Collisions In One Dimension

Equipment Capstone, meter stick, 2 photogates, air track, 2 gliders of same mass, 1 glider of different mass, 2 cards for top of gliders to interrupt photogate beam, plastic circular container with 2 pieces of inelastic connectors

1 Purpose

To investigate conservation of energy and momentum in one dimensional analysis using 2 body collisions.

2 Theory

2.1 Conservation of Momentum

Let the mass of a particle be $m$ and its velocity $\vec{v}$. Momentum $\vec{p}$ is defined as $\vec{p} = m\vec{v}$ and Newton’s 2nd law may be written as $\vec{F} = \frac{d}{dt}\vec{p}$ where $\vec{F}$ is the net force on $m$. If $\vec{F} = 0$ this yields $\vec{p}$ equals a constant which is Newton’s 1st law.

For a system of particles we identify each particle with a subscript $i$ so that $\vec{F}_i = \frac{d}{dt}\vec{p}_i$. If each side of this equation is summed over all the particles then,

$$\sum_{i=1}^{n} \vec{F}_i = \vec{F} = \frac{d}{dt} \sum_{i=1}^{n} \vec{p}_i = \frac{d}{dt} \vec{P},$$

where $n$ is the number of particles, $\vec{F}$ is the net force on the system, and $\vec{P}$ is defined as the momentum (or the total momentum) of the system. In the sum over $\vec{F}_i$ the internal forces (the forces between the particles) can be excluded as Newton’s 3rd Law guarantees that they will cancel out. We don’t even have to know them. A consequence of this equation is that if any component of $\vec{F} = 0$ then the component of $\vec{P}$ will be a constant. Why? This will be true even if some of the particles make violent collisions with each other. Collisions produce only internal forces. One can say that the component of momentum is the same before the collisions as it is after the collisions, or that the component of momentum is conserved. If all components of $\vec{F} = 0$ then all the components of momentum are conserved.

Conservation of energy and conservation of momentum are different statements. Why can momentum be conserved when energy is not?

2.2 Definitions

In this lab we restrict ourselves to two body collisions. A collision is said to be elastic if the combined KE of the two objects is the same before the collision and after the collision. If the KE is less after the collision the collision is called inelastic. If the two objects stick together the collision is called a completely or perfectly inelastic collision. In this last case, maximum KE is lost.
2.3 Two Particles: One Dimensional Collision

Elastic collisions. The 2 particles will be gliders on a horizontal air track and we will only be concerned with the motion along the air track. Vector notation is not needed. The gliders will be referred to by subscripts 1 and 2. Unprimed quantities will refer to time before the collision and primed quantities to times after the collisions. Before the collision particle 2 will be at rest. The direction of motion for particle 1 before the collision will be taken as positive. Let $m$ stand for mass and $v$ for velocity. For elastic collisions conservation of energy and momentum are given by,

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1' + \frac{1}{2}m_2v_2'^2,$$

and

$$m_1v_1 = m_1v_1' + m_2v_2'.$$

The quantity $v_1$ is assumed known. These equations can be solved (very easily if you are clever!) for $v_1'$ and $v_2'$ to give

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1,$$

and

$$v_2' = \frac{2m_1}{m_1 + m_2}v_1.$$

In what situation does the incident glider reverse its direction during the collision? Perfectly inelastic collisions. Energy is not conserved. Why? The gliders stick together during the collision and $v_1 = v_2$. Momentum conservation gives $m_1v_1 = (m_1 + m_2)v_1'$. This equation gives

$$v_1' = v_2' = \frac{m_1}{m_1 + m_2}v_1.$$

3 Collision Experiments

3.1 Description

One glider is at rest on the horizontal air track. Another glider, whose mass may be larger, smaller, or the same, strikes the glider at rest. Two photogates measure the velocities of the gliders before and after the collision.

3.2 Photogate Setup

Index cards inserted in the top of the gliders will serve to interrupt the photogate beams. The cards can be moved along the top of the gliders. To minimize the effects of friction arrange the photogates and cards so that the velocities are measured as close to the collision point as feasible. Keep in mind for elastic collisions one of the photogates will measure 2 velocities.

3.3 Programming

Program Capstone by clicking on the digital channel that has the photogate sensor plugged in. After select One Photogate (Single Flag) from the dropdown menu. Do this again for the second photogate sensor. For each photogate sensor enter the width of the card on top of the glider. To do this go to the gear icon next to properties in the lower right side of the hardware setup window. Next, adjust the Flag Width to the length of the index card on the glider. Now open the table display window by dragging the Table icon from the Displays column to the white screen of Capstone. Configure the data table as illustrated on the next page.
3.4 Elastic Collisions

The spring bumpers on the gliders are fairly but not completely elastic. Take data for $m_1 = m_2$, $m_1 < m_2$, and $m_1 > m_2$. Try a few velocities for $m_1$. Keep track of the gliders and which photogate they go through. Compare your data to the equations for elastic collisions. How well is energy conserved? How does your results differ from the theory? Do you expect a bit of KE to be lost in the collision?

An elastic collision is defined as a collision where the KE of the objects is the same before and after the collision. What is occurring during the collision? What happens to the energy during the collision? Explain for the case of spring bumpers. If you have used disk magnets how would they influence the experiment? Explain.

3.5 Perfectly Inelastic Collisions

In this section you will set up the gliders for Inelastic collisions. Insert the inelastic collision needle to the side of one glider. Next, insert the wax receptacle into the other glider. Below is an image of the inelastic needle inserted on the side of the glider.

Use the same combinations of masses as in the previous section and analyze your data in the same way. Use the equation for a perfectly inelastic collision to verify that KE is lost. What is occurring to the energy? How does the energy get dissipated?
4 Comment

When using the concepts of momentum and energy be aware that waves carry both of these quantities. When the gliders collide do you hear a sound? Could this contribute to the energy and momentum balance in your experiments? Hint an example is that sound from a “quiet conversation” has a sound intensity of about $10^{-6} \text{ W/m}^2$. If it is assumed that this level of sound is produced by the collision for 0.1 s at a distance of 1 m. The sound radiates isotopically from the collision point and the total energy in the sound wave would be $1.3 \times 10^{-6} \text{ J}$. How does this compare with the kinetic energy of the gliders?