
Problem 2: Imagine a particle of mass $m$ moving back and forth in the $x$ direction according to the equation

$$x(t) = A \cos(\omega t),$$

where $A$ and $\omega$ are constants. (a) What are the SI units for $A$ and $\omega$? (b) Derive an exact expression for the $x$ component $v_x(t)$ of the velocity as a function of time. (c) Derive an exact expression for the $x$ component $F_x(t)$ of the net force acting on the particle as a function of time. (d) What is the ratio $F_x/x$ as a function of time?

Problem 3: The example problem on pages 85 and 86 of Chabay & Sherwood gives masses for the Earth and Sun and an initial position and velocity for the Earth in its orbit. Create a spreadsheet or computer program that integrates the same trajectory as that integrated in the example problem, but use 1 d ($8.64 \times 10^4$ s) time steps instead of 1 mo time steps.

I recommend that your spreadsheet or program keep track of the time $t$, the $x$ component of position $x$, the $y$ component of position $y$, the magnitude of the position $r = \sqrt{x^2 + y^2}$, the $x$ component of momentum $p_x$, the $y$ component of momentum $p_y$, the magnitude of the force $F$, the $x$ component of the force $F_x$, and the $y$ component of the force $F_y$. You can test that you have your formulae correct by changing the timestep from 1 d up to 1 mo and seeing that you reproduce all of the numbers in the textbook example problem (though check the errata referred to on the web site for the course).

Integrate your orbit for 100 d and hand in a plot of $x$ vs $y$ for those 100 days. Define the time on the first line of your spreadsheet $t = 0$. On what day does the orbit cross the $y$ axis? That is, on what day does the $x$ position become negative?

Extra Problem (will not be graded for credit): Imagine you plan a interstellar voyage in which your space ship accelerates at $g = 10 \text{ m s}^{-2}$ for two years. What is wrong with your plan? Now imagine you increase your
momentum at a constant rate of momentum increase (that is, constant force) for two years, with your force set so that in the first few days (when you are not moving fast), you are accelerating at $g$. What speed do you end up going at the end of the two-year period, relative to the frame at which you were at rest when you started? *Note: This problem is very similar to the problem done in lecture.*

Recall that the change in momentum is the force times the time, and recall that momentum is $\gamma m v$. You will have to assume that your spacecraft has some mass $m$, but you will find that the mass cancels out in your final answer.