Equipment DataStudio, sonometer (with detector coil but not driver coil), set of 5 wires, voltage sensor, BNC to double banana plug adapter, weight hanger, set of weights

CAUTION

In this experiment a substantial mass, supported by a wire, hangs over the floor. If the wire should break the mass will fall to the floor. PLEASE KEEP YOUR FEET AWAY FROM UNDERNEATH THE MASS.

1 Introduction

This experiment is an extension of the Oscillations of a String experiment. The experiment uses a PASCO Sonometer to measure how the normal mode frequencies of metal wires fixed at both ends vary with the length, mode number, tension, and mass per unit length of the wires. A sonometer (or monochord) is a device that holds a single wire under a desired tension and can detect the normal mode vibrations of the wire.

2 Theory

The notation and assumptions are the same as the experiment “Oscillations of a String.” The wire tension is $T$ and the mass per unit length is $\rho$. Distance along the wire is denoted by $x$ and distance transverse to the wire by $y$. The coordinate $y$ satisfies the wave equation. Dispersionless waves propagate in the positive or negative $x$ direction with the velocity $v = \sqrt{T/\rho}$. For a wire fixed at $x = 0$ and $x = L$, a given normal mode $n$ of the string is described by the equation

$$y_n(x, t) = A_n \sin \frac{2\pi x}{\lambda_n} \cos(\omega_n t + \phi_n),$$

(1)

where the $\lambda_n = \frac{2L}{n}$ ($n = 1, 2, 3, \ldots$) are the normal mode wavelengths and the $\phi_n$ are constant phases. The normal mode frequencies $\nu_n$ in hertz ($\nu_n = \frac{\omega_n}{2\pi}$) are given by

$$\nu_n = n \frac{T}{2L \sqrt{\rho}}.$$  

(2)

This equation is the basis of this experiment. The normal mode frequencies depend on the 4 parameters $n, L, T, and \rho$. With the sonometer it is possible to keep three of the parameters constant and investigate how the normal mode frequencies depend on the 4th parameter.

3 Apparatus

A horizontal view of the sonometer is shown in Fig. 1. It is a long rectangular box that holds a wire in tension. Each wire used has a lug with a hole in it at one end and a small cylinder at the other end. The lug end of the wire is attached at one end of the sonometer
to a cylinder that can be moved along the length of the sonometer by turning a knob with a screw attached to it. The other end of the wire fits into a tensioning lever. This lever has 5 notches. Call the notch nearest the sonometer box notch 1, and the other notches successively 2, 3, 4, and 5. The lever pivot point is the same distance from the wire as it is from notch 1. If the lever is horizontal and a mass $M$ is hung from notch 1 the tension on the wire is $Mg$. If the mass $M$ is hung from notch 2 the wire tension is $2Mg$, and so forth. Always adjust the position of the wire so that the tensioning lever is horizontal.

There are 2 “bridges” which can be moved along the top of the sonometer which allow the points at which the wire is fixed to be changed. A scale along the top of the sonometer allows the wire length to be easily determined. The top of the sonometer has magnetic strips which keep the bridges and detector coil from slipping.

The sonometer has 5 different strings. We list the diameter and linear mass densities of only one string.

1. 0.022 inch (1.84 g/m)

The wire oscillations are observed with a detector coil. This is a small cylindrical magnet that has many turns of fine wire around it. The detector coil is placed directly beneath the wire. The wire is magnetic. When the string vibrates it slightly changes the magnetic field in the coil, and by induction produces a small ac voltage across the coil terminals. A voltage sensor is used to detect this signal.

Fortunately there is a way out. DataStudio comes with a FFT display, where FFT stands for fast fourier transform. This is an algorithm that computes the frequencies present in a signal. If the signal from detector coil is observed with a voltage sensor, the frequencies present in the signal can be determined by using the FFT display. The FFT display window is opened in the usual way: drag the FFT display icon to the voltage sensor icon.

4 Using the FFT Display

The procedures described here will show you how to use the FFT display to determine what frequencies a wire vibrates at when it is plucked. For these preliminary measurements insert a 0.022 inch wire in the sonometer. Place the bridges at 10 and 70 cm and place the detector coil midway between the bridges and directly under the wire. Place 1 kg in notch 3. Reminder: adjust the length of the string so that the tensioning lever is horizontal.

Check that the detector coil is connected to a voltage sensor by means of an adaptor plug, and that the voltage sensor is plugged into the interface. Program the interface for the voltage sensor. Open the FFT display window. Open the FFT setup window by clicking the settings button or by double clicking in the display area. Double click on the FFT settings window and a FFT settings window will pop up. Click on the axis (scaling / settings) tab and set the maximum frequency at 500 Hz. Leave the y axis (scaling / settings) alone and close the setup window. For parts of some of the experiments you will want to use other values. Generally, choose the lowest maximum frequency possible for the frequency you want to measure. Also, its possible to change the maximum frequency by scrolling the mouse to the x axis of the FFT display window. Click on the x axis and drag the end of the axis left and right.

Without plucking the string. Click Start. In a few seconds the spectrum of the stray voltages picked up by the detector coil and the wiring will appear on the FFT screen. After
the display has stabilized click STOP. Use the smart cursor to find out what the stray frequencies are and make a note of them. Remove the smart cursor. Click Start and again wait for the stray frequency spectrum to settle down. Then pluck the string near the middle with your thumb and forefinger and observe that new frequencies appear on the display. These are due to the oscillations of the wire. Initially the noise is suppressed by the stronger signal from the wire. Then as the wire damps the wire signal drops and the noise signal rises. When taking data, watch the display carefully and pick out the signals from the wire. As these signals start to drop note which peaks these are and click STOP. Again, you can use the smart cursor to determine the normal mode frequencies of the wire.

Practice a bit. It takes skill to pluck the string so as to get a good signal from the string oscillations, good judgement as to when to click STOP, and good discrimination to pick out the string oscillations from the noise. Higher frequency modes will be harder to spot as they damp more quickly. Remember that the frequency amplitudes you are looking for get smaller because of string damping. As this happens, the FFT display starts increasing the gain so that the noise amplitudes rise. After a while the string amplitudes will disappear and only the noise will be present. Note that the display stays constant for a few seconds and is then “updated” as the computer completes a calculation of the frequency spectrum.

5 Experiments

The experiments test the predictions of Eq.(2). In each experiment, 3 of the 4 parameters involved (n, L, T, and ρ) are held constant. The 4th parameter is varied. In the tables you are asked to construct, items in the last column should be equal to each other if the results are according to theory.

To measure ν1’s, the lowest frequency mode, a good place for the detector coil is in the middle of the wire. For some of the higher frequency modes you might want to use other positions.

In the FFT display, the maximum frequency should be the lowest one possible for the frequency you are measuring. This will give the greatest accuracy. This implies that you will have to use more than one maximum frequency for determining the different frequencies in a given experiment.

5.1 Mode frequencies versus n

Use the 0.022 inch wire with a 1 kg mass in notch 3. Set the bridges at 10 and 70 cm. Measure ν1 through ν4. How many higher normal mode frequencies can you determine? Make a table with three columns designated by n, νn (Hz), and νn/n (Hz). Comment on your results.

Use Eq.(2) to calculate ν1 and compare this theoretical result to your experimental one.

5.2 First Mode Frequency versus L

Use the 0.022 inch wire with a 1 kg mass in notch 3. Keep one bridge at 10 cm. Measure the ν1’s for the other bridge at 70, 60, 50, 40, and 30 cm. Make a table with columns labeled L (m), ν1 (Hz), and ν1L (m/s). Discuss your results.
5.3 First Mode Frequency versus Tension

Use the 0.022 inch wire. Set the bridges at 10 and 70 cm. Measure \( \nu_1 \) for a 1 kg mass in notches 1, 2, 3, 4, and 5. Make a table with columns labeled T (N), \( \nu_1 \) (Hz), \( \nu_1/\sqrt{T} \) (Hz/\( \sqrt{N} \)). Discuss your results.

5.4 Mode Frequencies and Calculating Mass Density

Remove the 0.022 inch wire and setup the second wire. Use a 1 kg mass in notch 3. Set the bridges at 10 and 70 cm. Measure \( \nu_1 \) through \( \nu_4 \). Calculate the mass density for all four modes. To calculate the mass density rearrange equation 2. Take the average of all four mass densities to find the experimental value of \( \rho \).

6 Questions

1. In the first 3 experimental sections, what was the motivation for the combination of parameters in the last column? Explain.

2. To measure \( \nu_1 \), why is the detector coil best placed midway between the bridges? Why is this not the best place for some higher modes?

3. Explain why the string tension is given by 1-5 Mg as the mass is moved from notch 1 to notch 5.

4. What would be the effect if the tensioning lever was not horizontal?

5. In regards to section 5.4. When compared to 0.022 inch wire why has \( \nu_1 \) through \( \nu_4 \) changed?

7 Historical Note

The Greek scholar Pythagoras (died about 497 BC) is well known for his theorem on right triangles. He is also credited with being the first person to observe consonances, or two tones that when played together have a pleasing sound. For this purpose he used a monochord or sonometer. He found that notes with frequency ratios of 2:1 (octave), 3:2 (fifth), and 4:3 (fourth), are consonances.

8 Finishing Up

Please leave the bench as you found it. Thank you.
Figure 1 The Sonometer and Suggested Accessories