Work-Energy

CAUTION For this lab, force sensors are mounted near the floor on rods that stick out into the aisle. Please be careful.

Equipment DataStudio, air track, 28.7 cm glider, set of kg masses with hooks, smart pulley, bench clamp, force sensor, 100 cm string with loops, 100 cm string with loops and 4 rubber bands, 130 cm string with loops, meter stick

1 Purpose
To investigate the validity of the work-energy relationship.

2 Theory
Consider a point mass \( m \) acted upon by a net force \( \vec{F} = \vec{F}(\vec{r}) \). The position, velocity, and acceleration of the mass are given by \( \vec{r}, \vec{v}, \) and \( \vec{a} \), and the time by \( t \). The force \( \vec{F} \) may be a function of \( \vec{r} \). Newton’s 2nd Law for the mass is \( \vec{F} = m\vec{a} \). This is integrated over position from an initial position \( (i) \) to a final position \( (f) \).

\[
\int_{i}^{f} \vec{F} \cdot d\vec{r} = m \int_{i}^{f} \vec{a} \cdot d\vec{r}.
\]

The left hand side is defined as the work \( W \) done on \( m \) as it moves from \( \vec{r}_i \) to \( \vec{r}_f \). It is only the component of the force parallel or anti-parallel to the motion of \( m \) that contributes to the work. The right hand side can be transformed to

\[
m \int_{i}^{f} \frac{d\vec{v}}{dt} \cdot d\vec{r} = m \int_{i}^{f} \vec{v} \cdot d\vec{v} = \left( \frac{1}{2} m \vec{v} \cdot \vec{v} \right)_{i}^{f} = \left( \frac{1}{2} m v^2 \right)_{f} - \left( \frac{1}{2} m v^2 \right)_{i}.
\]

The quantity \( \frac{1}{2}mv^2 \) is defined as the kinetic energy, or KE. In words, when \( m \) moves from \( \vec{r}_i \) to \( \vec{r}_f \), the net work \( W \) done on \( m \) between those two points is equal to the change in kinetic energy \( KE \) between those two points. More succinctly,

\[ W = \Delta KE. \]

This is the most basic form of the work-energy theorem. To be used correctly, \( W \) must include all the work done by all of the forces between the two points under consideration.

The work-energy theorem can be applied to a system of particles. The total work done is equal to the total change in KE. Care must be taken with the work done by the forces between the particles of the system, or the “internal” work. The internal work may not cancel out.

3 Does \( W = \Delta KE? \)

3.1 Description
A glider of mass \( m \) is on a horizontal air track. A horizontal string attached to the glider goes over a smart pulley. The end of the string that goes over the smart pulley is attached
to 3 or 4 rubber bands that are connected in series, and the last rubber band is attached to
a force sensor which is clamped to a rod near the floor. The glider is pulled back to stretch
the rubber bands and let go. The force on the glider and velocity of the glider are measured
as a function of the position of the glider. DataStudio integrates the force curve to give the
work, and the velocity curve can be examined to give the velocities at the chosen beginning
and end points. The change in KE is calculated and compared to W.

The force on the glider is not constant and as a consequence the acceleration is not
constant. The tension $T$ in the string will be the same on both sides of the pulley if the mass
of the string and pulley can be neglected and if the pulley is frictionless. The magnitude
of the force between the last rubber band and the force sensor will be the same as $T$ if the
rubber bands are massless.

## 3.2 Theory

The motion in this experiment is one dimensional and the vector notation used in defining
work is not needed. If the one dimension is taken to be $x$, the work becomes $\int F dx$. This
is equal to the area under the $F \text{ vs } x$ curve for the portion of the curve between the two
vertical lines defined by $x_i$ and $x_f$. The statistics function of DataStudio will calculate this
area for you.

## 3.3 Programming

Check that the (analog) force sensor and the (digital) smart pulley are plugged into the
interface, and noting which channels they are plugged into, program DataStudio for these
two sensors. Make sure that both velocity and position are checked in the experiment setup
window under ”measurements” for the smart pulley. Look at figure 1. Open up the graph
display by dragging its icon to the force sensor icon. Click Time on the horizontal axis and
choose position from the submenu. Drag the graph icon to the Velocity icon to add Velocity
to the window. On the graph display move your mouse to the horizontal axis of the graph
until the arrow icon changes to a mini table icon. Again click Time and choose Postion from
the submenu. Your graph will have position for the horizontal axis and force and velocity for
the two vertical axes. In the force sensor settings, under sample rate, increase the periodic
samples to 200 Hz.

## 3.4 Calibrating the Force Sensor

Click on Calibrate Sensors at the top of the experiment window to open up the setup dialogue
box. Calibrate the force sensor as follows.

- Leave the force sensor in the positon in which it will be used. With nothing touching
  the hook on the force sensor, push the tare button on the sensor.

- In the box labeled Calibration Point 1, first click the Read From Sensor button and
  then enter 0 in the box labeled Standard Value.

- Take a 100 cm string with loops at both ends and attach one end to the hook of the
  force sensor. Pass the other end of the string over the pulley and down through the
hole in the bracket holding the smart pulley. Hang a 100 g mass from that end of the string.

- In the box labeled Calibration Point 2, first click the Read From Sensor button and then enter the **weight** of the 100 g mass in the box labeled Standard Value.
- click OK.

Figure 1:

3.5 Taking Data

Remove the string used for calibrating the force sensor. Level the air track. Take the string with the rubber bands at one end and hook the last rubber band to the force sensor. Pass the string over the smart pulley and hook the end to the 28.7 cm glider. Pull the glider back to stretch the rubber bands but do not pull the rubber bands into the smart pulley. Click START, and let go of the glider. Click Stop just after the string goes slack.

3.6 Analysis

Click the statistics button and then the scale to fit button. Box in the part of the force curve you wish to integrate. Your beginning integration point \( x_i \) should exclude perhaps the first two data points. The end of your integration \( x_f \) should be just before the velocity curve levels off for the first time and before the force curve goes to zero for the first time. Click the statistics menu button and choose area. This is the value of the integral (area under the graph) which is the work \( W \) done between the limits chosen. Use the smart cursor to determine the velocities at the end points of your integration and calculate the change in KE. Compare to the measured work done.

3.7 Repeat

Take a few more data runs. Discuss random errors that contribute to the scatter of your results from run to run, and systematic errors which will effect all of your runs in the same
4  \( W = \Delta KE \) Applied to a System

4.1 Description

The work-energy theorem is tested for a system of two masses. The theorem is applied to the two masses separately and to the entire system.

A horizontal string is attached to a glider on a horizontal air track. The string goes over the smart pulley and a mass is attached to the end. The glider is released and the velocity of the glider is measured as a function of distance. The velocity of the glider is measured at two points and the change of KE is calculated for each mass (glider and hanging mass) and for both masses together. The work done on each mass and for the system is calculated.

4.2 Theory

The hanging mass is denoted \( M_1 \) and the glider mass by \( M_2 \). From the equations given in the Lab Newton’s 2nd Law the tension \( T \) in the string can be calculated as

\[
T = \frac{M_1 M_2 g}{M_1 + M_2}.
\]

This is a constant and the work done by \( T \) simplifies to \( T \times \text{(distance)} \) with an appropriate sign.

The work done on \( M_1 \) is \((M_1 g - T)(x_f - x_i)\). Explain the minus sign before the \( T \). The work done on \( M_2 \) is \( T(x_f - x_i) \).

The work done on the system is \( M_1 g(x_f - x_i) \). Explain clearly why \( T \) does not appear in this expression. (Due to Newton’s 3rd law, internal forces cancel out. Keep in mind internal work does not always cancel out.) What would be the consequence if the internal work doesn’t cancel out?

4.3 Programming

Program DataStudio for the smart pulley (linear). Remember to check both position and velocity under ”measurements” in the experiment set up window. Open a graph display by clicking on graph in the ”Displays” window and drag it to velocity of the smart pulley in the ”Data” window. On the graph display move your mouse to the horizontal axis of the graph until the arrow icon changes to a mini table icon. At this point click on the left side of the mouse and select ”Position” from the submenu. Your graph will have velocity on the vertical axis and position on the horizontal axis. Look at figure 2.

4.4 Taking Data

Move the air track and the smart pulley so that a string with weight hanging from the smart pulley will not hit the force sensor. Level the air track. Hook the string to the glider and pass the string over the smart pulley. Hang a 50 g mass from the string. Pull the glider back until the 50 g mass is close to the smart pulley, Click the START button, and let the glider go. Click Stop just before the hanging mass hits the floor.
4.5 Analysis

Pick suitable $x_i$ and $x_f$ and determine the velocities at those points. For $M_1$, $M_2$ and the system, calculate the work done and the change in KE. Compare the work done with the change in KE for these three items.

Is the velocity vs distance curve a straight line? If not, what do you think it is?