1 Purpose
To investigate the validity of Newton’s 1st Law.

2 Theory
An inertial coordinate system is one that is not accelerating or rotating with respect to the fixed stars. An inertial coordinate system will be either at rest or moving at a constant velocity with respect to the fixed stars. Newton’s 1st law states that if all the forces on a point mass add up to zero, then as measured in an inertial coordinate system the velocity of the point mass is a constant. This includes the case where the velocity is zero, in which case the point mass is at rest in the chosen inertial coordinate system. The particle is said to be in equilibrium.

Newton’s 1st law is contained in Newton’s 2nd law. Newton’s 2nd law for a point mass m states that if \( \vec{F} = \sum_i \vec{F}_i \) is the vector sum of all the forces acting on the mass, then \( \vec{F} = m\vec{a} \), where \( \vec{a} \) is the acceleration of the mass. If \( \sum_i \vec{F}_i = 0 \), then \( \vec{a} = \frac{d\vec{v}}{dt} = 0 \), where \( \vec{v} \) is the velocity of the mass. This implies that \( \vec{v} \) is a constant.

3 Description
See Fig.1. The apparatus consists of a force table. This is a circular table that has pulleys around the circumference that can be moved. There are markings around the circumference of the table that give the angular position of the pulleys and there is a centering pin that can be inserted in the middle of the circular table. A small ring near the center of the table acts as the point mass. Strings attached to the ring go over the pulleys. The strings have weight hangers attached to them. Weights can be added to the weight hangers. The angular position of the pulleys and the weights on the mass hangers are changed until the ring is centered on the table.

4 Procedures
Position 3 or 4 pulleys around the circumference of the table. Avoid making the position of the pulleys completely symmetric. There should be a string for each pulley. One end of each string is attached to the ring and the other end to a weight hanger. The centering pin should go through the ring. Add weights to the weight hangers until the ring does not touch the centering pin and is in the exact center of the table. You can also reposition the pulleys a bit. The greatest source of error is friction. To minimize this lift the ring slightly and release it. In recording the weight on each string, note that the weight hangers also have weight.
5 Three Forces

Set up the force table for 3 forces. Add weights to the weight hangers and adjust the positions of the pulleys until the ring is in equilibrium in the center of the table.

5.1 Graphical Analysis

See Fig. 2. Fold a piece of paper in half. Remove the centering pin and place the folded paper under the strings on the force table. For each string carefully make two widely spaced marks indicating the position of each string. Remove the paper and reconstruct the directions of the three forces by drawing lines through each pair of points. The lines should intersect at a common point (the center of the ring) and represent the directions of the three forces which we shall call $\vec{A}$, $\vec{B}$, and $\vec{C}$.

Now unfold the paper and use the other half of the paper to perform a vector addition of the three forces on the ring. A ruler and a triangle can be used to construct lines parallel to the 3 lines already on the paper. Place one leg of the triangle along one of the lines already on the 1st half of the paper. Place the ruler along another leg of the triangle and hold the ruler firmly to the paper. Slide the triangle along the ruler until you reach a convenient place on the unused half of your paper. Draw a line parallel to the original line and make the length of this line proportional to the weight on the string corresponding to that line. This line segment represents one of the 3 vectors. Put an arrowhead on one end of the line segment so that it represents a vector pointing away from the center of the table. Repeat this process twice, starting the new line segments so that the “tail” of the vector added is at the “head” of the vector already on the 2nd half of the paper. If there were no errors, the 3 line segments should close to form a triangle. The error is the small vector necessary to close the figure. Determine the direction and magnitude of your error force. Question: What thing(s) contribute to the error force?

5.2 Analytical Analysis

Consider the forces $\vec{A}$ and $\vec{B}$. The sum or resultant of these two forces must be equal and opposite to the 3rd force $\vec{C}$. From the law of cosines we can calculate the magnitude of the resultant force $\vec{R} = \vec{A} + \vec{B}$. Let $\theta$ be the interior angle between $\vec{A}$ and $\vec{B}$ legs of the triangle. This angle can be found by reading the angle between the $B$ and $C$ legs on the force table and substracting from 180°. Then

$$ R = \sqrt{A^2 + B^2 - 2AB \cos \theta}. \quad (1) $$

See whether $R$ is equal to the magnitude of $\vec{C}$ for your experiment. (For a complete check the direction of $\vec{R}$ would have to be found anti-parallel to $\vec{C}$.)

6 Four Forces

Set up the force table for 4 forces and adjust the apparatus so that the ring is in equilibrium in the center of the force table. Position one of the strings along the 0° line and call this the x axis.
6.1 Analysis By Components

See Fig. 3. Calculate the x components of the 4 forces and see how close their sum is to zero. Do the same for the y components of the 4 forces. Determine the error force (magnitude and direction). You can read the necessary angles off the force table.

7 Comment

The rules for adding vectors, tails to heads or by components, come from mathematics. Here we see they make sense in a physical situation described by a vector equation.
Equilibrium Of A Particle

Figure 1. Setup showing how three strings should run from the ring to three weight hangers.

Figure 2. Graphical description of the forces acting on the ring in the equilibrium condition.
Figure 3. Angles of four forces referenced to the x-axis of a Cartesian coordinate system.