Review of "Introduction to the Replica Theory of Disordered Statistical Systems", by Viktor Dotsenko

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Understanding statistical systems with quenched (i.e., frozen-in) disorder has proven surprisingly challenging. The "replica method", proposed by Mark Kac, remains one of the few successful theoretical tools for analyzing these systems. The method involves "creating" multiple copies of the system, averaging over the disorder, and then taking the number of copies to zero. But this turns out to be the least bizarre part.

The proving ground for the replica method was the spin glass, a disordered magnetic system in which the local couplings vary randomly in sign (and possibly magnitude). In a landmark paper, Sam Edwards and Philip Anderson boiled the problem down to its physical essentials. But even David Sherrington's and Scott Kirkpatrick's further simplification to an infinite-ranged model proved surprisingly difficult to treat. What is now believed to be its correct solution — the replica symmetry breaking (RSB) approach of Giorgio Parisi — took several years to appear and another several years to be interpreted.

The idea of breaking the replica symmetry (i.e., assuming that, somehow, different copies of the system should not be regarded as identical) was originally suggested by the work of Jairo de Almeida and David Thouless, but Parisi identified the correct pattern of RSB for the infinite-ranged model. His solution's eventual interpretation, as a new type of broken symmetry with an exotic arrangement of numerous low-temperature phases, generated widespread excitement.

While a description of the RSB approach has appeared in several excellent reviews and books, those have been devoted to the larger subject of spin glasses, and replica theory is but one part of that story. In contrast, Viktor Dotsenko's "Introduction to the Replica Theory of Disordered Statistical Systems" is entirely devoted to replica theory, from the point of view that the spin glass is but one part of *that* story.

Regardless of one's opinions on this last point, a knowledge of the replica approach and its technical machinery is indispensable for anyone aspiring to work in the field. In this regard, Dotsenko's book is a welcome addition.

The book is divided into three parts; the first covers spin glasses (mostly infinite-ranged models), the second the effects of quenched disorder on critical phenomena, and the third applications of RSB to other systems. The book's principal strength is that it provides a clear introduction to the mathematical machinery of RSB, accessible to an advanced graduate

student interested in working in the statistical mechanics of disordered systems. For this reason alone it should be part of the library of any theorist working in these areas. At the same time, it falls into the trap (particularly in the second and third parts) of descending into a heavily technical morass of detailed calculations that assume the reader already has a sophisticated statistical mechanical background. Dotsenko seems unsure whether his book intended for a beginner or an expert.

Along with inconsistency in the level of sophistication, the text's other problem is perhaps that of the RSB approach itself: It is quite good when it discusses mathematical procedures and algorithms, but obscure when it tries to interpret what these calculations mean physically. (At this point some truth-in-advertising is required: my collaborator Charles Newman and I have been vocal critics of traditional interpretations of RSB theory, and of attempts to apply it to more realistic spin glass models. For this reason I had particular problems with the book's discussion of the physical interpretation of RSB. But this perhaps is less the fault of the author than of the field — it has spawned a number of "urban legends" that are at best murky, often misleading, and sometimes simply incorrect.)

Dotsenko states in the preface, perhaps wisely, that he will avoid the contentious issue of the applicability of RSB theory to realistic spin glasses. But he's not completely consistent on this point. His discussions of aging experiments, for example, are interpreted solely from the RSB viewpoint. Such a presentation is fine, given the book's mission; but someone new to the field might be misled into thinking that these interpretations are the only game in town. (The Preface also notes that the scaling approach of William McMillan, Alan Bray and Michael Moore, and Daniel Fisher and David Huse, will not be discussed. Why not throw this assertion to the wind as well, and at least acknowledge that some influential theoretical studies of aging start from this very different picture of the spin glass phase?)

I would also like to have seen a few more areas covered. Mean-field dynamics is barely discussed. Some of the more recent applications of RSB theory, for example to quantum spin glasses, are missing entirely. I hope these can be included in an expanded second edition.

However, I think the book's biggest shortcoming is its bibliography. An introductory treatment should at least have an extensive, if not exhaustive, list of references. But the bibliography is highly selective and often very incomplete. Just two (of many) examples: the discussion of lower critical dimension of random field magnets neglects to mention the contribution of Michael Aizenman and Jan Wehr, and the Griffiths phase is covered without mentioning the work of Deepak Dhar and Mohit Randeria, James Sethna, and Richard Palmer. This problem recurs throughout the book. Fortunately, it is a shortcoming that is easily corrected in a second edition.

The final chapter returns to the difficult problem of the low temperature phase (assuming there is one) of real spin glasses: is RSB relevant or not? A book review is perhaps not the place to address an important issue theoretically — if not rigorously — avoided by the book itself. Dotsenko closes by asserting his belief in the affirmative; for myself, I believe that "interesting times" have arrived for RSB. But this does not diminish my appreciation of the book's value. RSB theory is a stunning achievement, even if more limited in applicability than some hope; and Dotsenko's book will regardless remain an important contribution.