment. The aim of this study is to determine the angular dependence of the average wavelength shift and compare it to that expected for Compton scattering, Eqn. 3. **Determine the wavelength shift for these additional angles and plot the wavelength shift versus the scattering angle θ. Plot the theoretically expected wavelength shift as a function of angle on the same graphic. Do the results agree? If not, what are possible reasons for the discrepancy?**

There are several other experiments that you can try. Here are some suggestions, but please feel free to propose and conduct your own experiment! Conduct at least one more experiment that uses the x-ray apparatus.

- Determine the average wavelength shift when a Bragg condition is satisfied for x-ray scattering from Al. Note that to find the Bragg condition you need to conduct a angle scan and also set the angle of incident to be equal to the angle of reflection, using the "COUPLED" mode of the setup. **How does the average wavelength shift compare to those of your previous studies? Is the wavelength shift smaller or larger and why?**

- Determine the average wavelength of the x-rays as a function of the tube voltage V_T. Are the results what you expect? Why or why not?

- Repeat the experiment without the copper filter 10 times and determine the standard deviation of the counting rate. Does your resulting standard deviation agree with that expectations for a Poisson distribution, $\sigma_R = \sqrt{R/t_c}$

4 Finishing Up

Turn off the x-ray apparatus and remove the crystals and place it back in plastic bin with the other components of the experiment.

5 Final Note

Include an analysis of the uncertainties in your determination of the wavelength shift in your lab reports. Also, answer all the questions in this lab writeup in your laboratory reports!