Physics envy in psychology: A cautionary tale

CCNY Physics Interdisciplinary Seminar
November 19, 2015

Alan Sokal

New York University and University College London

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A sociologist might say that this quote shows what is wrong with economists: they want a subject that is fundamentally about human beings to have the mathematical certainty of the hard sciences …

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Economics is *harder* than physics; luckily it is not quite as hard as sociology.

November 2011: An e-mail out of the blue

From: Nick Brown <u1109621@uel.ac.uk>
Date: Wed, 30 Nov 2011 01:34:32 +0100
Subject: Possible "intellectual impostures" in a key paper
To: sokal@nyu.edu

Dear Professor Sokal,

Please excuse me writing to you spontaneously like this. My name is Nick Brown and I am a student on the Masters in Applied Positive Psychology (MAPP) course at the University of East London, in England.

An e-mail out of the blue (2)

I am writing to you because I read your book "Intellectual Impostures" some years ago and I think that I may have found a related sort of case, although here the presumed abuse is in the field of psychology … and is in a peer-reviewed journal.

The paper to which I am referring is "Positive Affect and the Complex Dynamics of Human Flourishing", by Barbara Fredrickson and Marcial Losada, *American Psychologist* 60 (2005) 678–686.

An e-mail out of the blue (3)

This is the paper in which Fredrickson introduced the idea of an ideal positive-to-negative emotion ratio of 3:1, or more precisely, 2.9013:1. She went on to popularise it in a general-readership book, *Positivity* … Fredrickson & Losada (2005) is one of the most quoted papers in the new field of positive psychology.

[This paper] derives most of its legitimacy by copying ideas from "The complex dynamics of high performance teams", by M. Losada, *Mathematical and Computer Modelling* 30 (1999) 179–192. … The 1999 paper seems to have a number of issues, even from my uninitiated standpoint.

An e-mail out of the blue (4)

Two pages of detailed mathematical critique follow . . .

(from a self-proclaimed mathematical novice!)

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Now, here's my problem. I am just this grad student with no qualifications or credentials, starting out in the field. I don't know how to express this kind of idea especially coherently in academic written form, and I suspect that even if I did, it would be unlikely to be published. …

On the other hand, I don't think that I'm a crank, and this is starting to bug me. … I would therefore very much appreciate it if you could give me some advice on how to proceed.

Fredrickson & Losada 2005

Positive Affect and the Complex Dynamics of Human Flourishing

Barbara L. Fredrickson *University of Michigan*
Marcial F. Losada *Universidade Católica de Brasília*

*Extending B. L. Fredrickson's (1998) broaden-and-build theory of positive emotions and M. Losada's (1999) nonlinear dynamics model of team performance, the authors predict that a ratio of positive to negative affect at or above 2.9 will characterize individuals in flourishing mental health. Participants (*N *= 188) completed an initial survey to identify flourishing mental health and then provided daily reports of experienced positive and negative emotions over 28 days. Results showed that the mean ratio of positive to negative affect was above 2.9 for individuals classified as flourishing and below that threshold for those not flourishing. Together with other evidence, these findings suggest that a set of general mathematical principles may describe the relations between positive affect and human flourishing.*

Keywords: nonlinear systems, emotions, broaden-and-build theory, positive psychology, subjective well-being

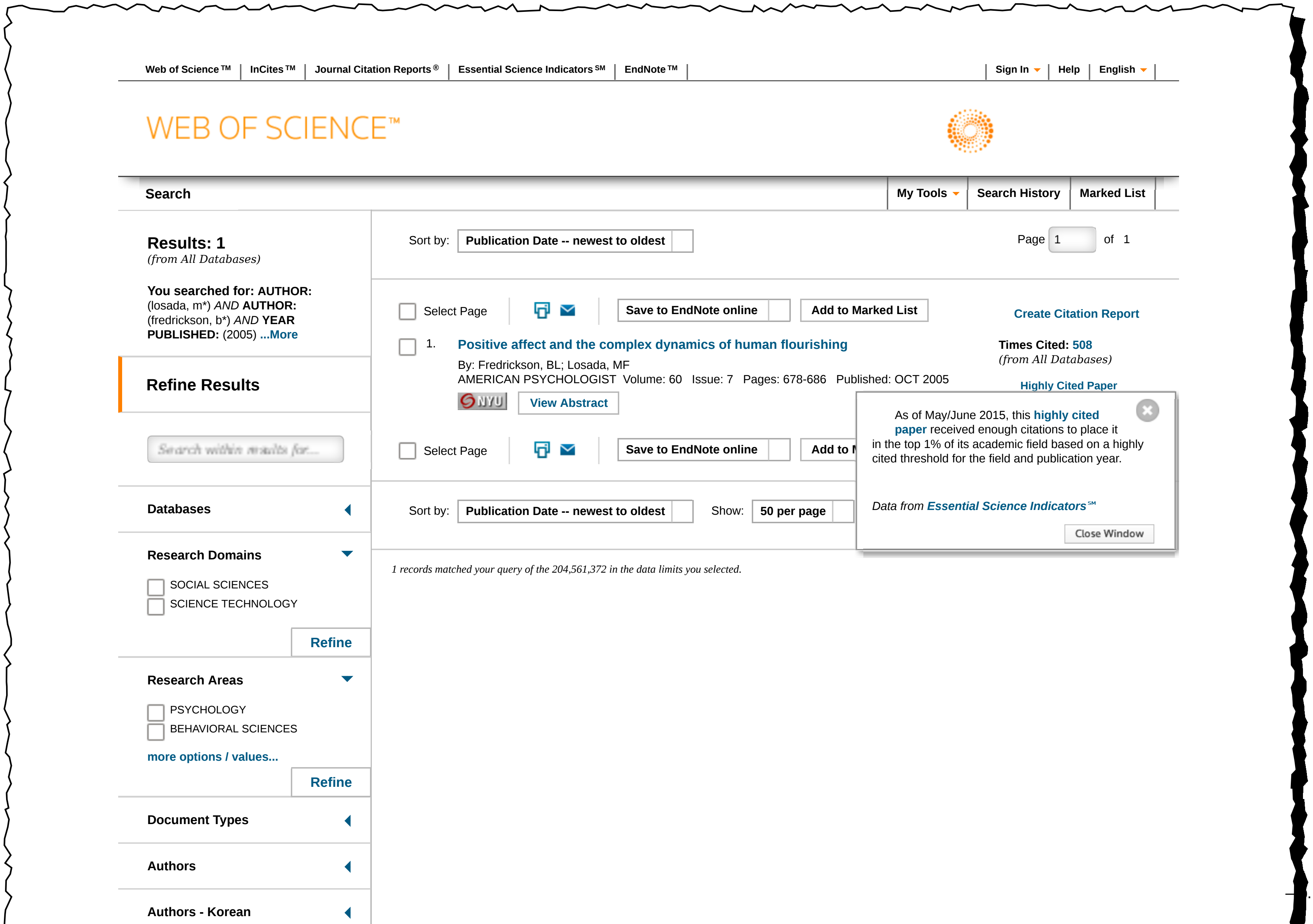
To *flourish* means to live within an optimal range of human functioning, one that connotes goodness, generativity, growth, and resilience. This definition builds on path-breaking work that measures mental health in positive terms rather than by the absence of mental illness (Keyes, 2002). Flourishing contrasts not just with

expressing appreciation, liking) and *negative affect* and *negativity* representing the unpleasant end (e.g., feeling contemptuous, irritable; expressing disdain, disliking). The affective texture of a person's life—or of a given relationship or group—can be represented by its *positivity ratio*, the ratio of pleasant feelings and sentiments to unpleasant ones over time. Past research has shown that for individuals, this ratio predicts subjective well-being (Diener, 2000; Kahneman, 1999). Pushing further, we hypothesize that—for individuals, relationships, and teams—positivity ratios that meet or exceed a certain threshold characterize human flourishing. Although both negative and positive affect can produce adaptive and maladaptive outcomes, a review of the benefits of positive affect provides a particularly useful backdrop for our theorizing.

Benefits of Positive Affect: Empirical Evidence

A wide spectrum of empirical evidence documents the adaptive value of positive affect (for a review, see Lyubomirsky, King, & Diener, in press). Beyond their pleasant subjective feel, positive emotions, positive moods, and positive sentiments carry multiple, interrelated benefits. First, these good feelings alter people's mindsets: Experi-

Fredrickson & Losada 2005



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- The *same* critical positivity ratios hold for individuals, couples, and groups of arbitrary size.

Fredrickson & Losada 2005: What do they claim? (2)

and testing. Uniting existing theory on positive emotion (Fredrickson, 1998, 2001) with the mathematics of nonlinear dynamics (Hirsch et al., 2004; Lai & Ye, 2003; Losada, 1999), we make the following seven predictions:

1. Human flourishing and languishing can be represented by a set of mathematical equations drawn from the Lorenz system.

2. The positivity ratio that bifurcates phase space between the limit cycle of languishing and the complex dynamics of flourishing is 2.9.

3. Positivity ratios at or above 2.9 are associated with human flourishing. Flourishing is associated with dynamics that are nonrepetitive, innovative, highly flexible, and dynamically stable; that is, they represent the complex order of chaos, not the rigidity of limit cycles and point attractors.

4. Human flourishing at larger scales (e.g., groups) shows a similar structure and process to human flourishing at smaller scales (e.g., individuals).

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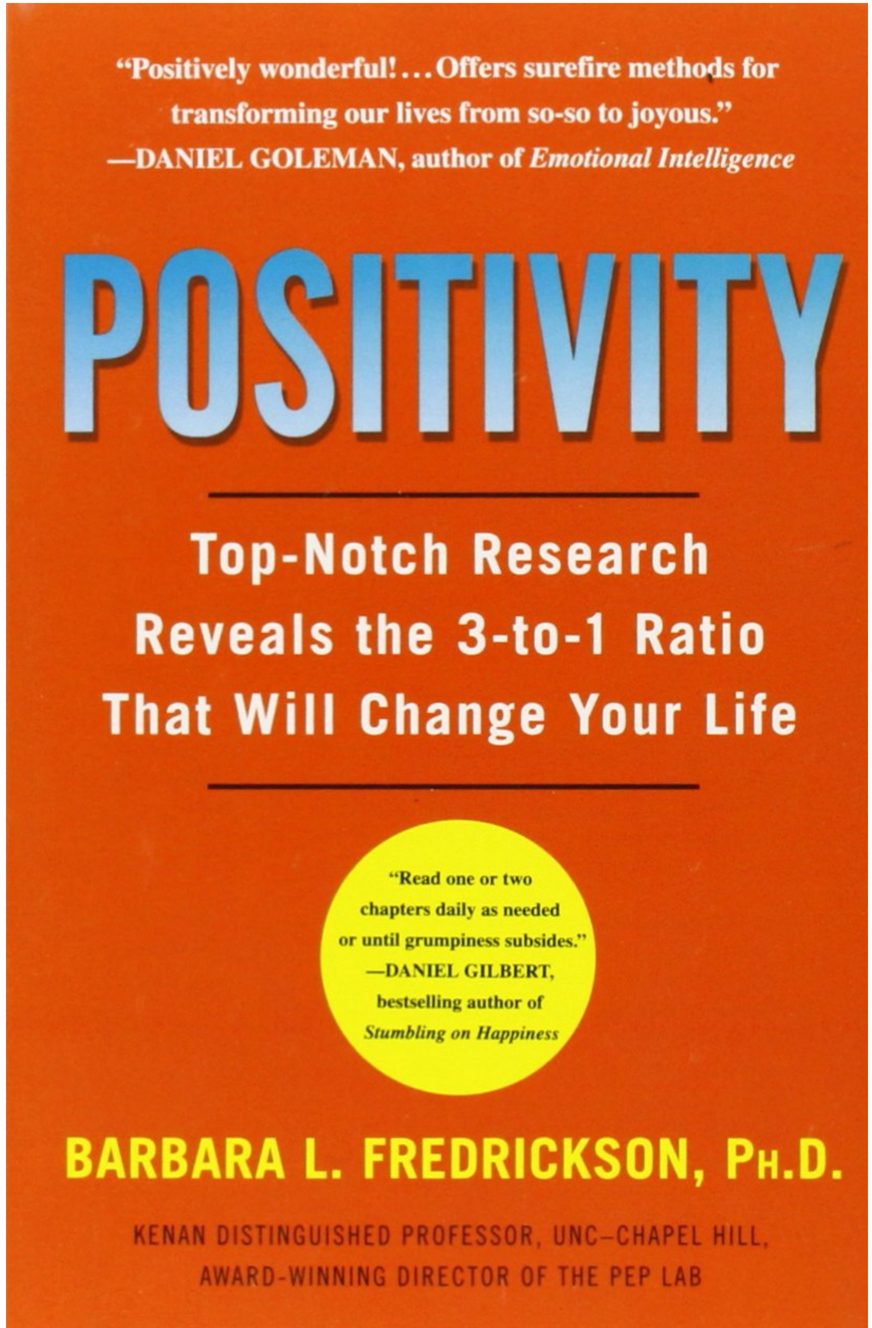
“Our discovery of the critical 2.9 positivity ratio may represent a breakthrough.”

Fredrickson 2009



Fredrickson 2009

CHAPTER 7

The Positivity Ratio

People think angels fly because they have wings.
Angels fly because they take themselves lightly.
—ANONYMOUS

In chapter 1, when I introduced the positivity ratio, I said it had a "tipping point." What exactly does this mean? What's a tipping point?

The best way to explain it might be to remind you of a tipping point you know well already. Consider ice and water. Look at these familiar and indispensable substances of life with fresh eyes. At one level they seem dramatically different. Ice is solid, rigid, and immobile. Water is liquid, flowing, flexible, and dynamic. Yet here's the marvel: to change one into the other simply requires a change in temperature. If you raise the ambient temperature above zero degrees Celsius, rigid ice melts into flowing water.

It's hardly magic, at least to most grown-ups. We know ice and water are chemically the same. Both are H_2O, two parts hydrogen and one part oxygen. But this common chemical compound is subject to a simple tipping point. You can change it from one state to another—from solid to liquid—by changing its temperature.

The differences between languishing and flourishing seem to show similar properties. If we "warm up" the emotional climate of your life by increasing your positivity ratio above the critical tipping point, you'll begin to flourish. Just as zero degrees Celsius is a special number in thermodynamics, the 3-to-1 positivity ratio may well be a magic number in human psychology.

Of course, there's nothing supernatural here, no real "magic." Even so, I do see reason for awe. The world obeys universal natural laws, and sometimes these laws are shockingly simple. Human psychology—complex as it is—may be no different. Perhaps we too are subject to universal laws that have never before been articulated. These laws may map out an escape from the rigid and confining ice block of languishing. They may equip us to find our way to the more flowing, flexible, and dynamic life of flourishing.

I'm not asking you to accept my claim on faith. Instead, I'd like you to appreciate it based on the supporting scientific evidence. In this chapter, I describe how that evidence came together for me.

Match Made

The origin of the positivity ratio begins with my good friend and University of Michigan colleague, Jane Dutton, an endowed professor at Michigan's Ross School of Business. Jane, a cutting-edge scholar of relationships in the workplace,[1] is also a self-described matchmaker, but she doesn't connect lonely hearts; she connects people with promising, interrelated ideas. She'd connected me to soon-to-be collaborators in the past, so I'd come to trust her intuition.

Early in 2003 I received an e-mail from Marcial Losada. He said he'd developed a mathematical model—based on nonlinear dynamics—of my broaden-and-build theory and that we should talk. It turned out that Jane, having seen several possible points of connection

Fredrickson 2009

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1963: The Lorenz equations

Deterministic Nonperiodic Flow[1]

EDWARD N. LORENZ

Massachusetts Institute of Technology

(Manuscript received 18 November 1962, in revised form 7 January 1963)

ABSTRACT

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

1. Introduction

Certain hydrodynamical systems exhibit steady-state flow patterns, while others oscillate in a regular periodic fashion. Still others vary in an irregular, seemingly haphazard manner, and, even when observed for long periods of time, do not appear to repeat their previous history.

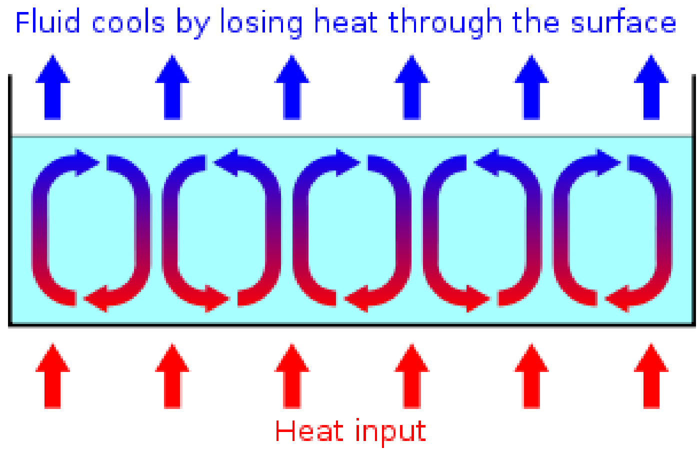
These modes of behavior may all be observed in the familiar rotating-basin experiments, described by Fultz, *et al.* (1959) and Hide (1958). In these experiments, a cylindrical vessel containing water is rotated about its

Thus there are occasions when more than the statistics of irregular flow are of very real concern.

In this study we shall work with systems of deterministic equations which are idealizations of hydrodynamical systems. We shall be interested principally in nonperiodic solutions, i.e., solutions which never repeat their past history exactly, and where all approximate repetitions are of finite duration. Thus we shall be involved with the ultimate behavior of the solutions, as opposed to the transient behavior associated with arbitrary initial conditions.

The Lorenz equations (2)

Rayleigh–Bénard convection



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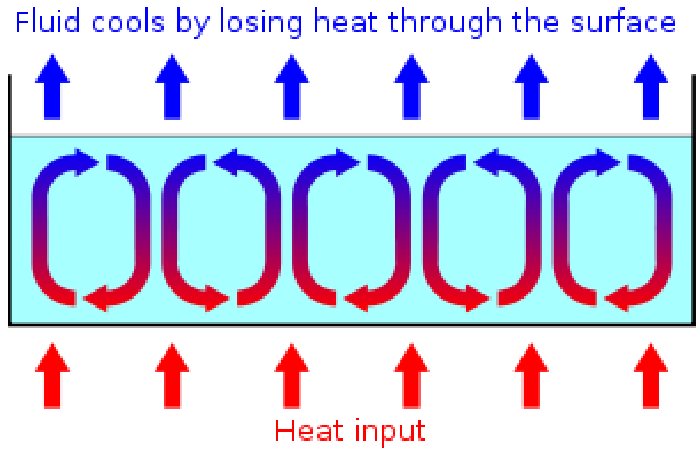

- Henri Bénard (1901 PhD thesis): "Les tourbillons cellulaires dans une nappe liquide propageant de la chaleur par convection en régime permanent"

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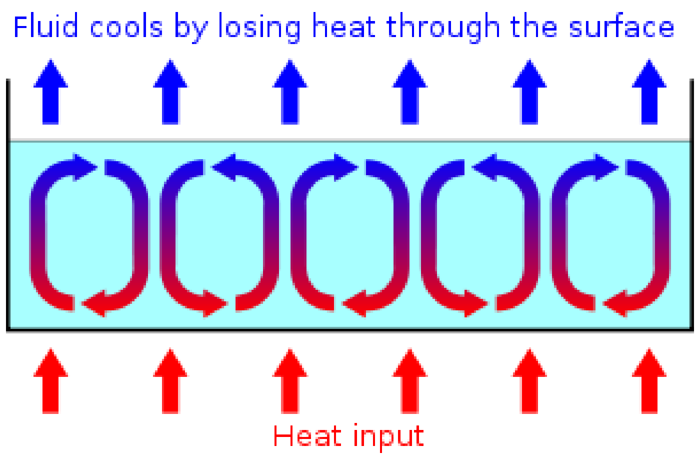

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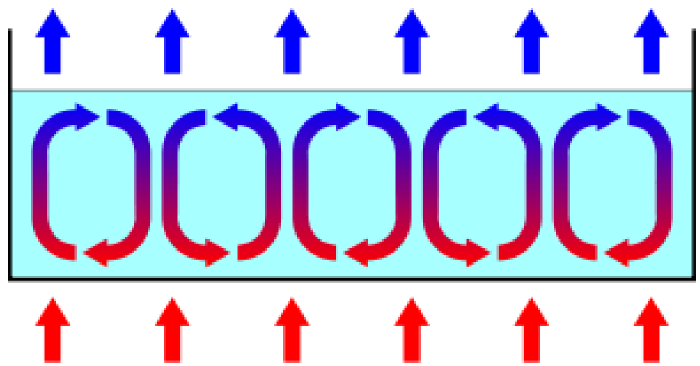

- Henri Bénard (1901 PhD thesis): "Cellular vortices in a liquid layer propagating heat by convection in steady state"

- John William Strutt, aka the 3rd Baron Rayleigh (1916): "On convection currents in a horizontal layer of fluid, when the higher temperature is on the under side"

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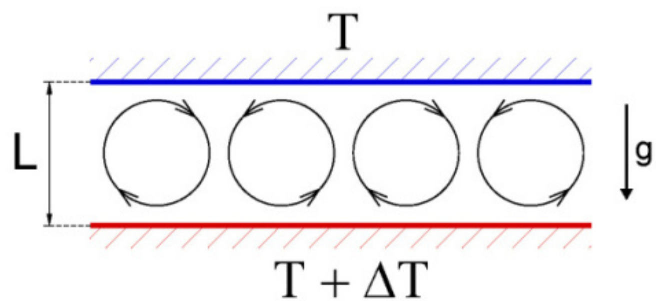

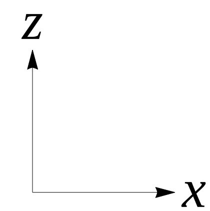

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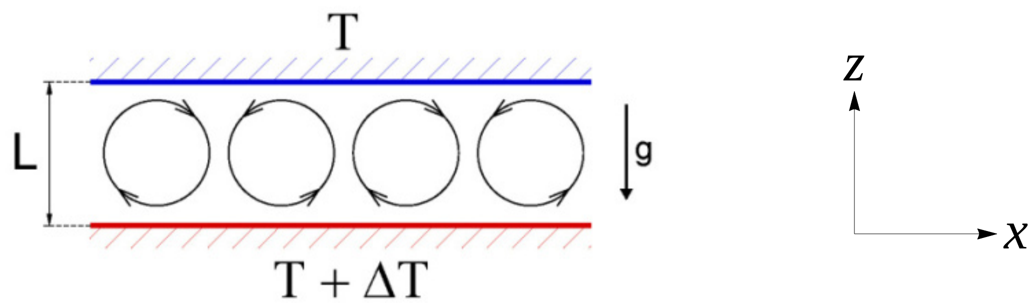

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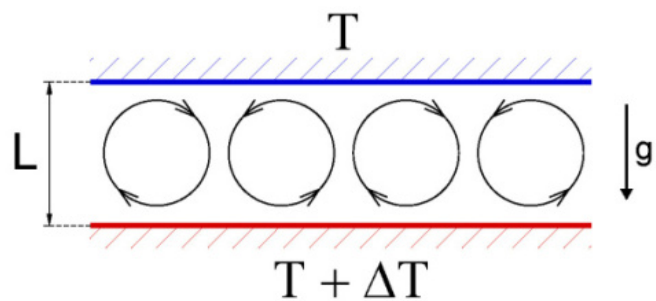

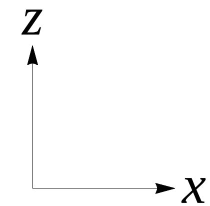

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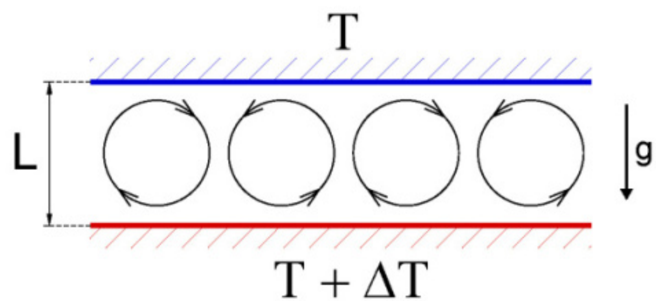
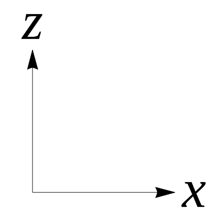
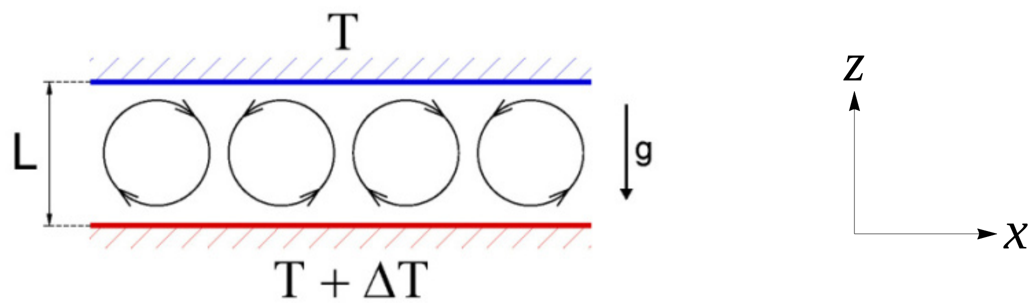




- Rayleigh (1916) wrote the partial differential equations governing the (two-dimensional) flow and temperature gradients.
- There is a steady-state solution in which there is no flow, and temperature varies linearly with depth.
- But this solution is *unstable* if ΔT exceeds a certain critical value. Then convection occurs.

The Lorenz equations (4)

Rayleigh–Bénard convection



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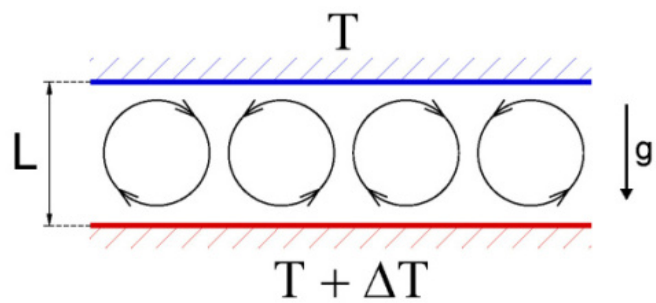

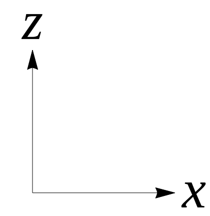

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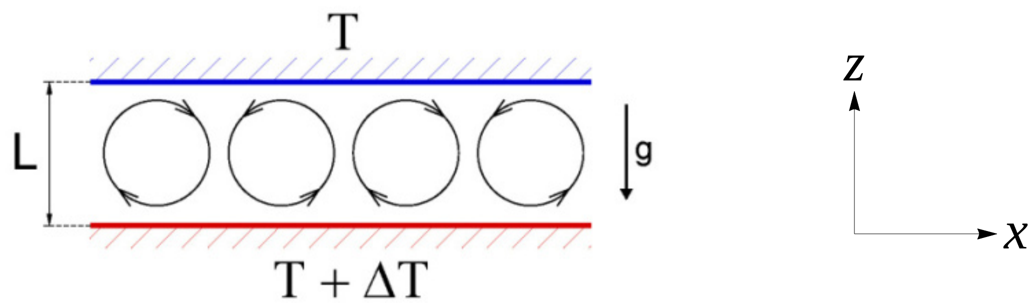

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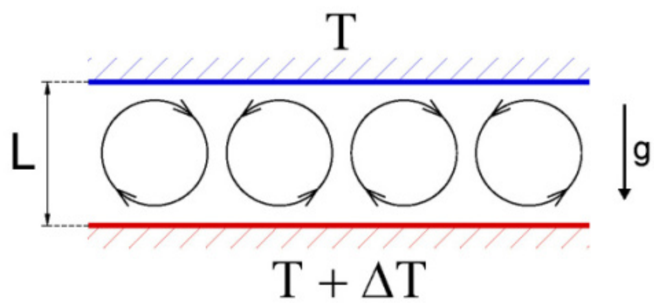

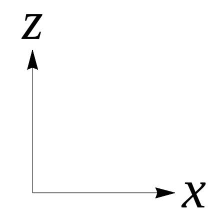

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- These three variables undergo highly irregular fluctuations (which appear to be non-periodic).

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$$\begin{aligned}\frac{dX}{d\tau} &= -\sigma X + \sigma Y \\ \frac{dY}{d\tau} &= rX - Y - XZ \\ \frac{dZ}{d\tau} &= -bZ + XY\end{aligned}$$

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- τ is dimensionless and is proportional to time t.
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- σ, b, r are dimensionless parameters. In particular, $r \propto \Delta T$ measures the strength of the tendency to develop convection.

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Solution:

I will do my best to explain these concepts (briefly) to you.

A crash course in differential equations for non-experts

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- One or more dependent variables $x_1, x_2, \ldots, x_n$
- *Evolve deterministically* as a function of time (t)
- The *rate of change* of each variable at each moment of time is a *known function* of the values of the variables at *that same moment of time*.

A crash course in differential equations (2)

Simplest case: *One* dependent variable x

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- x is assumed to *vary smoothly* as a function of t
- Calculus defines the *rate of change* of x, written dx/dt
- A (first-order) *differential equation* for the function $x(t)$ is an equation

$$\frac{dx}{dt} = F(x)$$

where F is a *known* (i.e. explicitly specified) function

A crash course in differential equations (3)

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(DE4) The rate of change of x at any given moment of time *depends only on the value of x itself* (i.e. not some additional variables), and only on the value of x at *that same moment of time* (i.e. not values in the past).

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(DE5) The rate of change of x at time t is exactly $F(x(t))$, where F is an *explicitly specified function*.

A crash course in differential equations (4)

Ex. 1: Bank account with continuously compounded interest

A crash course in differential equations (4)

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- Amount of money in account (x) increases with time (t) according to

$$\frac{dx}{dt} = rx$$

where r is the interest rate.

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- Has simple solution: $x(t) = x_0\, e^{rt}$ where $x_0 =$ account balance at time 0

A crash course in differential equations (4)

Ex. 1: Bank account with continuously compounded interest

- General principle: Solution $x(t)$ of differential equation is completely determined by the *initial conditions* (i.e. values of dependent variables at time 0)

A crash course in differential equations (4)

Ex. 1: Bank account with continuously compounded interest

- General principle: Solution $x(t)$ of differential equation is completely determined by the *initial conditions* (i.e. values of dependent variables at time 0)
- Usually the solution cannot be written down explicitly
- But it can be studied numerically by computer

A crash course in differential equations (5)

Ex. 2: Population biology

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- Population x of some species living in a limited territory
- Maximum sustainable population $X_{\max}$

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$$\frac{dx}{dt} = rx\left(1 - \frac{x}{X_{\max}}\right)$$

- This is a *nonlinear* differential equation
- Possible objection: Population is not a continuous variable.
- Answer: DE is a valid approximation if (and only if) the population is *large*.

A crash course in differential equations (6)

General case: *Several* dependent variables $x_1, \ldots, x_n$

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General case: *Several* dependent variables $x_1, \ldots, x_n$

- Independent variable t ("time") and dependent variables $x_1, \ldots, x_n$ are treated as *continuous* quantities
- $x_1, \ldots, x_n$ are assumed to *vary smoothly* as a function of t
- A *system of* (first-order) *differential equations* for the functions $x_1(t), \ldots, x_n(t)$ is a system of equations

$$\begin{aligned} \frac{dx_1}{dt} &= F_1(x_1, \ldots, x_n) \\ &\vdots \\ \frac{dx_n}{dt} &= F_n(x_1, \ldots, x_n) \end{aligned}$$

where $F_1, \ldots, F_n$ are specified functions.

A crash course in differential equations (7)

Ex. 3: Lorenz equations for Rayleigh–Bénard convection

A crash course in differential equations (7)

Ex. 3: Lorenz equations for Rayleigh–Bénard convection

$$\begin{aligned}
\frac{dX}{d\tau} &= -\sigma X + \sigma Y \\
\frac{dY}{d\tau} &= rX - Y - XZ \\
\frac{dZ}{d\tau} &= -bZ + XY
\end{aligned}$$

This is a system of differential equations with *three* dependent variables X, Y, Z.

Nonlinear dynamics and chaos

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Example: Double pendulum

The Lorenz equations, revisited

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We can use the Lorenz equations to illustrate some concepts from nonlinear dynamics.

- There is a *fixed point* at $X = Y = Z = 0$.
- Physical interpretation: Fluid at rest.
- It is *stable* if $r < 1$, *unstable* if $r > 1$.

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- For $r > 1$ there is another pair of fixed points, at $X = Y = \pm\sqrt{b(r-1)}, \quad Z = r - 1.$
- Physical interpretation: Steady-state convective flow.
- They are *stable* for $r < r_{\mathrm{crit}}$ and *unstable* for $r > r_{\mathrm{crit}}$, where $r_{\mathrm{crit}} = \sigma(\sigma + b + 3)/(\sigma - b - 1)$ [here we assume $\sigma > b + 1$].

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- Lorenz attractor is a *fractal*: neither 2-dimensional (a surface) nor 3-dimensional (a volume) but something in-between.
- Trajectories near the Lorenz attractor exhibit *sensitive dependence to initial conditions*.

When can differential equations validly be applied?

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(VA5) *Find the specific differential equation* giving (at least approximately) that evolution.

The trail leading to Fredrickson & Losada 2005

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1) Marcial Losada, "The complex dynamics of high performance teams", *Mathematical and Computer Modelling* 30, 179–192 (1999).

2) Marcial Losada and Emily Heaphy, "The role of positivity and connectivity in the performance of business teams: A nonlinear dynamics model", *American Behavioral Scientist* 47, 740–765 (2004).

3) Barbara Fredrickson and Marcial Losada, "Positive affect and the complex dynamics of human flourishing", *American Psychologist* 60, 678–686 (2005).

The trail leading to Fredrickson & Losada 2005

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- Here I will have to be brief, and refer you to BSF 2013 for details.

Losada 1999

PERGAMON

Mathematical and Computer Modelling 30 (1999) 179–192

MATHEMATICAL AND COMPUTER MODELLING

www.elsevier.nl/locate/mcm

The Complex Dynamics of High Performance Teams

M. Losada
Meta Learning, 2280 Georgetown Blvd.
Ann Arbor, MI 48105, U.S.A.

(Received and accepted November 1998)

Abstract—The connectivity of a team is highly correlated with its performance. Connectivity is measured by the strength and number of cross-correlations among time series of the coded speech acts of meeting participants. Connectivity is used as a control parameter in a nonlinear dynamical model derived from the observed time series. Different types of attractors occur in phase space depending on the team's connectivity and performance level: low performance teams show point attractors, medium performance teams show limit cycles, and high performance teams show low-dimensional chaotic attractors.

Keywords—Connectivity, Team performance, Nonlinear dynamics, Complexity, Team learning.

1. INTRODUCTION

For several years I had been the director of the Center for Advanced Research (CFAR), built by EDS in Ann Arbor and Cambridge, near the University of Michigan and MIT campuses. CFAR had a laboratory known as the Capture Lab. This lab had an observation room with a one-way mirror, computers, videotaping equipment, and other devices that allowed several observers to code speech acts of participants in a meeting using a specialized software [1,2]. As a speech act was coded, the computer put a time stamp indicating at what moment during the meeting the

Losada 1999 (2)

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- Observers behind one-way mirrors coded their "speech acts" as positive-negative, other-self, inquiry-advocacy.

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- From these time series, each team's "degree of connectivity" was determined by (unspecified) statistical tests.

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(VA1) Inquiry-advocacy and other-self: Ratios or differences?
How to convert discrete "speech acts" to continuous variables?

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This seems even more implausible than VA2.

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Tantamount to assuming that participants have no memory.

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(VA5) Let's examine Losada's "derivation" of his equations …

Losada 1999 (3)

Thinking about the model that would generate time series that would match the general characteristics of the actual time series observed at the Capture Lab, it was clear that it had to include nonlinear terms representing the dynamical interaction among the observed behaviors. One such interaction is that between inquiry-advocacy and other-self. If I call the first X and the second Y, their interaction should be represented by the product XY, which is a nonlinear term. I also knew from my observations at the lab, that this interaction should be a factor in the rate of change driving emotional space (which I will call Z). In addition, I would need a scaling parameter for Z. Consequently, the rate of change of Z should be written as

$$\frac{dZ}{dt} = XY - aZ,$$

where a is a scaling parameter that would be held constant.

From my observations at the lab, I also knew that connectivity had a critical incidence on the level of inquiry-advocacy and, consequently, it should interact with X and the product of this interaction should be part of the rate of change of Y, according to the characteristics of the time series observed, where there was a lead-lag relationship between Y and X. I also needed to discount the interaction between X and Z (which would be represented by the nonlinear term XZ) and Y with itself, so that the rate of change of Y should be written as

$$\frac{dY}{dt} = cX - XZ - Y,$$

where c is the control parameter representing connectivity, as measured by the nexi index, and should be varied according to the nexi number for each team performance category.

Finally, and in accordance with the characteristics of the time series generated at the lab, the rate of change of X should be a function of Y, discounting the level of X; so that with the inclusion of a scaling parameter, the rate of change of X should be written as

$$\frac{dX}{dt} = b(Y - X),$$

where is b is a scaling parameter to be held constant.

I realized that, except for some differences in the arrangement of the terms and the letters chosen to designate the parameters, these were the same set of coupled nonlinear differential equations that Lorenz had chosen for his model and published in one of the most often cited papers in science [15]. Lorenz obtained his equations from an idealized mathematical model of

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Losada 1999 has many further flaws; see BSF 2013 for details.

Losada 1999 (4)

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Conclusion:

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  No surprise. Why *should* they?

Losada & Heaphy 2004

The Role of Positivity and Connectivity in the Performance of Business Teams

A Nonlinear Dynamics Model

MARCIAL LOSADA
Meta Learning
EMILY HEAPHY
University of Michigan Business School

Connectivity, the control parameter in a nonlinear dynamics model of team performance is mathematically linked to the ratio of positivity to negativity (P/N) in team interaction. By knowing the P/N ratio it is possible to run the nonlinear dynamics model that will portray what types of dynamics are possible for a team. These dynamics are of three types: point attractor, limit cycle, and complexor (complex order, or "chaotic" in the mathematical sense). Low performance teams end up in point attractor dynamics, medium perfomance teams in limit cycle dynamics, and high performance teams in complexor dynamics.

Keywords: *positivity; connectivity; team performance; nonlinear dynamics*

Positive organizational scholars have made an explicit call for the use of nonlinear models stating that their field "is especially interested in the nonlinear positive dynamics . . . that are frequently associated with positive organizational phenomena" (Cameron, Dutton, & Quinn, 2003, pp. 4-5). This article answers this call by showing how a nonlinear dynamics model, the *meta learning* (ML) model, developed and validated against empirical time series data of business teams by Losada (1999), can be used to link the positivity/negativity ratio (P/N) of a team with its connectivity, the control parameter in the ML model. P/N was obtained by coding the verbal communication of the team in terms of approving versus disapproving statements. In the ML model, positivity and negativity operate as powerful feedback systems: negativity dampens deviations from some standard, while positivity acts as amplifying or reinforcing feedback that

Losada & Heaphy 2004 (2)

Main steps of Losada–Heaphy reasoning

(as best I can reconstruct it, anyway . . .)

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- Conclusion: $P/N = (c - i - 1)/b$ where

$$\begin{aligned} c &= \text{"connectivity"} \\ i &= \text{initial value of the } P/N \text{ state variable} \end{aligned}$$

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- Does any of this make sense? Not as far as I can tell . . .

Losada & Heaphy 2004 (2)

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(as best I can reconstruct it, anyway ...)

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- Does any of this make sense?
  But you should judge for yourself ...

Putting it all together: Fredrickson & Losada 2005

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F&L 2005 derived the critical positivity ratio of 2.9013 as follows:

> Subsequent work on the model (Losada & Heaphy, 2004) revealed that the positivity ratio relates directly to the control parameter by the equation $P/N = (c - Y_0 - 1)b^{-1}$ ... Past mathematical work on Lorenz systems by Sparrow (1982) and others (Frøyland & Alfsen, 1984; Michielin & Phillipson, 1997) has established that when r, the control parameter in the Lorenz model, reaches 24.7368, the trajectory in phase space shows a chaotic attractor. Losada (1999) established the equivalence between his control parameter, c, and the Lorenzian control parameter, r. Using the above equation, it is known that the positivity ratio equivalent to $r = 24.7368$ is 2.9013.

Fredrickson & Losada 2005 (2)

F&L didn't explain where $r_{\text{crit}} = 24.7368$ comes from or how it leads to $(P/N)_{\text{crit}} = 2.9013$ …

Fredrickson & Losada 2005 (2)

F&L didn't explain where $r_{\text{crit}} = 24.7368$ comes from or how it leads to $(P/N)_{\text{crit}} = 2.9013\ldots$ but we can!

Fredrickson & Losada 2005 (2)

- Accept uncritically the main "result" of Losada & Heaphy:
  $P/N = (c - i - 1)/b$

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  the boundary between chaos and non-chaos in the Lorenz system

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  the boundary between chaos and non-chaos in the Lorenz system
- Simple algebra then yields

$$(P/N)_{\text{crit}} = \frac{\sigma(\sigma + b + 3)}{b(\sigma - b - 1)} - \frac{i + 1}{b}$$

Fredrickson & Losada 2005 (3)

- Specializing to $\sigma = 10$, $b = 8/3$, $i = 16$ yields

$$(P/N)_{\mathrm{crit}} \;=\; \frac{441}{152} \;=\; 2.901\overline{3157894736842105 26}$$

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- But where did $\sigma = 10$, $b = 8/3$, $i = 16$ come from?
- Saltzman (1962) chose $\sigma = 10$, $b = 8/3$ for illustrative purposes.
- Lorenz (1963) followed him; Losada (1999) followed Lorenz.
- There is nothing special about these numbers!
  Any other values within a wide range would produce qualitatively similar behavior — but *completely different* predictions for $(P/N)_{\mathrm{crit}}$.

Fredrickson & Losada 2005 (4)

Conclusion: Even if we accept for the sake of argument that

- Every single claim made in Losada (1999) and Losada and Heaphy (2004) is correct;
- The Lorenz equations provide a valid and universal way of modeling human emotions;

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nothing more than an artifact of the arbitrary choice of an illustratively convenient value made by a geophysicist in Hartford in 1962.

July 2, 2012: Submission to *American Psychologist*

Dear Dr. Anderson,

We are enclosing a submission to *American Psychologist* entitled "The Complex Dynamics of an Intellectual Imposture: The Critical Positivity Ratio", by Nicholas J.L. Brown, Alan D. Sokal and Harris L. Friedman. The manuscript is 35 pages long.

We are happy for the manuscript to be given the customary masked review, and we have deleted all identifying information from the manuscript.

All three authors have agreed to the byline order and to the submission of the manuscript in this form. *ETC ETC*

10 days later: Summary rejection

Thank you for submitting your manuscript, "The Complex Dynamics of an Intellectual Imposture: The Critical Positivity Ratio," to the *American Psychologist* (AP). I am sorry to inform you that it will not be sent out for formal peer review. …

[Y]our manuscript is really a commentary on a manuscript previously published in the AP. The AP has a standard commentary policy, and it involves a timely response … Proposed AP comments are expected within 2–3 months after the publication of an article in the AP. … [T]hus, the manuscript file will be closed.

Sincerely, Gary R. VandenBos, PhD, Managing Editor

July 13, 2012: Appeal to the Editor-in-Chief

Dear Dr. Anderson,

… Of course, you have a perfect right to apply your "standard commentary policy" as rigidly as you wish; it is not our role to tell you how to run your journal. But as should be obvious from the title, introduction, content and conclusion of our manuscript, *this is no ordinary comment.* Rather, we are contending … that a highly-cited article published 7 years ago in *American Psychologist* … is an out-and-out intellectual imposture. …

July 13, 2012: Appeal to the Editor-in-Chief

This situation is quite likely *unprecedented* in the history of *AP*, and for this reason you might wish to be a bit flexible in your response. Otherwise, fair-minded observers will take home the following message about *AP*'s editorial practices: it is acceptable for *AP* to publish an article that is, in reality, an intellectual imposture; but unless the imposture is discovered within 2–3 months of publication, *AP* will not deign to publish a corrective. This is an absurdly restrictive "statute of limitations", and your reliance on it will not enhance the public image of *AP*.

July 13, 2012: Appeal to the Editor-in-Chief

Of course, the foregoing *presumes* the correctness of our claim that the article of Fredrickson and Losada (2005) is indeed an intellectual imposture. Perhaps you doubt this claim. Fair enough: then send our manuscript out for review, and let us see whether any of the reviewers can come up with any valid scientific criticisms of our reasoning.

Let us be clear: we are *not* begging you to publish our manuscript. …

Three hours later: Reversal by *American Psychologist*

Dear Dr. Sokal,

I received your letter of appeal of the decision to reject without review your manuscript, "The Complex Dynamics of an Intellectual Imposture: The Critical Positivity Ratio" … I have carefully reviewed your letter and have decided to grant your appeal. We will begin processing your manuscript shortly.

Best wishes,

Norman Anderson, Ph.D.
Chief Executive Officer
American Psychological Association

One year later . . .

The Complex Dynamics of Wishful Thinking

The Critical Positivity Ratio

Nicholas J. L. Brown — *Strasbourg, France*
Alan D. Sokal — *New York University and University College London*
Harris L. Friedman — *Saybrook University and University of Florida*

We examine critically the claims made by Fredrickson and Losada (2005) concerning the construct known as the "positivity ratio." We find no theoretical or empirical justification for the use of differential equations drawn from fluid dynamics, a subfield of physics, to describe changes in human emotions over time; furthermore, we demonstrate that the purported application of these equations contains numerous fundamental conceptual and mathematical errors. The lack of relevance of these equations and their incorrect application lead us to conclude that Fredrickson and Losada's claim to have demonstrated the existence of a critical minimum positivity ratio of 2.9013 is entirely unfounded. More generally, we urge future researchers to exercise caution in the use of advanced mathematical tools, such as nonlinear dynamics, and in particular to verify that the elementary conditions for their valid application have been met.

Keywords: positivity ratio, broaden-and-build theory, positive psychology, nonlinear dynamics, Lorenz system

The "broaden-and-build" theory (Fredrickson, 1998, 2001, 2004) postulates that positive emotions help to develop broad repertoires of thought and action, which in turn build resilience to buffer against future emo- … those who were "flourishing" had an average positivity ratio of 3.2.

The work of Fredrickson and Losada (2005) has had an extensive influence on the field of positive psychology. This article has been frequently cited, with the Web of Knowledge listing 322 scholarly citations as of April 25, 2013. Fredrickson and Kurtz (2011, pp. 41–42), in a recent review, highlighted this work as providing an "evidence-based guideline" for the claim that a specific value of the positivity ratio acts as a "tipping point beyond which the full impact of positive emotions becomes unleashed" (they now round off 2.9013 to 3). An entire chapter of Fredrickson's (2009) popular book (Chapter 7) is devoted to expounding this "huge discovery" (p. 122), which has also been enthusiastically brought to a wider audience by Seligman (2011a, pp. 66–68, 2011b). In fact, the paperback edition of Fredrickson's (2009) book is subtitled *Top-Notch Research Reveals the 3-to-1 Ratio That Will Change Your Life*.

It is worth stressing that Fredrickson and Losada (2005) did not qualify their assertions about the critical positivity ratios in any way. The values 2.9013 and 11.6346 were presented as being independent of age, gender, ethnicity, educational level, socioeconomic status or any of the many other factors that one might imagine as potentially

. . . and simultaneously . . .

Updated Thinking on Positivity Ratios

Barbara L. Fredrickson
University of North Carolina at Chapel Hill

This article presents my response to the article by Brown, Sokal, and Friedman (2013), which critically examined Losada's conceptual and mathematical work (as presented in Losada, 1999; Losada & Heaphy, 2004; and Fredrickson & Losada; 2005) and concluded that mathematical claims for a critical tipping point positivity ratio are unfounded. In the present article, I draw recent empirical evidence together to support the continued value of computing and seeking to elevate positivity ratios. I also underscore the necessity of modeling nonlinear effects of positivity ratios and, more generally, the value of systems science approaches within affective science and positive psychology. Even when scrubbed of Losada's now-questioned mathematical modeling, ample evidence continues to support the conclusion that, within bounds, higher positivity ratios are predictive of flourishing mental health and other beneficial outcomes.

Keywords: positivity ratio, broaden-and-build theory, positive psychology, nonlinear dynamics, Lorenz system

In their lively article "The Complex Dynamics of Wishful Thinking: The Critical Positivity Ratio," Brown, Sokal, and Friedman (2013) offered a critique of the application of nonlinear dynamics and differential equations in two of Marcial Losada's foundational papers (Losada, 1999; Losada & Heaphy, 2004). They also identified additional logical errors that permeate an article that use of nonlinear differential equations, particularly chaotic ones such as the Lorenz equations, is appropriate.

My aim in this response article is not to defend Losada's mathematical and conceptual work. Indeed, I have neither the expertise nor the insight to do so on my own. My aim, rather, is to update the empirical evidence for the value and nonlinearity of positivity ratios. My intent is to offer a steadying counterpoint to Brown and colleagues' (2013) article. Absorbing their many critiques of Losada's work might tempt a reader to throw out the proverbial baby with the bath water. Even while Brown and colleagues have called into question some of the claims Losada and I made in 2005, in the intervening years, others of our claims not only remain unchallenged but stand now on even firmer empirical footing.

It bears underscoring that the claims Losada and I made in our 2005 *AP* article (Fredrickson & Losada, 2005) were supported by three interwoven elements: psychological theory, mathematical modeling, and quantitative data. Here I unthread the now-questionable element of mathematical modeling from this braid, which leaves us in territory familiar to most psychological scientists, that at the interface of theory and data. While perhaps not as compelling as the trio of theory and data buttressed by mathematical modeling, the resulting duo nevertheless remains a strong and dynamic one.

Before illuminating the logic and importance of positivity ratios, I lay the necessary theoretical and empirical

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But Fredrickson 2013 is extremely unclear about

- which claims she has opted to renounce
- which claims she has chosen to reaffirm

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Let's try to disentangle it.

Fredrickson 2013 (2)

To clarify what is at stake, consider the following sequence of successively weaker claims for the behavior of “degree of flourishing” as a function of the positivity ratio:

Fredrickson 2013 (2)

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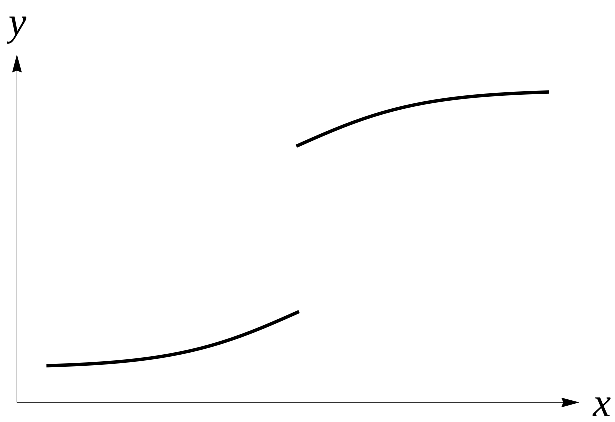
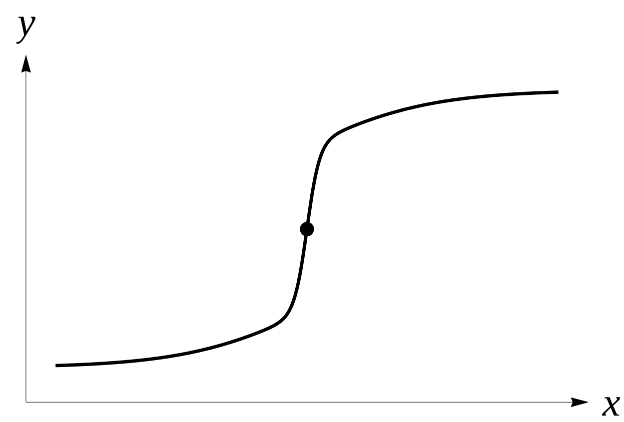
1. There is a discontinuous phase transition (“tipping point”) exactly at 2.9013.
2. There is a discontinuous phase transition somewhere around 3.
3. There is a rapid change somewhere around 3.
4. There is an inflection point (separating convexity from concavity) somewhere around 3.
5. There is an inflection point (separating convexity from concavity) somewhere.
6. There is some nonlinearity somewhere.

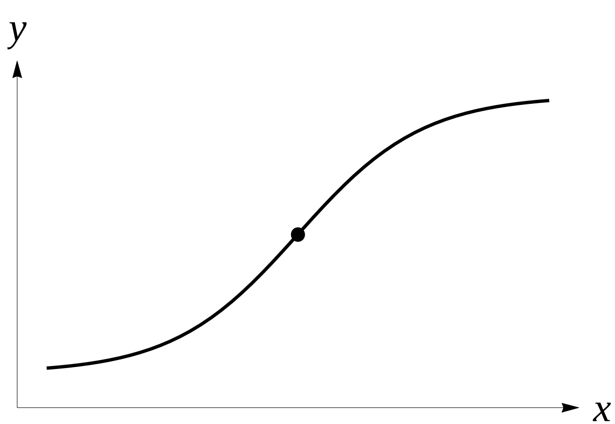
Fredrickson 2013 (2)



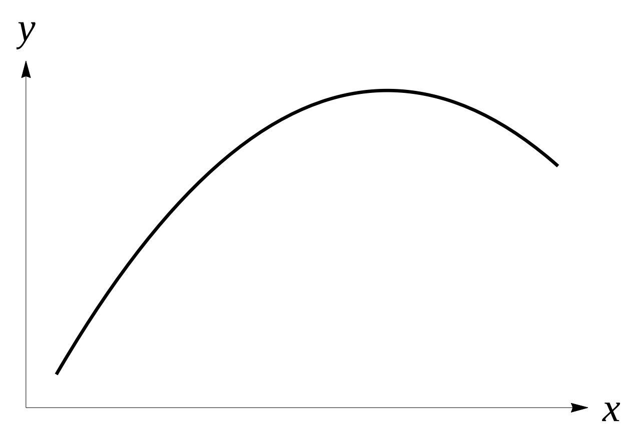
(1,2)



(3)



(4,5)



(6)

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- What does Fredrickson (2013) assert?

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- What does Fredrickson (2013) assert?
  Alas, this is shrouded in confusion.

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- Fredrickson and Losada (2005) made claim #1.
- Fredrickson (2009) reaffirmed claim #1 but noted that, because of "impurities" and measurement imprecision, the data might look in practice more like claim #2 or #3.
- Perhaps still #1?

  > The question … is whether positivity ratios obey one or more critical tipping points, and if so, whether those critical tipping points coincide with the ones identified by Losada's mathematical work for all individuals, samples, and subgroups. Clearly, these questions merit further test.

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  [Puzzling because their mathematical model does not make *any* definite prediction for the "critical tipping points": it depends on completely arbitrary choices of σ, b and i.]

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  > Whether the Lorenz equations … can be fruitfully applied to understanding the impact of particular positivity ratios merits renewed and rigorous inquiry.

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- Or maybe #2?

  > Whether the outcomes associated with positivity ratios show discontinuity and obey one or more specific change points, however, merits further test.

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  > Whether the outcomes associated with positivity ratios show discontinuity and obey one or more specific change points, however, merits further test.

  "On empirical grounds, yes, tipping points are highly probable."
  (Fredrickson to a British journalist, January 2014)

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- Fredrickson and Losada (2005) made claim #1.
- Fredrickson (2009) reaffirmed claim #1 but noted that, because of "impurities" and measurement imprecision, the data might look in practice more like claim #2 or #3.
- But Fredrickson did not present *any* evidence that such a discontinuity occurs or is even plausible.

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- But Fredrickson did not present *any* evidence that such a discontinuity occurs or is even plausible.
- Rather, in summarizing recent empirical work, she appeared to be arguing for claim #4, #5 or #6 (it is not clear which).

Analysis of Fredrickson 2013’s empirical evidence

Fredrickson & Losada (2005) empirical study

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  because the information that might provide this evidence was discarded at an early stage, when participants were dichotomized as "flourishing" or "nonflourishing".

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- Mean positivity ratio for "flourishers" was 3.2; for "nonflourishers" it was 2.3.

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- Mean positivity ratio for "flourishers" was 3.2;
  for "nonflourishers" it was 2.3.
- This provides some evidence of a positive correlation
  between positivity ratio and "degree of flourishing".

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- This provides some evidence of a positive correlation between positivity ratio and "degree of flourishing".
- It provides *no* evidence about whether that correlation is linear or nonlinear.

Analysis of Fredrickson 2013's empirical evidence

Fredrickson & Losada (2005) empirical study

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- It provides *no* evidence about whether that correlation is linear or nonlinear.
- It certainly does not provide any evidence of a discontinuity!

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- Purported to find empirical evidence in favor of claim #1
- Mean positivity ratio for "flourishers" was 3.2; for "nonflourishers" it was 2.3.
- The fact that "these mean ratios flanked the 2.9 ratio", which F&L considered "critical to our hypothesis" — reiterated by Fredrickson (2013) — is utterly irrelevant:

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- The fact that "these mean ratios flanked the 2.9 ratio", which F&L considered "critical to our hypothesis" — reiterated by Fredrickson (2013) — is utterly irrelevant:

  it provides no evidence even for claim #6 (nonlinearity), much less for claim #1 (discontinuity).

Analysis of Fredrickson 2013's empirical evidence (2)

Rego *et al.* (2012)

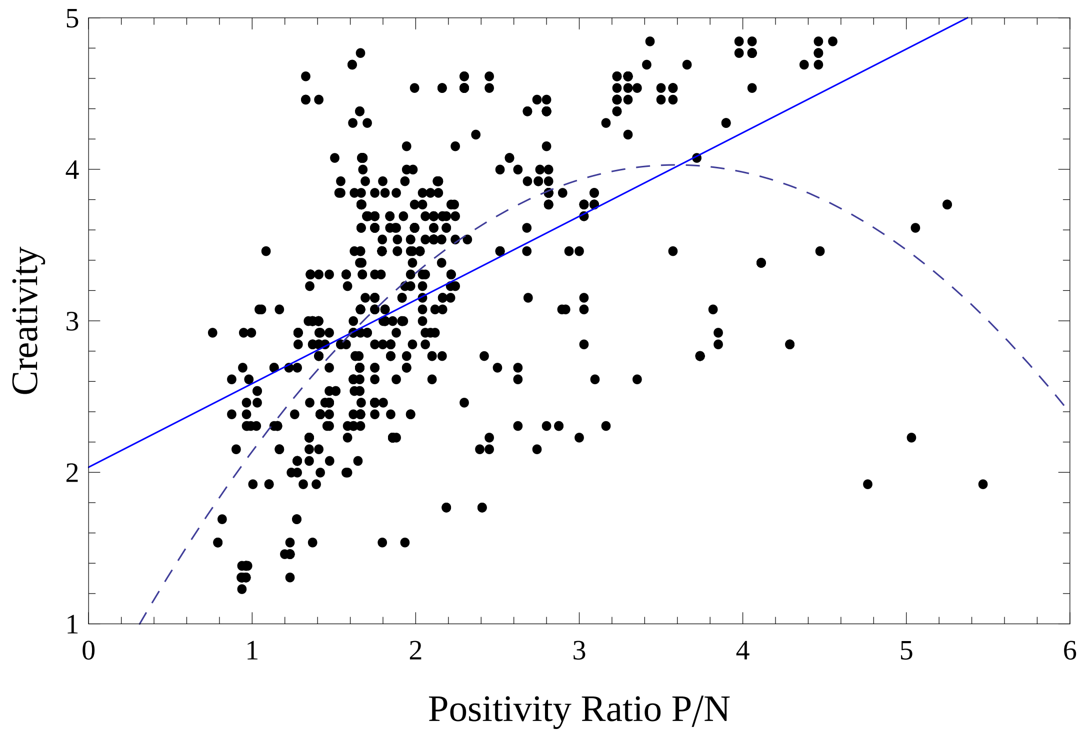
Analysis of Fredrickson 2013's empirical evidence (2)

Rego *et al.* (2012)

- Studied Portuguese retail workers ($n = 595$)
- Measured positivity ratio and "creativity"
- Data were quantitative, not dichotomized
- Performed linear and quadratic regressions

Analysis of Fredrickson 2013's empirical evidence (2)

Rego *et al.* (2012)



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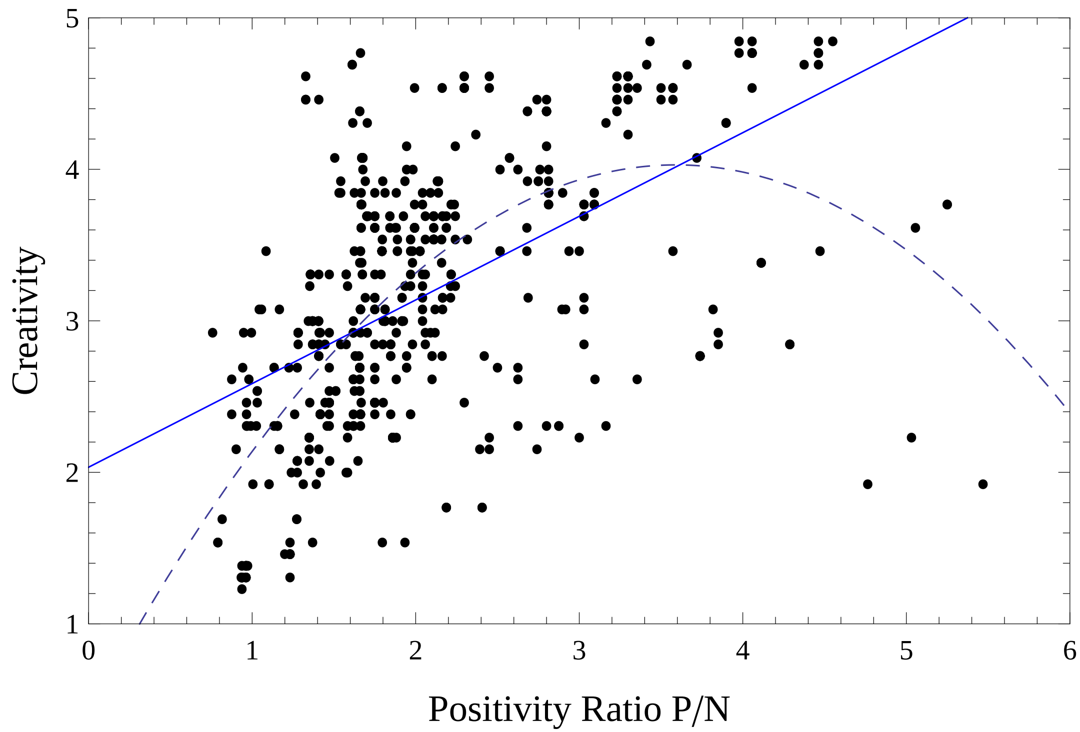
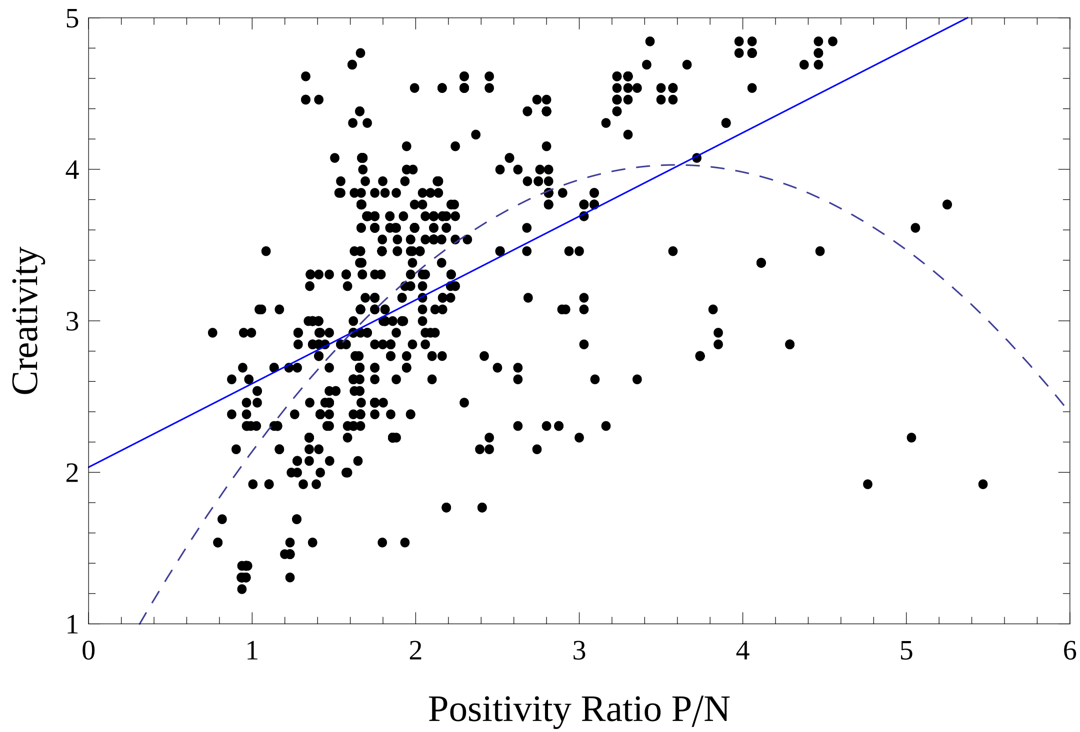


No hint of any inflection point, much less any discontinuity!

Analysis of Fredrickson 2013's empirical evidence (2)

Rego *et al.* (2012)



Does provide evidence of

- Positive correlation between positivity ratio and "creativity"
- Concave nonlinearity in this correlation

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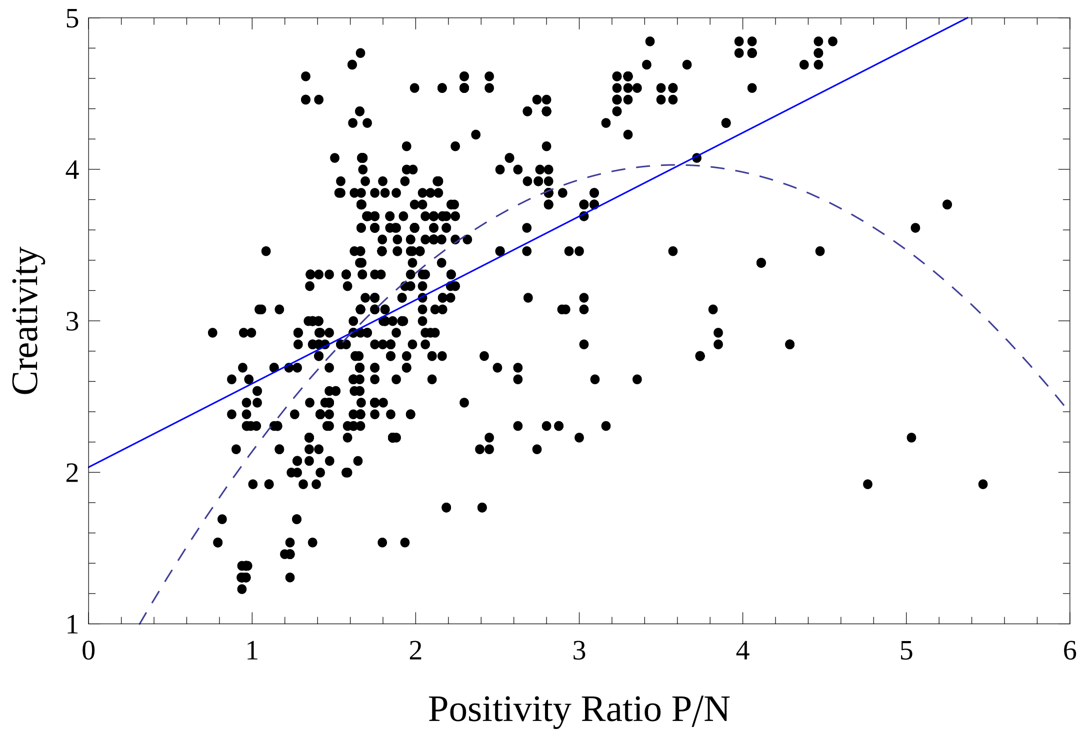


But in retrospect such concave nonlinearity is *inevitable*, since "creativity" is bounded (from 1 to 5 in Rego *et al.*) while positivity ratio (P/N) is unbounded (from 0 to ∞)

Analysis of Fredrickson 2013's empirical evidence (2)

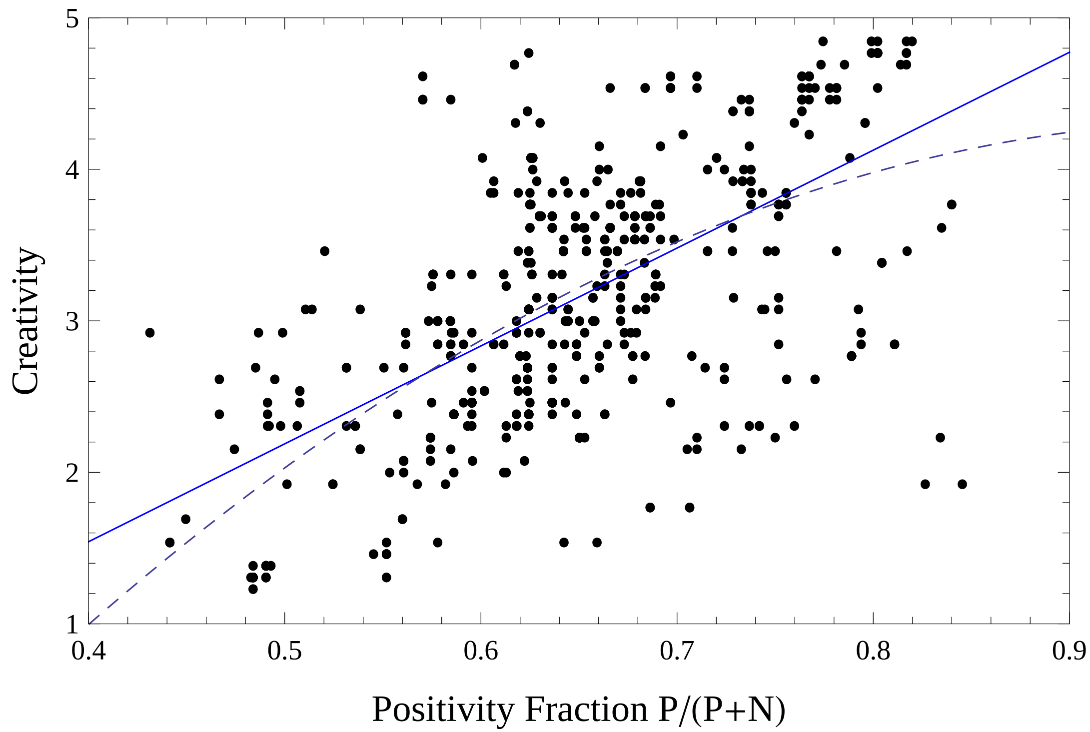
Rego *et al.* (2012)

Better approach:

- Use "positivity fraction" P/(P+N) as independent variable
- Runs from 0 to 1

Analysis of Fredrickson 2013’s empirical evidence (2)

Rego *et al.* (2012)



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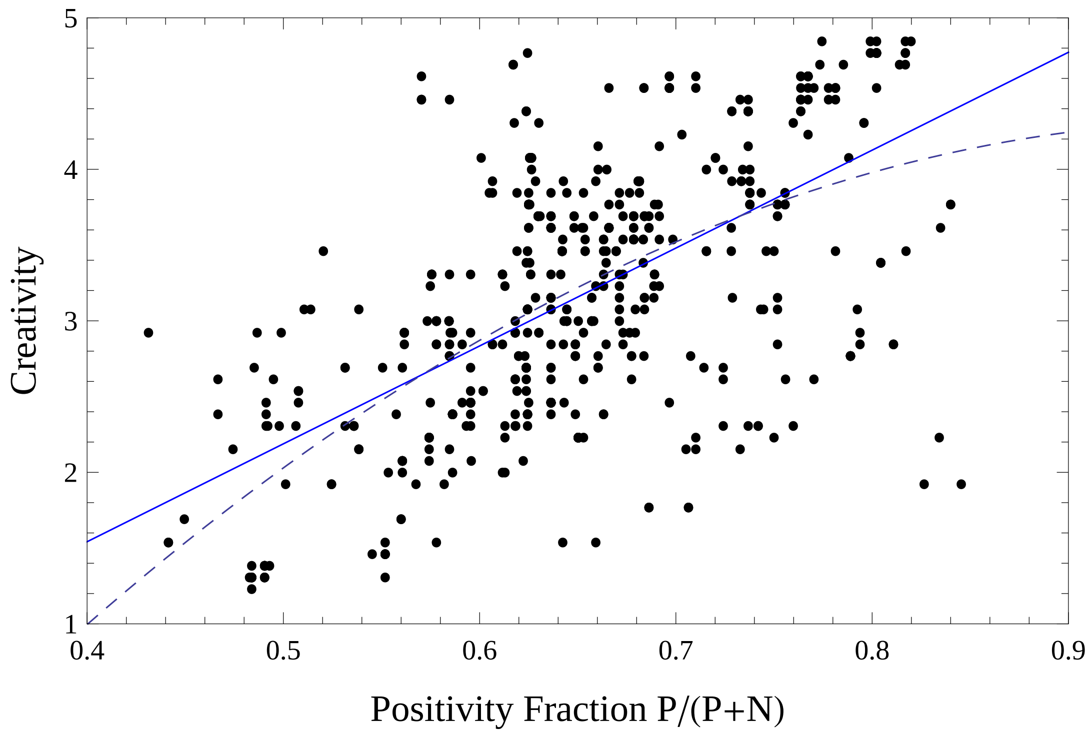

Now correlation is almost linear.

Analysis of Fredrickson 2013's empirical evidence (3)

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Conclusions:

- No evidence whatsoever for any tipping points
- Significant evidence against tipping points
  (Rego *et al.* 2012, Shrira *et al.* 2011)
- Significant evidence for positive correlations
  between positivity ratio and various other things
  (but the direction of causality, if any, is still uncertain)
- Weak evidence for concave nonlinearity in these correlations

Response from Fredrickson

Let’s give the last word to a sociologist . . .

The process that has taken place in this trio of articles was presciently foreseen four decades ago by the sociologist Stanislav Andreski:

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> The recipe for authorship in this line of business is as simple as it is rewarding: just get hold of a textbook of mathematics, copy the less complicated parts, put in some references to the literature in one or two branches of the social studies without worrying unduly about whether the formulae which you wrote down have any bearing on the real human actions, and give your product a good-sounding title, which suggests that you have found a key to an exact science of collective behaviour.
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- An exaggeration? Yes, in most cases.

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- But in this case *literally accurate*.

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without *anyone* calling it into question … until a first-term part-time Masters' student at an obscure London university came along and expressed his doubts?

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- Where were all the leaders in positive psychology?
- The leaders in applying nonlinear-dynamics models to psychology?
- Was everyone *really* so credulous?
- Or were some people less credulous but politely silent, for reasons of internal politics?

For further reading (1)

- Marcial Losada, "The complex dynamics of high performance teams", *Mathematical and Computer Modelling* 30, 179–192 (1999).
- Marcial Losada and Emily Heaphy, "The role of positivity and connectivity in the performance of business teams: A nonlinear dynamics model", *American Behavioral Scientist* 47, 740–765 (2004).
- Barbara Fredrickson and Marcial Losada, "Positive affect and the complex dynamics of human flourishing", *American Psychologist* 60, 678–686 (2005).
- Barbara Fredrickson, *Positivity* (Crown, New York, 2009).
- Barbara Fredrickson, "The dynamics of positive opposites", lecture (March 2010), `http://www.youtube.com/watch?v=`

For further reading (2)

- Nick Brown, Alan Sokal and Harris Friedman, "The complex dynamics of wishful thinking: The critical positivity ratio", *American Psychologist* 68, 801–813 (2013), arXiv:1307.7006 [nlin.CD].
- Barbara Fredrickson, "Updated thinking on positivity ratios", *American Psychologist* 68, 814–822 (2013).
- Brown–Sokal–Friedman, "The persistence of wishful thinking: Response to ‘Updated thinking on positivity ratios’", *American Psychologist* 69, 629–632 (2014), arXiv:1409.4837 [stat.AP].
- Brown–Sokal–Friedman, "Positive psychology and romantic scientism: Reply to comments", *American Psychologist* 69, 636–637 (2014), arXiv:1409.5172 [stat.AP].

For further reading (3)

- Tom Bartlett, "The magic ratio that wasn't", *Chronicle of Higher Education*, August 5, 2013.
- Vinnie Rotondaro, "Nick Brown smelled bull", *narrative.ly*, October 17, 2013.
- Andrew Anthony, "The British amateur who debunked the mathematics of happiness", *The Observer* [London], January 19, 2014.

Thanks to my collaborators

Nick Brown

Harris Friedman