Conservation of Energy Lab 11

Equipment SWS, ruler 2 meters long, 6 in ruler, heavy duty bench clamp at corner of lab bench, 90 cm rod clamped vertically to bench clamp, 2 double clamps, 40 cm rod clamped horizontally to 90 cm vertical rod, brass spring clamped to horizontal rod 25 cm from vertical rod, 100 g mass, 0.5 kg mass with 3 in cardboard square taped to bottom, motion sensor II, photogate sensor, non-paper tube, paper tube about 2.5 cm in diameter, bubble wrap, 30 cm string with loops at both ends, calipers.
Comments: Check that 0.5 kg mass hangs so cardboard is horizontal. If it is not, carefully bend hook on mass so it is. The bench clamp screw holding the vertical rod should be perpendicular to the horizontal rod holding the spring (to avoid acoustic reflections from the bench clamp screw).

1 Purpose

The total energy E of a simple mechanical system is the sum of the potential energy PE and the kinetic energy KE. In the absence of friction the total energy E of the system is a conserved quantity so that $E = KE + PE$. In the absence of friction, if the KE and PE change, they must change so that their sum is equal to the total energy E. In this experiment several simple mechanical systems will be examined for this property.

2 Theory

The work-energy theorem is obtained from a spatial integration of Newton's Second Law. Let $\vec{F}$ be the force on a point mass m whose position and velocity are $\vec{r}$ and $\vec{v}$. If the mass moves from an initial position (i) to a final position (f) the work-energy theory states that

$$\int_i^f \vec{F} \cdot d\vec{r} = \left(\frac{1}{2}mv^2\right)_f - \left(\frac{1}{2}mv^2\right)_i .$$

The left hand side of this equation is defined as the work W and the right hand side as the change in the kinetic energy △KE. For the work, given by an integral called a line integral, there are 2 possibilities.

1. The value of the work or line integral depends on the path or route taken by the mass as it moves from (i) to (f). In this case the force is called non-conservative. An example is the force of friction. A consequence of this is that there is no function whose differential equals the integrand of the line integral and a definite path for the mass m must be specified to evaluate the work.

2. The value of the work or line integral does not depend on the path or route taken by the mass as it moves from (i) to (f). In this case the force is called conservative. Examples are the the force exerted by a linear spring and the uniform gravitational force. In this case there is a function called $-U$ whose differential $-dU$ is equal $\vec{F} \cdot d\vec{r}$. The function U is called the potential energy (PE).

For case 2 the work-energy theorem can now be written as

$$-\int_i^f dU = -(U)_f + (U)_i = \left(\frac{1}{2}mv^2\right)_f - \left(\frac{1}{2}mv^2\right)_i = (KE)_f - (KE)_i.$$

As U is evaluated only at the points (i) and (f) it is clear that for conservative forces the work depends only on the end points and not on the particular path traversed by m. This equation can be written as

$$(U)_f + (KE)_f = (U)_i + (KE)_i.$$

Each side of this equation is called the total (mechanical) energy E of the mass. On the left E has been evaluated at point (f) and on the right at point (i). The quantity E has been conserved and $E_f = E_i$. This statement is called the conservation of energy. Energy is not conserved if friction or other non-conservative forces are present.

It is convenient to refer the PE at any point to a fixed reference point (o). For a conservative force the work integral from (i) to (f) is independent of path. Let that path go through the reference point (o). The work integral becomes

$$W = \int_i^f \vec{F} \cdot d\vec{r} = \int_i^o \vec{F} \cdot d\vec{r} + \int_o^f \vec{F} \cdot d\vec{r} = -\int_o^i \vec{F} \cdot d\vec{r} + \int_o^f \vec{F} \cdot d\vec{r} = +U_{io} - U_{fo}, \ where$$

$$U_{io} = -\int_o^i \vec{F} \cdot d\vec{r} \qquad and \qquad U_{fo} = -\int_o^f \vec{F} \cdot d\vec{r}.$$

U_{fo} is called the potential at (f) relative to (o) and U_{io} is called the potential at (i) relative to (o). The conservation of energy statement may be written

$$U_{fo} + (KE)_f = U_{io} + (KE)_i.$$

From this last equation it is evident that any value can be given to the PE at the reference point, for this value will appear on both sides of the equation and effectively cancels out. For simplicity, the value of the PE at the reference point is usually taken as zero and in what follows we assume this to be the case. The subscript o referring to the reference point is often omitted but the PE is always with respect to a reference point and the location of the reference point should be clearly stated.

In one dimension, if that dimension is taken to be x, $dU = -Fdx$, and

$$F = -\frac{dU}{dx}.$$

2.1 Linear Spring

Let a spring lie on the x axis and assume one end of the spring is fixed. If x_o is the position of the free end of the unstretched linear spring, a mass attached to the spring will experience a restoring force $F = -k(x - x_o)$ where x is the position of the free end of the stretched spring, $(x - x_o)$ is the length the spring has been stretched and k is the spring constant. Take the reference point for the PE at x_o. The PE for a mass attached to the spring will be

$$U(x) = -\int_{x_0}^{x} F dx' = -\int_{x_o}^{x} -k(x' - x_o)dx' = \frac{1}{2}k(x - x_o)^2,$$

where the potential energy at $x = x_0$ has been taken as zero. Often, x_o is chosen as the origin and $U(x) = \frac{1}{2}kx^2$.

2.2 Uniform Gravitational Field

For a mass m in the uniform gravitational field on the surface of the earth the force of gravity is $F = -mg$ where g is the acceleration of gravity and up is positive. Take up as the positive y coordinate and y_o as the PE reference height. The PE at any height y is given by

$$U(y) = -\int_{y_0}^{y} F dy' = -\int_{y_o}^{y} -mg dy' = mg(y - y_o),$$

where the potential energy at $y = y_0$ has been taken as zero. If y_0 is taken at the origin of the coordinate, the PE becomes $U = mgy$. This PE is often written as mgh where h is the height above the reference point. (h is negative if the mass is below the reference point.) The PE depends only on y and not how far the mass moves horizontally. Horizontal motion does not contribute to the PE. Why?

2.3 Multiple Masses and Conservative Forces

For a system consisting of a number of masses and forces, the analysis is easily extended. If all the forces, both internal and external, are conservative, the work done by all the forces can be represented by potential energies. The total energy is the sum of all the PE's and KE's of all the masses and this quantity is conserved.

3 Free Fall

3.1 Description

A tube with mass m is held horizontally a distance h above the level beam of a photogate sensor. The tube is dropped (without rotation) and its velocity is measured as it passes through the photogate. The total energy at the time of dropping is compared to the total energy as the tube passes through the photogate.

3.2 Theory

Take the reference point for the gravitational PE as the level of the photogate beam. At the time of dropping the total energy is mgh. If v is the velocity of the tube as it passes through the photogate the total energy at that time is $\frac{1}{2}mv^2$. Equating the energy at the time of dropping to the energy as the tube passes through the photogate, $mgh = \frac{1}{2}mv^2$, which gives $v = \sqrt{2gh}$.

3.3 Programming

Measure the diameter of the tube with the calipers, checking for uniformity. If the diameter of the tube is not uniform, discuss this in your error analysis. Program SWS for the digital photogate sensor & solid object. In the sensor setup window enter the diameter of the tube. Open the digits display window and choose velocity. Double click in the display area of the digits display to open the display setup window. Program for 3 decimal digits.

3.4 Taking Data

Move the photogate so that its plane is horizontal and is opposite the base plate that holds the vertical rod supporting the photogate. Adjust the height of the photogate beam to be 25 cm above the table and place the photogate over the edge of the table next to the vertical rod in the bench clamp. Drop the non-paper tube so that it is horizontal, does not rotate, and is perpendicular to the photogate beam. Use the vertical rod next to the photogate as a guide for dropping the tube through the photogate without hitting it. Click Rec, and using the meter stick drop the tube onto bubble wrap from a height $h = 10$ cm above the beam, or a height of 35 cm above the table. Measure height to the middle of the tube. Record the velocity through the photogate. You will want to take some practice runs and a few runs for real. Repeat for $h = 20$ and 30 cm.

3.5 Analysis

Compare your measured velocities with that predicted by the theory. Is there any friction in this experiment? If so, how would it affect your data? Try dropping the paper tube from a height of 50 cm above the photogate beam and analyze the data as before. Is energy conserved? If not, what happens to the energy?

4 Pendulum

4.1 Description

A 100 g mass is hung from a 30 cm string and used as a pendulum. The weight is pulled to one side and let go. At the lowest point the weight passes through the beam of a photogate sensor and its velocity is measured.

4.2 Theory

As the pendulum swings down, PE is converted into KE. The change in PE is given by mgh where h is the vertical distance that the weight has moved. The weight travels the arc of a circle, and this distance is longer than h. The velocity is given by exactly the same expression as in the previous experiment, $v = \sqrt{2gh}$. The weight has a force exerted on it by the string, but this force does not contribute to the PE. Why?

4.3 Programming

Measure the diameter of the 100 g weight with calipers. Follow the procedures in the previous section.

4.4 Taking Data

Adjust the photogate so that the legs point up and the beam is 10 cm above the bench. The photogate beam should be directly below the suspension point of the pendulum. Adjust the height of the suspension point of the pendulum so that the middle of the weight swings through the center of the photogate beam. Pull the weight to one side so that the middle of the weight is 10 cm above the level of the beam. Let go and record the velocity of the weight at its lowest point. Repeat for $h = 8$, 6, and 4 cm.

4.5 Analysis

Compare your measured velocities to the theoretical values. Note that it is only the vertical distance through which the weight falls that contributes to the change in PE.

5 Gravity + Spring

5.1 Description

A vertical spring has a 0.5 kg mass m hanging from it. The mass is set into vertical oscillatory motion and its position is measured as a function of time by a motion sensor. The velocity and acceleration are calculated by SWS. The total energy of the mass is compared at different points in the motion. The PE of the spring-mass system is due to both gravitational PE and the PE of the spring.

5.2 Theory

The mass m performs simple harmonic motion with an angular frequency given by $\omega = \sqrt{k/m}$, where k is the spring constant.

Let y be the vertical coordinate (up positive) that gives the position of *the end of the spring* from which m is hung. Take the coordinate origin as the position of this end of the spring when there is no mass on the spring (spring unstretched). When the spring is stretched downward by hanging a mass on it (y will be negative) the PE of the spring will be $\frac{1}{2}ky^2$ where k is the force constant of the spring. Let the gravitational PE of the mass m be zero when $y = 0$ (spring unstretched). The gravitational PE for m is then mgy, which will be negative. (While y is not the coordinate of the mass, the end of the spring and the mass move the same amount.) When the end of the spring is at a point given by y and has a velocity $v = dy/dt$ (also the velocity of the mass), The total energy E of the spring-mass system is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}ky^2 + mgy.$$

E should be the same at any point in the motion, assuming no friction.

5.3 Programming

Program SWS for the motion sensor, using the default values. Open the graph window, choosing position, velocity, and acceleration for the vertical axes.You will probably find the default sampling rate of 20 Hz to be satisfactory, but feel free to experiment.

5.4 Preliminaries

The spring should be hanging over the edge of the bench. Adjust the height of the spring so that with the 0.5 kg mass attached the bottom of the mass is about 65 cm from the floor. With the mass removed from the spring use the two meter stick oriented vertically and the 6 in ruler oriented horizontally to determine the height of the unclamped end of the spring from the floor. Attach the mass to the spring and measure the height of the same end of the spring from the floor. Determine k for the spring. Also, let the amount the spring stretches be y_0, a negative number. Then, using the coordinates already introduced, the position of the end of the spring with the mass attached and at rest is given by $y = y_0$. Place the motion sensor on the floor directly beneath the the mass. Check that the face of the motion sensor is horizontal. There is a switch on the motion sensor labeled narrow or std (standard) which sets the width of the acoustic beam of the sensor. You will probably find that std works the best but feel free to experiment. The motion sensor should be plugged into the interface so that position increases as the mass goes higher.

Set the mass into motion by pulling it down a reasonable amount and letting go. (the motion sensor does not record distances less than 0.15 m from its face). Do not set the mass in motion by lifting it up. If you lift it too far it will crash into the motion sensor. After the motion settles down, click REC and do the following.

- Simultaneously observe the motion of the mass and the graphs of position, velocity, and acceleration. Does your intuition about the motion correspond to what the graphs are displaying? For example, is the acceleration maximum or minimum when the velocity is zero? When the velocity is maximum is the acceleration maximum or minimum?
- Observe the spring-mass system. Is there kinetic energy that is not given by $\frac{1}{2}mv^2$? If so, this would be worth while mentioning in your error analysis. (Hint: There are at least two items one might notice here.)

5.5 Experiment and Analysis

Set the mass into motion, and when the motion settles down click REC, and click STOP after 3 or 4 cycles of the motion. Using the cross hairs on the position graph, determine the distance the mass has traveled from a chosen highest position (H) to the following lowest position (L) on the position graph. This distance will be 2A, where A is the amplitude of the motion and is a positive number. Determine the total energy E for the following 4 positions of m during the motion: the highest and lowest positions of the mass utilized in finding the amplitude A, and the two positions of maximum velocity following these highest and lowest positions of the mass. Recall that the coordinate y gives the position of the end of the spring, not the position of the mass. The highest position of the mass occurs when $y = y_0 + A$ and the lowest position of the mass when $y = y_0 - A$. Maximum velocity occurs when $y = y_0$.

1. When the mass is at the highest chosen position and the KE is zero, the energy E_H will be
$$E_H = \frac{1}{2}k(y_0 + A)^2 + mg(y_0 + A). \quad (1)$$
2. When the mass has the maximum down velocity following the highest chosen position of the mass the energy E_{01} is given
$$E_{01} = \frac{1}{2}mv^2 + \frac{1}{2}ky_0^2 + mgy_0. \quad (2)$$
3. When the mass is at the lowest chosen point and the KE is zero, the energy is given by
$$E_L = \frac{1}{2}k(y_0 - A)^2 + mg(y_0 - A). \quad (3)$$
4. When the mass has the maximum up velocity following the lowest chosen position of the mass the energy E_{02} is given by
$$E_{02} = \frac{1}{2}mv^2 + \frac{1}{2}ky_0^2 + mgy_0. \quad (4)$$

Obtain the velocity from the velocity curve of the graph display.

Compare the total energy at these 4 points. Is energy conserved? What factors might introduce error into your measurements and calculations?

5.6 Question

1. The analysis presented in this section assume no friction. How will friction affect your experimental results? Do you see evidence of friction in the curve of position vs time?

6 Finishing Up

Please return the bench to the condition in which you found it. From the File menu click New and then click Don't Save. Thank you.