Lab 3: Collisions In One Dimension

1 Purpose
To investigate conservation of energy and momentum in one dimensional two-body collisions.

2 Theory
2.1 Conservation of Momentum
Let the mass of a particle be $m$ and its velocity $v$. The momentum $p$ is defined by $p = mv$ and Newton’s 2nd law may be written as $F = dp/dt$ where $F$ is the net force on $m$. If $F = 0$ this yields $p = \text{constant}$, which is Newton’s 1st law.

For a system of particles we identify each particle with a subscript $i$ so that $F_i = dp_i/dt$. If each side of this equation is summed over all the particles

$$\sum_{i=1}^{n} F_i = F = \frac{d}{dt} \sum_{i=1}^{n} p_i = \frac{d}{dt} P,$$

where $n$ is the number of particles, $F$ is the net force on the system, and $P$ is defined as the total momentum of the system. In the sum over $F_i$, the internal forces, the forces between the particles, can be excluded as Newton’s 3rd Law guarantees that they will cancel out. We don’t even have to know them. A consequence of this equation is that if any component of the force $F$ is zero, that component of the momentum $P$ will be a constant. This is true even if some of the particles make violent collisions with each other, because collisions involve only internal forces. One can say that the component of momentum is the same before the collisions as it is after the collisions, or that the component of momentum is conserved. If all components of $F = 0$ then all components of the momentum are conserved.

Conservation of energy and conservation of momentum are different statements. Momentum may be conserved when energy is not.

2.2 Definitions
In this lab we restrict ourselves to two body collisions. A collision is said to be elastic if the combined KE of the two objects is the same before the collision as after the collision. If the KE is less after the collision the collision is called inelastic. If the two objects stick together the collision is called completely or perfectly inelastic. In this last case maximum KE is lost.
2.3 Two Particle One Dimensional Collisions

*Elastic collisions.* The 2 particles you will work with are gliders on a horizontal air track. As we are concerned only with motion along the air track, vector notation is not needed. The two gliders are denoted by subscripts 1 and 2. Unprimed quantities refer to times before the collision and primed quantities to times after the collisions. Before the collision particle 2 is at rest. The direction of motion for particle 1 before the collision is taken as positive. Let \( m \) stand for mass and \( v \) for velocity. For elastic collisions conservation of energy and momentum give

\[
\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1' + \frac{1}{2} m_2 v_2' \tag{2}
\]

\[
m_1 v_1 = m_1 v_1' + m_2 v_2' \tag{3}
\]

The initial velocity \( v_1 \) is assumed to be known. These equations can be solved (very easily if you are clever!) for \( v_1' \) and \( v_2' \) to give

\[
v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad \text{and} \quad v_2' = \frac{2m_1}{m_1 + m_2} v_1. \tag{4}
\]

For what circumstances does the incident glider reverse its direction during the collision?

*Perfectly inelastic collisions.* Energy is not conserved. The gliders stick together during the collision and \( v_1' = v_2' \). Momentum conservation gives \( m_1 v_1 = (m_1 + m_2) v_1' \). This equation gives

\[
v_1' = v_2' = \frac{m_1}{m_1 + m_2} v_1. \tag{5}
\]

3 Collision Experiments

3.1 Description

One glider is at rest on the horizontal air track. Another glider, whose mass may be larger, smaller, or the same, strikes the glider at rest. Two photogates measure the velocities of the gliders before and after the collision.

3.2 Photogate Setup

Cards inserted in the top of the gliders will serve to interrupt the photogate beams. The cards can be moved along the top of the gliders. To minimize the effects of friction arrange the photogates and cards so that the velocities are measured as close to the collision point as feasible. For elastic collisions one of the photogates will measure 2 velocities.

3.3 Programming

Program DataStudio by clicking on the digital channel that the photogate sensor is plugged in. After select photogate under ScienceWorkshop Digital Sensors from the dropdown menu. Do this again for the second photogate sensor. For each photogate sensor enter the width of the card on top of the glider under the constants tab inputing the width where it says Flag Length. Open the table display window by dragging its icon to one of the photogate’s velocity icon. Drag the Table 1 icon to the Velocity icon for the second Photogate so that both velocities are in the same table.
3.4 Elastic Collisions
The spring bumpers on the gliders are nearly but not completely elastic. If two small disk magnets are available, these can be mounted in the appropriate orientation on the bumpers for repulsion. Take data for \( m_1 = m_2 \), \( m_1 < m_2 \), and \( m_1 > m_2 \). Try a few velocities for \( m_1 \). Compare your data to the equations for elastic collisions. How well is energy conserved? Do your results differ from the theory in the way you might expect if a bit of KE is lost in the collision?

An elastic collision is defined as a collision where the KE of the objects is the same before and after the collision. Is the KE the same during the collision? If not, where does the energy go during the collision? Explain for the case of spring bumpers and magnet bumpers.

3.5 Perfectly Inelastic Collisions
To make the two gliders stick together during the collision use 4 paper clips to attach 5-cm pieces of velcro to one bumper of each glider. Use the same combinations of masses as in the previous section and analyze your data in the same way, using the equation for a perfectly inelastic collision. Verify that KE is lost. Where does the energy go?

4 Comment
When using the concepts of momentum and energy be aware that waves carry both of these quantities. When the gliders collide do you hear a sound? Could this contribute to the energy and momentum balance in your experiments. Probably not. For example the sound from a “quiet conversation” has a sound intensity of about \( 10^{-6} \) W/m\(^2\). If it is assumed that this level of sound is produced by the collision for 0.1 s at a distance of 1 m from the collision, and that the sound radiates isotropically from the collision point, the total energy in the sound wave would be \( 1.3 \times 10^{-6} \) J. How does this compare with the kinetic energy of your gliders?

5 Error analysis
As usual, take care to make estimates of the uncertainties involved in all of your measurements, including the length of the cards used to interrupt the photogate, the masses, and any other quantities you measure. When making statements about conservation of momentum and energy, be sure to specify the uncertainties in all quantities of consequence.