Collisions In One Dimension

1 Purpose
- To investigate conservation of energy and momentum in one dimensional two-body collisions.
- Learning to make histograms Gaussian distribution

2 Theory
2.1 Conservation of Momentum
Let the mass of a particle be $m$ and its velocity $v$. The momentum $p$ is defined by $p = mv$ and Newton’s 2nd law may be written as $F = dp/dt$ where $F$ is the net force on $m$. If $F = 0$ this yields $p = \text{constant}$, which is Newton’s 1st law.

For a system of particles we identify each particle with a subscript $i$ so that $F_i = dp_i/dt$. If each side of this equation is summed over all the particles

$$
\sum_{i=1}^{n} F_i = F = \frac{d}{dt} \sum_{i=1}^{n} p_i = \frac{d}{dt} P,
$$

where $n$ is the number of particles, $F$ is the net force on the system, and $P$ is defined as the total momentum of the system. In the sum over $F_i$, the internal forces (the forces between the particles), can be excluded, as Newton’s 3rd Law guarantees that they will cancel out. We don’t even have to know them. The terms $F_i$ in the sum of the above equation can therefore be taken to refer only the forces outside of the system acting on the system. A consequence of the equation is therefore that if any component of the sum of external forces applied to a system is zero, that component of the system’s momentum $P$ will be a constant. This is true even if some of the particles that make up the system make violent collisions with each other, because collisions involve only internal forces. One can say that the component of momentum is the same before the collisions as it is after the collisions, or that the component of momentum is conserved. If all components of $F = 0$ then all components of the momentum are conserved.

Conservation of energy and conservation of momentum are different statements. Momentum may be conserved when energy is not and vice versa.

2.2 Some Definitions
In this lab we restrict ourselves to two body collisions. A collision is said to be elastic if the combined KE of the two objects is the same before the collision as after the collision. If the
KE is less after the collision the collision is called inelastic. If the two objects stick together the collision is called completely or perfectly inelastic. In this last case the maximum KE that can be lost (consistent with other conservation laws) is lost.

2.3 Calculations for Two-Particle One-Dimensional Collisions

Consider a situation with two particles where the first, of mass $m_1$ and initial velocity $v_1$, collides with a second particle of mass $m_2$ which is initially at rest. Denote the velocities after the collision with primes. As we are working only in one dimension, we will deal only with the components in that direction and vector notation is not needed.

For an elastic collision, conservation of energy and momentum give

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_{1}'^2 + \frac{1}{2} m_2 v_{2}'^2$$

$$m_1 v_1 = m_1 v_{1}' + m_2 v_{2}' .$$

(2)

These equations can be solved (very easily if the right method is chosen!) for $v_{1}'$ and $v_{2}'$ to give

$$v_{1}' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

and

$$v_{2}' = \frac{2 m_1}{m_1 + m_2} v_1 .$$

(4)

For perfectly inelastic collisions, energy is not conserved, and the corresponding conservation equation cannot be used. However, since the particles stick together, there is only one unknown final velocity. Momentum conservation then gives $m_1 v_1 = (m_1 + m_2) v_{1}'$, which can be solved to give

$$v_{1}' = v_{2}' = \frac{m_1}{m_1 + m_2} v_1 .$$

(5)

3 Equipment Used

This experiment uses gliders as colliding objects, floating on an air track to approximate frictionless motion. Photogate sensors are used to measure the velocity of the gliders. Stands are provided to set up the photogate sensors in the appropriate orientation. Cardboard sheets are mounted on the gliders to provide the start and stop signals which the sensors use to calculate velocity. As was the case when you used the photogate in other experiments, you must provide DataStudio with the size of the cards for the velocity measurements to be meaningful.

As collisions between the cars’ spring bumpers are nearly - but not completely - elastic, magnets may be provided. These can be mounted on the bumpers to provide repulsion without friction. Velcro strips can be attached to the glider bumpers with paper clips so that colliding gliders stick together, in a completely inelastic collision.

Remember to be careful with the gliders, and not to place them on the air track unless the blower is on.
4 Experiment Procedures

4.1 General Procedures

Turn on the air blower. Set up the photogates so that the velocity of the carts can be measured before and after collisions. Choose the position of sensors to make the best possible measurement of velocities immediately before and immediately after collisions.

Plug the two photgate sensors into digital channels. Program DataStudio for the first sensor by clicking on the corresponding digital channel and selecting photogate from the drop down menu of ScienceWorkshop Digital Sensors. Enter the width of the card on top of the glider under the constants tab inputing the width where it says Flag Length. Follow the same procedure for the second sensor. Open the table display window by dragging its icon to one of the photgate’s velocity icon. Drag the Table 1 icon to the Velocity icon for the second Photogate so that both velocities are in the same table.

4.2 Elastic Collisions

4.2.1 Procedures

Perform collisions where one of the carts, of mass $m_2$ is initially at rest. Take data for $m_1 = m_2$, $m_1 < m_2$, and $m_1 > m_2$. Try a few velocities for $m_1$. Try collisions with and without magnets on the bumpers.

4.2.2 Analysis and questions

1. For what circumstances does the incident glider reverse its direction during the collision?

2. Explain how you chose the position of the sensors to make the best possible measurement.

3. Make estimates of the uncertainties involved in all of your measurements, including the length of the cards used to interrupt the photogate, the masses, and any other quantities you measure.

4. Compare your data to the equations for elastic collisions. Are momentum and energy conserved within the uncertainties of your measurement?

5. Do your results differ from the theory in the way you might expect if a bit of KE is lost in the collision?

6. If energy appears to be lost, where did the energy go? Explain any differences between results with spring bumpers and magnets on the bumpers.

4.3 Perfectly Inelastic Collisions

4.3.1 Procedures

To make the two gliders stick together during the collision use 4 paper clips to attach 5-cm pieces of Velcro to one bumper of each glider. Use the same combinations of masses as in the previous section.

Finally, for one combination of masses, do the experiment 25 times.
4.3.2 Analysis and questions

1. Compare your data to the equations for perfectly inelastic collisions. Are differences consistent within the uncertainties in your measurement?

2. Verify that KE is lost. Where does the energy go?

3. This next analysis is lengthy, so do the rest of the experiment first and come back to this later, in lab if you have the time, otherwise after class.

   For the experiment you did 25 times:

   (a) Write a python program that histograms the values of \( v' / v_1 \) you obtained. Choose the bin size of the histograms so that you get multiple entries in at least a few bins. That is, if your bin size is too small then all bins will have either 0 or 1 entry, and if your bin size is too big all entries will be in one bin.

   (b) Have the program calculate the average and standard deviation of your values of \( v' / v_1 \).

   (c) What fraction of the measurements are outside the range average \( \pm \) standard deviation and average \( \pm \) two times the standard deviation?

5 Comment

When using the concepts of momentum and energy be aware that waves carry both of these quantities. When the gliders collide do you hear a sound? Could this contribute to the energy and momentum balance in your experiments. Probably not. For example the sound from a “quiet conversation” has a sound intensity of about \( 10^{-6} \) W/m\(^2\). If it is assumed that this level of sound is produced by the collision for 0.1 s at a distance of 1 m from the collision, and that the sound radiates isotropically from the collision point, the total energy in the sound wave would be \( 1.3 \times 10^{-6} \) J. How does this compare with the kinetic energy of your gliders?