A) Dp branes describe gauge theories.

We will consider mostly the example D3 (4-d gauge theories)

\[ \text{Lowest lying states have mass } M \sim \delta_{ij} M_5^2 \]

They form an \( \mathcal{N} = 4 \) D=4 multiplet (1 (1), 4 (\( \frac{1}{2} \)), 5 (01)) \( \text{massive} \)

(1 (1), 4 (\( \frac{1}{2} \)), 6 (01)) \( \text{massless} \).

Fields: \( A_\mu, \Phi^{IJ}, \Phi^{IJ} \)

\[ (\Phi^{IJ})^4 = \frac{1}{2} \epsilon_{IJKL} \Phi^{KL} \]

- All fields in adjoint of \( U(N) \)
- When branes are at different positions

\[ U(N) \rightarrow U(1)^N \]

Special point: All branes at same position i.e.

\[ \langle \Phi^{IJ} \rangle = 0 \]

In this case \( U(N) \) is unbroken.
Also: LAGRANGIAN IS $N=4$ SUGER YANG-MILLS

$$S = \frac{1}{g_s} \int d^4x \left( \text{Tr} \ F_{\mu\nu} F^{\mu\nu} + \ldots \right) e^{-\phi(x,0)}$$

$\phi(x,y)$ IS THE DILATON. WE DEFINED IT SO THAT

$$\phi(x,y) \to 0 \quad \text{as} \quad |y| \to \infty$$

$g_s$ IS THE STRING COUPLING AWAY FROM THE BRAVE.

$N=4$ D=4 HAS ALL $\beta$ FUNCTIONS $= 0$ ED $\langle \phi^2 \rangle = 0$ IMPLIES THAT THE THEORY IS CONFORMAL.

LARGE N LIMIT: $g_s = \frac{g_s^2}{g_{YM}} \to 0$ \[ g_s N = \text{constant} \]

$N \to \infty$ \[ \equiv \text{'t Hooft parameter} \]

IN THIS LIMIT CLOSED STRINGS DO NOT INTERACT.

AT ENERGIES $\mathcal{E} \gg N$, THE $N$ D3 BRAVE SYSTEM DESCRIBES JUST A $U(N)$ GAUGE THEORY WITHOUT GRAVITY.

Example: DILATON ABSORPTION.

$$S = S_{\Pi B} + S_{D3} = \frac{1}{g_s^2} \int d^4x \sqrt{-g} e^{-2\phi} \left( \partial \phi \partial \phi + \partial \phi \partial \phi \right) + \frac{1}{g_s N} \int d^4x e^{-\phi(x,0)} \left[ \text{Tr} F_{\mu\nu} F^{\mu\nu} + \ldots \right]$$
When $\delta s \to 0$, $\phi$ interacts only with the open string fields ($A_\mu + \text{SUSY partners}$).

Absorption $\mathcal{L}$:

$$A \sim \text{Im} \int d^4x e^{iK_\mu X^\mu} \left< \text{Tr} F_{\mu\nu}^2(x) \left. \text{Tr} F_{\mu\nu}^2 \right|_0 \right>$$

Notice: Dilaton momentum $K_\mu, K^\mu \in \mathbb{C}$, $I_{4,\ldots,9}$

Obey:

$$K_\mu K^\mu + K_\mu K^\mu = 0$$

The 4 components of the momentum along the D3 brane directions are unconstrained 2D $A(K_\mu)$ is proportional to an off-shell Green's function of the gauge theory. Perturbative when $\delta s \ll 1$

Other description:

D3 branes have nonzero tension and charge. They curve space-time.

$$ds^2 = f^{-\frac{1}{2}} dx_\mu^2 + f^{\frac{1}{2}} (dv^2 + r^2 d\Omega_5^2)$$

Line-element radial of sphere of coord. $L_{0,1,2,3}$ unit radius in 5-D, $\Sigma_5$

$$A_{0123} = -\frac{1}{2} (f^{-1} - 1)$$

$f = 1 + \frac{4\pi g_5 N}{(r \Sigma_5)^4}$
For \( r \to \infty \) we recover flat space. For \( r \to 0 \) the metric is regular:

\[
\frac{1}{r^2} \sim \frac{4 \pi g_s N}{(\alpha M_s)^4}.
\]

Define \( T \)-coordinate as \( r = R e^{T/R} \):

\[
R = (4 \pi g_s N)^{1/4} \left( \frac{1}{M_s} \right).
\]

Metric:

\[
ds^2 = dt^2 + e^{2T/R} \, dx^2 + R^2 \, dr^2
\]

This is the metric of the homogeneous space \( \text{AdS}_5 \times S_5 \) \( \uparrow \) Anti-de Sitter space.

Both \( \text{AdS}_5 \) and \( S_5 \) have constant curvature. Curvature radius for both is \( R \).

\[
g_s N = g_{YM} N = x \left( \text{'t Hooft coupling} \right)
\]

\( x \gg 1 \) \( \Rightarrow \) \( R \gg \frac{1}{M_s} = L_s \). This ensures that the supergravity action receives only small higher-curvature corrections.

Equivalently: \( \sigma \)-Model corrections to the metric are negligible when \( x \gg 1 \).
CLOSED STRING LOOP CORRECTIONS NEGLECTIBLE WHEN $\delta_s \rightarrow 0$.

**IN SUMMARY:** IIb SUPERGRAVITY METRIC ACCURATE EVERYWHERE IN THE LARGE-$N$ LIMIT FOR $X = \delta_s N \gg 1$.

**NOTICE:** THIS REGIME IS THE OPPOSITE OF THE PERTURBATIVE REGIME FOR GAUGE THEORY ($X \ll 1$).

**FUNDAMENTAL CONSEQUENCE:** WHEN $X \gg 1$ WE CAN COMPUTE THE ABSORPTION CROSS-SECTION ALSO USING CLASSICAL (SUPER) GRAVITY, SINCE THAT DESCRIPTION IS EVERYWHERE ACCURATE.

**THUS:**
- WHEN $X \ll 1$ $A \propto$ PERTURBATIVE OFF-SHELL CORRELATOR IN SYM.
- WHEN $X \gg 1$ $A \propto$ CLASSICAL (GEOMETRICAL) SCATTERING AMPLITUDE

**NOTICE:** BY COMPUTING $A$ AT $X \gg 1$ WE MAY BE ABLE TO SAY SOMETHING ABOUT NON-PERTURBATIVE, STRONGLY COUPLED SYM.

- LET US COMPUTE NEXT $A$ USING CLASSICAL GEOMETRY.
DILATON IN D3 BACKGROUND OBEYS E.O.M.

$$\nabla_M \phi^{MN} \sqrt{-g} \nabla_N \phi = 0.$$

CLASSICALLY: A PART OF INCOMING WAVE IS REFLECTED AND A PART IS ABORBED.

\begin{center}
\begin{tikzpicture}
  \draw[->, >=stealth, line width=1pt] (-1,0) -- (2,0);
  \fill[black!20] (0,0) ellipse (0.5 and 0.5);
  \node at (0,0) {\text{IN}};
  \node at (2,0) {\text{OUT}};
  \node at (-1,0) {\text{ABSORBED}};
  \node at (0,2) {\text{AdS}_5 \times S^5};
  \node at (2,2) {\mathbb{R}^{(1,9)}};
  \draw[->, >=stealth, line width=1pt] (0,2) .. controls (-0.5,1.5) and (-0.5,1.5) .. (0,1);
  \node at (0,1) {\text{TRANSITION POINT}};
  \node at (2,1) {\text{BEFORE } \text{ AdS}_5 \times S^5 \text{ GEOMETRY AND \text{FLAT } } \mathbb{R}^{(1,9)} \text{ GEOMETRY}};
  \node at (0,-2) {R_0 = \mathbb{R}^+ \text{ RE}};
\end{tikzpicture}
\end{center}

SIMPLEST case: $\phi$ INDEPENDENT OF ANGULAR VARIABLES (CONSTANT ON $S^5$)

$$\phi = e^{i K x} \phi(T)$$

$$\phi(T_0) = 1$$

FLUX IN: $F_{IN} = K$

$$F_{IN} = \text{FLUX OUT: } \sum_{T_0} F_{OUT} = \int e^{i \frac{T}{R} \phi(T)} \partial_T \phi(T)\bigg|_{T_0}$$

$$A = 1 - \frac{F_{OUT}}{F_{IN}} = i \frac{K}{e^{i \frac{T}{R} \phi(T)} \partial_T \phi(T)\bigg|_{T_0}}$$
II B STRINGS ON $AdS_5 \times S^5 \equiv SU(N)$ SYM $N=4$

AT $g_s \to 0$

$g_s N = \text{constant} \gg x$

$g_s \to 0$

AT $g_s^2 N = x$

FOR ANY $x$

0) IT IS A DUALITY: $x \ll 1$ WEAKLY COUPLED

DESCRIPTION IS PERTURBATIVE SYM. II B ON $AdS_5 \times S^5$

IS STRONGLY COUPLED SINCE CURVATURE RADIUS

$R \propto (x)^{1/4} L_5 \ll L_5$

$x \gg 1$ $R \gg L_5$, II B SUPERGRAVITY DESCRIPTION

IS WEAKLY COUPLED. SYM IS STRONGLY COUPLED.

1) (SUPER) SYMMETRIES IN BOTH DESCRIPTIONS ARE THE SAME

- BOJOWIC SYMMETRIES OF SYM: $SO(2,4) \otimes SU(4)$

\[\text{CONFORMAL} \quad \overset{\text{R-SYMMETRY}}{\text{GROUP IN 4-D}}\]

= BOJOWIC SYMMETRIES OF II B ON $AdS_5 \times S^5 \equiv$ ISOMETRIES.

$S^5$ HAS ISOMETRY GROUP $SU(4)$ (COVER OF $SO(6)$; COVER NEEDED BECAUSE II B HAS FERMIONS)

ISOMETRY OF $AdS_5$ IS $SO(2,4)$.

TO SEE THIS DEFINE $AdS_5$ AS HYPERBOLOID
\[ \mathbf{\nabla} \mathbf{V} + X^\mu A_{\mu \nu} X^\nu = -R^2 \quad \gamma_{\mu \nu} = (-1, 1, 1, 1) \]

**Hyperboloid has Isometry SO(2, 4).**

**Parametrize Hyperboloid as:**

\[ X^\mu = U x^\mu \quad V = -\frac{R^2}{U} - U x^\mu \gamma_{\mu \nu} x^\nu \]

\[ dV = R^2 \frac{dU}{U^2} - dU \quad x^\mu \gamma_{\mu \nu} x^\nu - 2U \quad x^\mu \gamma_{\mu \nu} dx^\nu \]

\[ dU dV = R^2 \frac{dU^2}{U^2} - dU^2 \quad x^\mu \gamma_{\mu \nu} x^\nu - 2 U dU \quad x^\mu \gamma_{\mu \nu} dx^\nu \]

\[ dX^\mu = dU x^\mu + U dx^\mu \]

\[ \gamma_{\mu \nu} dX^\mu dX^\nu = dU^2 x^\mu \gamma_{\mu \nu} x^\nu + U^2 dx^\mu dx^\nu \gamma_{\mu \nu} + 2 U dU x^\mu \gamma_{\mu \nu} dx^\nu \]

**Induced Metric:**

\[ dU dV - dX^\mu dX^\nu \gamma_{\mu \nu} = R^2 \frac{dU^2}{U^2} + U^2 \quad dx^2 \]

Set \( U = e^{TR} \). Metric \( ds^2 = dt^2 + e^{2TR} \quad dx^2 \).

**= Supersymmetries.**

**In AdS_5 \times S_5 32 SUSY:** \( Q_\alpha^I \quad \mathcal{Q} \)

**In SYM 32 SUSY** \( Q_\alpha^I + \text{Conformal SUSY} \mathcal{Q}_\alpha^I \)

**Super Group is** \( U(2, 2/4) \)
2) To make further checks of the AdS/CFT duality we must define it more precisely:

**Conjecture:** \( \phi(T_1 x, \theta) = \text{field on } AdS_5 \times S_5 \)

(Scalar, for simplicity)

\[
\phi(T_1 x, \theta) = \sum_M \phi_M(T_1 x, \theta) Y^M(\theta)
\]

\( \phi_M(T_1 x) \) is a 5-D field of mass \( m \).

**E.O.M.**

\[
\left( - \partial_T e^{T/R} \partial_T - e^{2T/R} \partial_\mu \partial^\mu + (mR)^2 e^{T/R} \right) \phi_M = 0.
\]

In the limit \( T \to +\infty \)

\[ \phi_M(T_1 x) \approx e^{-\Delta T/R} \phi_M(x) \]

Substitute in E.O.M.:

\[ \Delta(\Delta - 4) = (mR)^2 \]

Two roots: \( \Delta_+ > \Delta_- \)

Choose the (generic) solution \( e^{-\Delta T/R} \phi_M(x) \)

Since \( AdS_5 \) has a boundary, the partition function of \( \text{IIB Superstring on } AdS_5 \times S_5 \) depends on 4-D fields \( \phi_M(x) \)

\[ \mathcal{Z}_{\text{IIB}} \left[ \phi_M(x) \right] \]

Notice: At \( x \gg 1 \) supergravity O.K., semiclassical
\[ Z_{\mathrm{IIB}} \left( \varphi_M(x) \right) \approx e^{-S_{\mathrm{IIB}} \left( \varphi \right)} \bigg|_{\text{on-shell}} \]

**On-shell** means \( S_{\mathrm{IIB}} \) evaluated at the stationary point:

\[ \frac{\delta S_{\mathrm{IIB}}}{\delta \varphi} = 0 \quad \Rightarrow \quad \varphi_M(T, x) \rightarrow e^{-A_T R} \varphi_M(x) \quad T \rightarrow +\infty \]

Also, at \( x \ll 1 \)

\[ Z_{\mathrm{IIB}} \left( \varphi_M(x) \right) = \left\langle e^{-\int d^4 x \varphi_M(x) O_M(x)} \right\rangle \]

(Open-string at \( E \ll M_s \))

(VEV over SYM \( N=4 \))

Composite operator in \( SU(N) \) \( N=4 \) SYM

**Example:** When \( \varphi \) is the dilaton \( O_0(x) = \text{Tr} F_{\mu \nu}^2 \)

**Conjecture:** The equality holds for any \( x \).

In particular, at \( x \gg 1 \) the generating functional of the SYM \( SU(N) \) \( N=4 \) gauge theory is given by the classical on-shell action of IIB supergravity.
**Question:** What is the conformal dimension of $O_M$?

**Answer:**

$$ds^2 = dT^2 + R e^{2T/R} dx^a dx^b \eta_{ab}$$

**Symmetry:** $x^a \rightarrow \lambda x^a$, $T \rightarrow T - R \log \lambda$ acts as a dilation on the boundary.

Thus:

$$\phi^M_\lambda(T - R \log \lambda, \lambda x) = \phi^M(T, x)$$

In $\mathbb{R}^D$:

$$\lambda^{-\Delta_-} \phi^M_\lambda(\lambda x) = \phi^M(x)$$

$$\phi^M_\lambda(\lambda x) = \lambda^{-\Delta_-} \phi^M(x)$$

$\phi^M$ has conformal dimension $\Delta_-$.  

$$\int d^Dx \; \phi^M(x) O_M(x)$$  has conformal dimension $0$.

Thus $O_M$ has dimension $4 - \Delta_- = \Delta_+$.  

**Exception to this rule:**

When $1 \leq \Delta_- \leq 2$, we can define a source that behaves as:

$$\phi^M(T, x) \rightarrow e^{-\Delta_+ T/R} \phi^M(x)$$

and still get a meaningful correspondence with $\text{SYM}$. In that case, the dimension of the operator $O_M$ is $\Delta_- \geq 1$; allowed by unitarity.
Since unitary representations of $\mathfrak{so}(2,4)$ have conformal dimension $\geq 1$.

Example: Break $SU(N) \to U(1)^N$ (i.e. choose $\langle \Phi^{IJ} \rangle \neq 0$) then the operator

$$\text{Tr} \langle \phi^I \phi^{IJ} \rangle$$

has dimension 1.

When $SU(N)$ unbroken, $\langle \phi^{IJ} \rangle = 0$ and the minimum conformal dimension of any operator in $\text{SYM}$ is $\Delta \equiv 2$.

Another check of AdS/CFT.

In $\mathcal{N} = 4$, $D = 4$ the fields $A^a_i$, $\lambda^I$, $\phi^{IJ}$ fit into a chiral superfield:

$$\phi^{IJ} + \theta^I \lambda^J + \theta^I \psi \gamma^M \phi^{IJ} F_{MN} + \ldots$$

up superspace (Grassmann) coordinate $\theta^I$ in this expansion.

$[IJ] = 6$ of $SU(4)$ = vector rep. of $\mathfrak{so}(6)$

label 6 of $\mathfrak{so}(6)$ with index $A$.

Superfield:

$$\Phi^A = \phi^A + \ldots$$

$\Phi^A$ is the simplest example of chiral (short)
MULTIPL ET OF $U(2,2/4)$. IN GENERAL THESE SHORT MULTIPL ET S BELONG TO THE $m$-TIMES
SYMMETRIC IRREP OF $SO(6)$:
AND HAVE CONFORMAL
DIMENSION $\Delta = n$.

$$\Phi_{A_1 \ldots A_n} = \Phi_{A_1 \ldots A_n} + \ldots$$

$$\text{Tr} \Phi_{A_1 \ldots A_n} \text{TRACESLESS}$$

IS CHIRAL.

IMPORTANT CONSEQUENCE:

AT $x \equiv g^2 N = 0$ CONFORMAL DIMENSION OF THIS
MULTIPL ET IS $\Delta = n$

BUT SINCE $\Delta$ IS PROTECTED, BECAUSE THE MULTIPL ET IS
CHIRAL, $\Delta = n$ ALSO AT $x \gg 1$.

ADS/CFT CONJECTURES THAT AT $x \gg 1$ $N=4$ SYM IS
THE SAME AS SEMICLASSICAL IIB SUPERGRAVITY ON $AdS_5 \times S_5$.
WE MUST FIND IN IIB SUGRA ALL CHIRAL MULTIPL ET S
OF $N=4$ SYM.

IN PARTICULAR SCALARs WITH MASS $m$ SUCH THAT:

$$m(m-4) = (Rm)^2 \quad \text{FOR} \quad n = 1, 2, 3, \ldots$$

+ SUPERPARTNERS.

THEY EXIST! THEY ARE THE KALUZA-KLEIN MODES
OF IIB ON $AdS_5 \times S_5$ COMPUTED IN PHYS. REV. D 32
389 (1985)
Example: \[ \text{Tr } \Phi^A \Phi^B \] 

The scalar components of the multiplet are: 

- \[ \text{Tr } \phi^A \phi^B \] with \( \Delta = 2 \), in \( \frac{20}{R} \) of \( SU(4) \times SO(6) \)
- \[ \text{Tr } \lambda^I \lambda^J \] with \( \Delta = 3 \), in \( \frac{10}{C} \) of \( SU(4) \)
- \( \mathcal{L}_{N=4} + i \text{Tr } F_{\mu} \tilde{F}^{\mu} \) with \( \Delta = 4 \), in \( \frac{1}{C} \) of \( SU(4) \)

\( N=4 \) Lagrangian

Spectrum of KK Modes

\[ \begin{array}{c}
(Rm)^2 \\
\end{array} \]

\[ \begin{array}{c}
\bullet 1c \\
\bullet 4.5c \\
\bullet 10c R \\
\bullet 2.5c R \\
\bullet 5c R \\
\bullet 10c R \\
\bullet 50c R \\
\bullet 105c R \\
\bullet 20c R \\
\bullet 205c R \\
\end{array} \]

\[ \Delta (\Delta - 4) = -9 \quad \text{and} \quad \Delta, = 2 \]
\[ \Delta (\Delta - 2) = -3 \quad \text{and} \quad \Delta, = 3 \]
\[ \Delta (\Delta - 4) = 0 \quad \text{and} \quad \Delta, = 4 \]
1) R-symmetry $SU(4)$ is anomalous in SYM since $\lambda^i_\alpha$ are chiral fermions in the 4 of $SU(4)$.

In supergravity on $AdS_5 \times S_5$, there exist gauge vectors of $SO(6) \sim SU(4)$. They are sources for $J^{ij}_\mu$ ($SU(4)$ R-current).

$$\langle e^{-i\int d^4x A^{i\mu}_I J^{ij}_\mu} \rangle = e^{-W[A]}$$

Under $SU(4)$, $SW = N\frac{1}{16\pi^2} \int d^4x \text{Tr}_{SU(4)} W F \wedge F$

**AdS/CFT:** $W[A] = S_{g_{\mu\nu}}$ on $AdS_5 \times S_5$

$S_{g_{\mu\nu}} = \ldots + \frac{N}{16\pi} \int d^5x C$

$C = \text{Chern-Simons term} = \text{Tr}_{SU(4)} A \wedge dA \wedge dA + \ldots$

$$\delta S_{g_{\mu\nu}} = \frac{N}{16\pi} \int d^4x \text{Tr}_{SU(4)} \omega \wedge F \wedge F = \frac{N}{16\pi} \int d^4x \text{Tr}_{SU(4)} \omega \wedge F \wedge F$$

2) Scale transformation is also anomalous

$$\delta S_{g_{\mu\nu}} \frac{\delta}{\delta g_{\mu\nu}} = \text{const} \cdot E_4 + \text{const}' A$$

$E_4 = \text{Euler density} = \int d^4x \sqrt{g} (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - R_{\mu\nu} R^{\mu\nu} + R^2)$

$A = \int d^4x \sqrt{g} C_{\mu\nu\rho} C^{\mu\nu\rho} \quad C_{\mu\nu\rho} = \text{Weyl tensor}$
How the anomaly works in $S_{\Pi B}$ on $AdS_5 \times S^5$:

$$S_{\Pi B} = \frac{M_5^8}{g_5^2} \; V_{S_5} \int d^5x \sqrt{-g} \; (R - 2\Lambda) + \ldots$$

$$V_{S_5} \sim R^5, \quad \text{Einstein Eqs.:} \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G_{\mu\nu}$$

$$(1 - \frac{\xi}{2}) R = -5 \Lambda$$

$$R = \frac{10}{3} \Lambda$$

$$S_{\Pi B} = \frac{4}{3} \frac{M_5^8}{g_5^2} R^5 \int d^5x \sqrt{-g} \quad \text{on shell}$$

$$\int d^5x \sqrt{-g} = \int_{-\infty}^{+\infty} e^{4T/R} dT \int d^4x$$

Thus the integral diverges must be regularized by cutting off the integral in $T$, i.e., integrating only up to $T_0$

$$S_{\Pi B}^{T_0} = \int_{-\infty}^{T_0} dT \int d^4x \sqrt{-g} \left( -\frac{4}{3} \frac{M_5^8}{g_5^2} \frac{R^5}{R^2} \right) \quad \Lambda = -\frac{6}{R^2}$$

Near $AdS_5$ boundary at $T = +\infty$ any metric can be cast in the form

$$ds^2 = dT^2 + g_{\mu\nu}(T, x) \; dx^\mu \; dx^\nu$$

$$g_{\mu\nu}(T, x) = e^{2T/R} g_{\mu\nu}^{(0)}(x) + \frac{g_{\mu\nu}(x)}{T} + \frac{T}{R} e^{-2T/R} g_{\mu\nu}^{(2)}(x) + e^{-2T/R} h_{\mu\nu}(x)$$

$g_{\mu\nu}^{(0)}(x) = \text{boundary value of the metric}$
$g^{(1)}_{\mu \nu}(x), g^{(2)}_{\mu \nu}(x) = \text{local functions of } g^{(0)}_{\mu \nu}(x) \text{ only.}$

$h_{\mu \nu}$ depends on the metric inside $AdS_5$

$S_{PB} = C \rho e^{2 T_0 / R} a_0 + C \rho e^{2 T_0} a_1 + C T_0 a_2 + S_{\text{finite}}$

$S_{\text{finite}}$ independent of $T_0$ in the limit $T_0 \to \infty$

$a_0 = \int d^4 x \sqrt{-g} R (g^0) = \int d^4 x \sqrt{-g} R (g^0)$

$a_1 = \rho \int d^4 x \sqrt{-g} R (g^0)$

$a_2 = \rho \int d^4 x \sqrt{-g} \left( \frac{1}{3} R^2 - R_{\mu \nu} R^{\mu \nu} \right)$

Asymptotic form of the metric $\sim e^{2 T_0 / R} g^{(0)}_{\mu \nu}(x)$

$R \frac{\partial}{\partial T_0} = \int d^4 x \sqrt{-g} g_{\mu \nu} \frac{\delta}{\delta g_{\mu \nu}}

R \frac{\partial}{\partial T_0} S_{PB} = C R a_2 + 2 C \rho e^{2 T_0 / R} a_1 + 4 C \rho e^{4 T_0 / R} a_0

\int d^4 x \sqrt{-g} g_{\mu \nu} \frac{\delta}{\delta g_{\mu \nu}} S_{PB} = 2 C \rho e^{2 T_0 / R} a_1 + 4 C \rho e^{4 T_0 / R} a_0

\int d^4 x \sqrt{-g} g_{\mu \nu} \frac{\delta}{\delta g_{\mu \nu}} S_{\text{finite}}$

Thus:

$\int d^4 x \sqrt{-g} g_{\mu \nu} \frac{\delta}{\delta g_{\mu \nu}} S_{\text{finite}} = -8 M_5^2 R_5^2 \frac{\delta}{\delta S_5} a_2$

$R = \frac{1}{M_5} \left( 4 \pi g_5 N \right)^{1/4}$

Central charge proportional to # degrees of freedom of SYM
TWO SAMPLE DYNAMICAL CALCULATIONS

I) TWO POINT FUNCTION OF A SCALAR OPERATOR

\( \mathcal{O}(x) = \) operator of dimension \( \Delta \).

Source \( \phi(x) \) of dimension \( 4-\Delta \).

In AdS/CFT 3 5-d field \( \phi(T,x) \) such that

\[
\phi(T,x) \rightarrow e^{-(4-\Delta)T} \phi(x), \quad T \rightarrow \infty
\]

Simplest case: \( \phi(T,x) \) free scalar of mass

\[
(M_R)^2 = \Delta(\Delta-4)
\]

Action:

\[
S = \int_{\text{AdS}_5} d^5x \sqrt{-g} \left[ g^\mu_\nu \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \right]
\]

E.O.M.:

\[
\left[ -\frac{1}{r^2} \partial_\mu \sqrt{-g} g^\mu_\nu \partial_\nu + m^2 \right] \phi = 0
\]

\[
S|_{\text{on-shell}} = \int_{\partial \text{AdS}_5} d^4x \sqrt{g} g^{TT} \partial_T \phi \phi
\]

Jet:

\[
\phi(T,x) = \sum_k \frac{e^{i k x \cdot \xi}}{(2\pi)^4} \phi_k(x) \epsilon_k
\]

To solve the E.O.M. define new coordinate \( \xi \):

\[
e^{T/R} = R^{-1/2}
\]
\[ ds^2 = \frac{R^2}{z^2} \left( dz^2 + dx^\mu dx^\nu \eta_{\mu \nu} \right) \]

**In terms of new coordinates E.O.M. read**

\[ \left[ - \partial_\tau \left( \frac{R}{z} \right)^3 \partial_\tau + k^2 \frac{R^3}{z^3} + m^2 \left( \frac{R}{z} \right)^5 \right] \tilde{\phi}_k(z) = 0 \]

**Set:** \( \tilde{\phi}_k(z) = z^2 \psi_k(z) \)

\[ \partial_\tau \frac{1}{z^3} \partial_\tau \left( z^2 \psi_k \right) = \partial_\tau \frac{1}{z^3} \left( 2z \psi_k + z^2 \partial_\tau \psi_k \right) \]

\[ = \partial_\tau \left( \frac{2}{z^2} \psi_k + \frac{1}{z} \partial_\tau \psi_k \right) = -\frac{4}{z^3} \psi_k + \frac{1}{z^2} \partial_\tau^2 \psi_k + \frac{1}{z} \partial_\tau \psi_k \]

**Equation for** \( \psi_k : \)

\[ z^2 \partial_\tau^2 \psi_k + 2 \partial_\tau \psi_k - 4 \psi_k - k^2 z^2 \psi_k - (mR)^2 \psi_k = 0 \]

**Euclidean momenta :** \( k^2 \geq 0 \)

**Define** \( k^2 = y \)

\[ \left\{ y^2 \partial_y^2 + y \partial_y - y^2 - [(mR)^2 + 4] \right\} \psi_k = 0 \]

\( \uparrow \text{modifed Bessel eq.} \)

**The solution that vanishes at** \( y \to \infty \) **is** \( \psi_k = K_y(y) \)

\[ y = \sqrt{4 + (mR)^2} = \Delta - 2 \]

\[ \tilde{\phi}_k = z^2 K_y(\Delta z) \sim z^{4 - \Delta} k^{2 - \Delta} \quad z \to 0 \]
\[ \tilde{\phi}_K(z) = \frac{2^{3-\Delta}}{\Gamma(\Delta-2)} K^{\Delta-2} z^2 K_\nu(kz) \]

For small the expansion of reads

\[ K_\nu(kz) = z^{4-\Delta} + \sum_{n < \Delta-2} \alpha_n z^{4-\Delta+2n} \]

\[ + \frac{2^{-2\Delta+4}}{\pi \sin \nu \pi} \frac{1}{\Gamma(\nu) \Gamma(\nu+1)} K^{2\Delta-4} z^{\Delta} \]

\[ + O(z^{\Delta+2}) \]

From \( \text{AdS}/\text{CFT} \)

\[ \frac{S^2}{\delta \phi_k \delta \phi(0)} \bigg|_{\text{on-shell}} = \langle \tilde{O}(K) \tilde{O}(0) \rangle \]

\[ = \lim_{z \to 0} \left( \frac{R}{z} \right)^3 \tilde{\phi}_K(z) \frac{\tilde{\phi}_K(zt)}{z^2} \int P(K^2) \] 

\[ \downarrow \text{PREDOMINANTLY POLYNOMIAL IN } K^2 \quad \text{LOCAL DIVERGENT TERM} = \text{HARMLESS CONTACT TERM} \]
\[ \langle \hat{D}(u) \hat{D}(0) \rangle = \text{const} \cdot k^{24-4} + \text{contact term} \]

Invariance

C.O.U. with conformal

Dynamical information.

II) Wilson loop

At \( g_s = 0 \)

\[ \begin{align*}
\text{ENERGY OF THIS CONFIGURATION: } & E = g \frac{N}{L} \text{ (Coulomb)} \\
\text{ADS dual picture at } g_s \neq 0 \quad g_s N \gg 1 & \rightarrow \text{MINIMAL ENERGY STRING.}
\end{align*} \]

\[ E_{\text{string}} \approx M_s^2 \left[ \int_{-L/2}^{L/2} dx \sqrt{e^{2TR} \left[ (\partial_x T)^2 + e^{2TR} \right]} \right] \]
CHANGE VARIABLE: $U = R \exp{\frac{T}{R}}$

$$E_{\text{string}} \approx 2 M_s^2 \int_0^{L/2} \! dx \sqrt{(\partial_x U)^2 + \frac{U^4}{R^4}}$$

IT DIVERGES. WE MUST SUBTRACT THE MASS OF THE 2 W'S

$$E - 2 M_w = 2 M_s^2 \int_0^{L/2} \! dx \left[ \sqrt{(\partial_x U)^2 + \frac{U^4}{R^4}} - \partial_x U \right]$$

DEFINE DIMENSIONLESS VARIABLES $U = \frac{R^2 U}{L}$, $x = \frac{Lx}{L}$

$$E - 2 M_w = 2 M_s^2 L \frac{R^2}{L^2} \int_0^{L/2} \! dx \left[ \sqrt{(\partial_x \hat{U})^2 + \hat{U}^4} - \partial_x \hat{U} \right]$$

DIMENSIONLESS CONSTANT, INDEPENDENT OF $R$ & $L$!

$$R^2 = (4\pi g_s N)^{1/2} \frac{1}{M_s^2}$$

$$E - 2 M_w = \text{CONST} \cdot (g_s N)^{1/2} \frac{1}{L} \quad \text{COULOMB}$$

$$= \sqrt{2 g_s N} \quad \text{VERY NON-PERTURBATIVE}!$$

HEP-TH/9711200, HEP-TH/9802109, HEP-TH/9802150, HEP-TH/9806087
HEP-TH/9803002
A Glimpse of Non-Conformal Holography

Deformations of $N=4$.

If we want $N=4$ in the UV we must deform with operators with UV dimension $< 4$ (scalar to preserve Lorentz)

\[ L_{N=4} \rightarrow L_{N=4} + \lambda \mathcal{O} \]

$\Delta = 2$ : $\mathcal{O} = \text{Tr} \phi^I (\text{Tr} \phi^J)^T \quad I, J = 1, \ldots, 6 \quad \text{for} R$

$\Delta = 3$

$\mathcal{O} = \text{Tr} \chi^I \chi^J + O(\phi^3) \quad I, J = 1, \ldots, 4 \quad \text{for} \ C$

IIB Sugra on $AdS_5 \times S_5$ is complex, but for these deformations we can study instead $d=5$ gauged supergravity.

Fields: $g_{\mu\nu} + 42$ scalars + $SU(4)$ gauge fields + ...  

\[ L_{d=4 \text{ sugra}} = R(g) + G_{\Sigma \pi}^2 \partial_\mu \lambda^\Sigma \partial^\nu \pi + V(\lambda) + ... \]

L scalars

4-d Poincare invariant metric

\[ ds^2 = dt^2 + e^{2\phi(t)} \delta_{\mu\nu} dx^\mu dx^\nu \]

$\lambda = \lambda(t)$

E.O.M.: \[ \frac{d}{dt} \left[ G_{\Sigma \pi} \dot{\lambda}^\pi \right] + 4 \phi G_{\Sigma \pi} \dot{\lambda}^\pi = \frac{\partial V}{\partial \lambda^\Sigma} \]

\[ G_{\Sigma \pi} \]

$\phi$
$G_{\mu\nu} = 8\pi G_5 T_{\mu\nu}$

$2D \quad G_{TT} = 6 \dot{\phi}^2 = G_{\Sigma \Pi} \dot{\lambda}^2 \dot{\tau}^2 - 2 V \leftrightarrow T_{TT}$

$G_{00} = -3 (2 \dot{\phi}^2 + \ddot{\phi}) e^{2\phi} = G_{\Sigma \Pi} \dot{\lambda}^2 \dot{\tau}^2 + 2 V \leftrightarrow \nabla^2_{00}$

Solutions: \( \dot{\lambda} = \dot{\tau} = 0 \), \( \frac{\partial V}{\partial \lambda} = 0 \), \( \phi = \frac{T}{R} \quad (AdS_5) \)

CONFORMALLY INVARIANT SOLUTION

\[ R^3 \sim \phi^{-3} \sim G_{TT}^{-3} = \text{CENTRAL CHARGE} \]

C-THEOREM: CENTRAL CHARGE IS MONOTONIC IN T

LIGHT-LIKE VECTOR: \( V^M = (e^{-\phi}, 1, 0, 1, 0, 0) \)

\[ \phi_{\mu\nu} V^M V^\nu = -e^{-2\phi} e^{2\phi} + 1 \cdot 1 = 0. \]

NULL POSITIVITY (WEAKEST THAN WEAK ENERGY CONDITION)

\[ V^M G_{\mu\nu} V^\nu \geq 0 \quad \Leftrightarrow \quad V^0 G_{00} V^0 + V^T G_{TT} V^T = \]

\[ = e^{-2\phi} G_{00} + G_{TT} = -3 \ddot{\phi} \geq 0 \]

\[ \ddot{\phi} \leq 0 \quad \Leftrightarrow \quad \frac{d}{dT} (\phi^{-3}) = -3\ddot{\phi}(\phi)^{-4} \geq 0 \]

CENTRAL CHARGE INCREASES MONOTONICALLY FROM IR (\( T=0 \)) TO UV (\( T=\infty \))

HOLOGRAPIC C EXISTS, DOES AN R.G. EQUATION EXISTS.
Define \( \mu = \frac{1}{e^\phi(T)} \), cut off sugra action by imposing \( \mu \leq \lambda \).

\[
\mathcal{L} = \left( \mu^{-1} \frac{d T}{d \phi} \right)^2 d \mu^2 + \mu^2 \delta_{\mu \nu} (\mu) \frac{d x^\mu}{d \mu} \frac{d x^\nu}{d \mu}
\]

\( \delta_{\mu \nu} (\mu,x) \rightarrow \eta_{\mu \nu} \)

\( \lambda^\Sigma_B = \lambda^\Sigma (\mu=\lambda, x) \quad \delta^B_{\mu \nu} (x) = \delta_{\mu \nu} (\mu=\lambda, x) \)

\[
S = \int d\mu \mathcal{L} [\delta_{\mu \nu}, \lambda] \quad \text{is independent of } \mu:
\]

\[
0 = \mu \frac{d S}{d \mu} = \mu \frac{d S}{d \mu} + \int d^4 x \left[ \mu \frac{d \lambda^\Sigma}{d \mu} \frac{d \lambda^\Sigma}{d \lambda^\Sigma} + \mu \frac{d \delta_{\mu \nu} \delta_{\mu \nu}}{d \mu} \right]
\]

Define \( \lambda^R = \lambda^\Sigma (\mu,x) \), "renormalized field".

The equation for \( S \) is not a tautology.

Once the equations for \( \lambda^\Sigma_R \) and \( \delta_{\mu \nu} = \delta_{\mu \nu} (\mu, x) \).

\[
\dot{\lambda}^\Sigma_R = \mu \frac{d}{d \mu} \lambda^\Sigma_R \quad \text{by definition are the } \beta \text{ functions}
\]

We obtain them as follows

\[
\dot{S} = S_{0UV}(\mu) + S_{1R}(\mu)
\]

\[
S_{0UV}[\mu] = \int d^4 x \int_{\mu}^{\lambda} d \lambda^\Sigma \mathcal{L} (\lambda^\Sigma, x), \quad S_{1R}[\mu] = \int d^4 x \int_{\mu}^{\lambda} \frac{d \lambda^\Sigma}{d \mu} \mathcal{L} (\lambda^\Sigma, x)
\]

\[
\dot{\lambda}^R_R = - \frac{\delta S_{0UV}}{\delta \lambda^R} \quad \dot{\delta}_{\mu \nu} = - \frac{\delta S_{0UV}}{\delta \delta_{\mu \nu}}
\]

\( \beta \text{-functions} \)
How do we find condensates \( \langle 0 \rangle \neq 0 \) ?

\[ L \rightarrow L + \hat{\lambda} \hat{O}, \quad \hat{\lambda} \text{ is the boundary value} \]

\[ \lambda \rightarrow \hat{\lambda} \hat{z}^4 + \ldots \quad z \rightarrow 0 \]

E.O.M. For \( \lambda : \left[ -\partial_z \left( \frac{R}{z} \right)^3 \partial_z + m^2 \left( \frac{R}{z} \right)^5 \right] \lambda = 0 \)

\[ \Delta (\Delta - 4) = (RM)^2 \]

\[ S \propto \int \frac{d^2z}{z} \left( \frac{R}{z} \right)^5 \left[ \left( \frac{z}{R} \right)^2 \left( \partial_z \lambda \right)^2 + m^2 \lambda^2 + \ldots \right] \]

\( z_0 = 1\)R boundary, there \( \partial_z \lambda = 0 \)

\[ S \bigg|_{\text{on-shell}} = \lim_{\epsilon \to 0} \left( \frac{R}{\epsilon} \right)^3 \lambda \partial_z \lambda \bigg|_{z = 0} \]

Holography:

\[ S[\lambda] \bigg|_{\text{on-shell}} = W[\hat{\lambda}] \]

And:

\[ \frac{\partial W}{\partial \hat{\lambda}} \bigg|_{\hat{\lambda} = 0} = \langle 0 \rangle \]

\[ S \bigg|_{\text{on-shell}} = \lim_{\epsilon \to 0} \left( \frac{R}{\epsilon} \right)^3 \left[ \hat{\lambda} \in \hat{g} - \Delta + B \in \hat{A} \right] \partial_\epsilon \left[ \hat{\lambda} \in \hat{g} - \Delta + B \in \hat{A} \right] \]

\[ = R^3 \left[ \hat{\lambda} \in \hat{g} - 2\Delta + (4 - \Delta + \Delta) \hat{B} + \Delta B^2 \right] \epsilon^{2\Delta - 4} \]

\( \Delta < 4 \) (UV soft operator)

\[ \frac{\partial S}{\partial \hat{\lambda}} \bigg|_{\hat{\lambda} = 0} = \frac{\partial W}{\partial \hat{\lambda}} \bigg|_{\hat{\lambda} = 0} = \langle 0 \rangle = 4 R^3 B \epsilon^{\text{condensate}} \)
\textbf{Goldstone Theorem.}

\textbf{Add 5-D Gauge Field } \hat{A} = 0.

\[ \lambda(z) = z^\Delta \left[ B + \int d^4k \, \hat{A}_\mu^*(k) C^\mu(k) \right] \]

\[ \langle O \rangle \hat{A}_\mu^* = 4 \left[ B + \int d^4k \, \hat{A}_\mu^*(k) C^\mu(k) \right] \]

By construction \[ \frac{\delta}{\delta \hat{A}_\mu^*(k)} \langle O \rangle = \langle O \rangle \hat{J}_\mu^*(k) \]

Choose:\[ \hat{A}_\mu(k) = i K_\mu \, \hat{A}(k) \]

Cancel \( A_\mu \) with a gauge transformation \[ A_\mu \to A_\mu - \partial_\mu \lambda = 0 \]

\textbf{Note 5-D Gauge Transformation Acts as 4-D Global Symmetry} (\( SU(4) \) is R-Symmetry in \( \mathcal{N}=4 \) Case)

\[ B \to B + \int d^4k \, \delta B(k) \hat{A}_\mu^*(k) + O(\hat{A}^2) \]

\[ \frac{\delta}{\delta \hat{A}_\mu^*} \langle O \rangle = -i K_\mu \langle O \hat{J}_\mu^*(k) \rangle = \delta B(k) \]

\[ \lim_{k \to 0} \delta B(k) = \delta B \neq 0 \quad (\text{Spontaneous Symmetry Breaking}) \]

\[ \langle O \hat{S}(k) \rangle = C(k) K_\mu \quad (\text{Lorentz}) \]

\[ \lim_{k^2 \to 0} -i K_\mu K_\mu C(k) = \delta B \neq 0 \quad \text{ED} \; C(k) \text{ Has A Pole At} \quad k^2 = 0 \]

\textbf{ED Massless Particle} (Goldstone Theorem)
\[ \lambda = \text{5-D Massless Field} \quad ds^2 = e^{-2\phi(z)} (dz^2 + dx^\mu dx^\nu g_{\mu\nu}) \]

\[ S = \int d^4x \int_{z_0}^{z_0} d\tau \quad e^{-3\phi(z)} (\partial_\tau \lambda \partial_\tau \lambda + \partial_\mu \lambda \partial_\nu \lambda g^{\mu\nu}) \]

\[ \tilde{\lambda}(z, k) = \lambda(k) \left[ 1 + \alpha k^2 z^2 + \beta k^4 z^4 \log \frac{z}{R} + \gamma k^4 A(k) \right] \]

\[ S|_{\text{on-shell}} = \lim_{z \to 0} e^{-3\phi(z)} \tilde{\lambda}^\ast(\tau) \partial_\tau \tilde{\lambda}(\tau) \]

\[ e^{\phi(z)} \to \frac{z}{R} \quad z \to 0 \]

\[ A(k) = \langle \tilde{\delta}(k) O(0) \rangle \quad \text{ED poles of } A(k) \text{ imply} \]

\[ \text{that 2-point function has a pole i.e. a particle} \quad \text{of mass } m^2 = -k^2 \text{ at pole}. \]

\[ \text{POLE: } A(k) \to \infty \quad \frac{1}{A(k)} \tilde{\lambda}(z, k) \to \frac{z^4}{k^2 - m^2} \tilde{\lambda}(k) . \]

\[ \exists \tilde{\lambda} \sim z^4 \quad z \to 0 \]

\[ \text{At } z_0 (k), \quad \partial_\tau \tilde{\lambda} = 0 \quad \text{so} \quad e^{-3\phi(z)} \tilde{\lambda}^\ast \partial_\tau \tilde{\lambda} \quad \bigg|_{z_0} = 0. \]

\[ \text{Thus:} \quad S = \int d\tau \quad e^{-3\phi(z)} \left[ \partial_\tau \tilde{\lambda}^\ast \partial_\tau \tilde{\lambda} - k^2 (\tilde{\lambda}^2) \right] = \]

\[ = e^{-3A(z)} \tilde{\lambda}^\ast \partial_\tau \tilde{\lambda} \quad \bigg|_{0}^{z_0} = 0 \]

\[ \text{ED } k^2 < 0 \quad \text{Strictly: } \text{Mass Gap} \]
AN EXACT $N=1$ R.G. FLOW

$N = 4 \rightarrow N = 1$

WRITE $N = 4$ IN $N = 1$ SUPERFIELDS.

$$
S = \frac{1}{N=4} \text{Tr} \left[ \int d^4 \theta \left( \Phi^+ e^V \Phi + \sum_i \lambda_i \mathcal{W}_i \mathcal{W}^i \right) + \sum_i \int d^2 \theta \left( \Phi^I \Phi^J \Phi^K \varepsilon_{IJK} \right) \right] \quad \Phi, \lambda, \mathcal{W}, \mathcal{W}^i = 1, 2, \ldots
$$

$$
S = \frac{4\pi}{\alpha'} + \frac{i\theta}{2\pi}
$$

$V = \text{VECTOR MULTIPLETS}$ $\Phi^I = 3$ $\text{(CHIRAL MULTIPLETS)}$

BREAK $N = 4$ TO $N = 1$ BY ADDING $\sum_i M^{i\bar{j}} \int d^2 \theta \text{Tr} \Phi^i \Phi^{\bar{j}}$

EXAMPLE: $M^{i\bar{j}} = \delta^{i\bar{j}} m$

PURE $N = 1$ SYM AT $E < m$

$N = 4$ INTERPRETATION: $\int d^2 \theta \text{Tr} \Phi^i \Phi^{\bar{j}} \sim \text{Tr} \lambda^i \lambda^{\bar{i}} \epsilon^{i\bar{j}}$ OF $SU(4)$

IT BELONGS TO THE $6$ OF $SU(3)$ INSIDE $10$ OF $SU(4)$

$$
10 \rightarrow 6 + 3 + 1
$$

$M^{i\bar{j}} = \delta^{i\bar{j}} m = 1$ OF $SO(3) \subset SU(3)$ $6 \rightarrow 1 + 5$

HOLOGRAPHIC $N = 1$ FLOW: IN 5-D GAUGED SUGRA WE MUST PRESERVE $N = 1$. WE NEED:

$$
\delta \Phi^k = 0 \quad \delta \psi = 0
$$

UP 5-D FERMIONS
\[ \text{WE CAN SET TO ZERO ALL SCALES EXCEPT } z : \mu, \sigma \]

\[ 10 \rightarrow 6 + 4 + 1 \approx 8 \text{ SINGLET} \]

\[ L : 1 + 5 \]

\[ \mu \text{ SINGLET} \]

\[ \sigma = \text{SOURCE FOR } \text{Tr} \lambda^4 \lambda^4 = \text{Tr} \lambda^4 \text{ (GAUGINO CONDENSATE)} \]

\[ S \rightarrow S + m \int d^2 \theta \text{Tr} \lambda^4 \lambda^4 + \sigma \text{Tr} \, W^a W^a \bigg|_{\theta = 0} \]

\[ N=1 \text{ FLOW}: \delta Y = \delta Y = 0 \]

\[ \text{METRIC}: \quad ds^2 = dt^2 + e^{-2 \phi(t)} \, d\lambda^4 \, d\lambda^4 \]

\[ \text{potential}: \quad V = \frac{1}{8} \left( \frac{\partial W}{\partial \lambda^4} \right)^2 - \frac{1}{3} |W|^2 \]

\[ N=1 \text{ FLOW EQUATIONS}: \quad \frac{d \lambda^4}{dT} = \frac{1}{2} \frac{\partial W}{\partial \lambda^4}, \quad \phi = -\frac{1}{3} W \]

\[ \text{KEEP ONLY M & } \sigma: \quad W = \frac{3}{4} \left( \cosh \frac{2 \mu}{\sqrt{3}} + \cosh 2 \sigma \right) \]

\[ \text{solution}: \quad M(T) = \frac{\sqrt{3}}{2} \log \frac{1 + e^{-3(T-C_1)/R}}{1 - e^{-3(T-C_1)/R}}, \quad \sigma(T) = \frac{1}{2} \log \frac{1 + e^{-3(T-C_2)/R}}{1 - e^{-3(T-C_2)/R}} \]

\[ M(T) \rightarrow \sqrt{3} e^{-T/R} e^{C_1/R} \quad T \rightarrow +\infty \]

\[ \sigma(T) \rightarrow e^{-3T/R} e^{3C_2/R} \quad T \rightarrow +\infty \]

\[ \text{COEFFICIENT OF DEFORMATION}: \quad 4 - \Delta = 1, \quad \Delta = 3 \quad \text{OK} \]

\[ \text{GAUGINO CONDENSATE} \quad \text{CONDENSATE} \quad \text{Tr} \lambda^4 \lambda^4 \neq 0 \]

\[ \text{HEP-TH/9310126, HEP-TH/9903026, HEP-TH/9909047, HEP-TH/9903035} \]

\[ \text{HEP-TH/9912012} \]