

Work-Energy Lab

CAUTION For this lab, force sensors are mounted near the floor on rods that stick out into the aisle. Please be careful.

Equipment SWS, air track, 28.7 cm glider, set kg masses with hooks, smart pulley, bench clamp, force sensor, 100 cm string with loops, 100 cm string with loops and 4 rubber bands, 130 cm string with loops, meter stick

1 Purpose

To investigate the validity of the work-energy relationship.

2 Theory

Consider a point mass m acted upon by a net force $\vec{F} = \vec{F}(\vec{r})$. The position, velocity, and acceleration of the mass are given by \vec{r} , \vec{v} , and \vec{a} , and the time by t . The force \vec{F} may be a function of \vec{r} . Newton's 2nd Law for the mass is $\vec{F} = m\vec{a}$. This is integrated over position from an initial position (i) to a final position (f).

$$\int_i^f \vec{F} \cdot d\vec{r} = m \int_i^f \vec{a} \cdot d\vec{r}.$$

The left hand side is defined as the work W done on m as it moves from \vec{r}_i to \vec{r}_f . It is only the component of the force parallel or anti-parallel to the motion of m that contributes to the work. The right hand side can be transformed to

$$m \int_i^f \frac{d\vec{v}}{dt} \cdot d\vec{r} = m \int_i^f \vec{v} \cdot d\vec{v} = \left(\frac{1}{2} m \vec{v} \cdot \vec{v} \right)_i^f = \left(\frac{1}{2} m v^2 \right)_f - \left(\frac{1}{2} m v^2 \right)_i.$$

The quantity $\frac{1}{2} m v^2$ is defined as the kinetic energy, or KE. In words, when m moves from \vec{r}_i to \vec{r}_f , the net work W done on m between those two points is equal to the change in kinetic energy KE between those two points. More succinctly,

$$W = \Delta KE.$$

This is the most basic form of the work-energy theorem. To be used correctly, W must include all the work done by all the forces between the two points under consideration.

The work-energy theorem can be applied to a system of particles. The total work done is equal to the total change in KE. Care must be taken with the work done by the forces between the particles of the system, or the "internal" work. The internal work may not cancel out.

3 Does $W = \Delta KE$?

3.1 Description

A glider of mass m is on a horizontal air track. A horizontal string attached to the glider goes over a smart pulley. The end of the string that goes over the smart pulley is attached

to 4 rubber bands that are connected in series, and the last rubber band is attached to a force sensor which is clamped to a rod near the floor. The glider is pulled back to stretch the rubber bands and let go. The force on the glider and velocity of the glider are measured as a function of the position of the glider. SWS integrates the force curve to give the work, and the velocity curve can be examined to give the velocities at the chosen beginning and end points. The change in KE is calculated and compared to W .

The force on the glider is not constant and as a consequence the acceleration is not constant. The tension T in the string will be the same on both sides of the pulley if the mass of the string and pulley can be neglected and if the pulley is frictionless. The magnitude of the force between the last rubber band and the force sensor will be the same as T if the rubber bands are massless.

3.2 Theory

The motion in this experiment is one dimensional and the vector notation used in defining work is not needed. If the one dimension is taken to be x , the work becomes $\int_i^f F dx$. This is equal to the area under the F vs x curve for the portion of the curve between the two vertical lines defined by x_i and x_f . The statistics function of SWS will calculate this area for you.

3.3 Programming

Check that the force sensor and the smart pulley (linear) are plugged into the interface, and noting which channels they are plugged into, program SWS for these two sensors. Open up the graph display by dragging its icon to the force sensor icon. Click the horizontal input menu button on the graph display, access the digital submenu, and click position. Click the add plot button on the graph display, access the digital menu, and click velocity. Your graph will have position for the horizontal axis and force and velocity for the two vertical axes. Click the sampling options button in the left experiment setup window and use the slider to change periodic samples to 200 Hz.

3.4 Calibrating the Force Sensor

Double click on the force sensor icon to open up the setup dialogue box. Calibrate the force sensor as follows.

- Leave the force sensor in the position in which it will be used. With nothing touching the hook on the force sensor, push the tare button on the sensor.
- In the row labeled Low Value, first click the Read button and then enter 0 in the column labeled Newtons.
- Take a 100 cm string with loops at both ends and attach one end to the hook of the force sensor. Pass the other end of the string over the pulley and down through the hole in the bracket holding the smart pulley. Hang a 100 g mass from that end of the string.
- In the row labeled High Value, first click the Read button and then enter the weight of the 100 g mass in the column labeled Newtons.
- click OK.

3.5 Taking Data

Remove the string used for calibrating the force sensor. Level the air track. Take the string with the rubber bands at one end and hook the last rubber band to the force sensor. Pass the string over the smart pulley and hook the end to the 28.7 cm glider. Pull the glider back to stretch the rubber bands but do not pull the rubber bands into the smart pulley. Click Rec, and let go of the glider. Click Stop just after the string goes slack.

3.6 Analysis

Click the statistics button and then the auto scale button. Box in the part of the force curve you wish to integrate. Your beginning integration point x_i should exclude perhaps the first two data points. The end of your integration x_f should be just before the velocity curve levels off for the first time and before the force curve goes to zero for the first time. Click the statistics menu button and choose integration. Record the value of the integral which is the work W done between the limits chosen. Use the smart cursor to determine the velocities at the end points of your integration and calculate the change in KE. Compare to the measured work done.

3.7 Repeat

Take a few more data runs. Discuss random errors which contribute to the scatter of your results from run to run, and systematic errors which will effect all your runs in the same way.

4 $W = \Delta KE$ Applied to a System

4.1 Description

The work-energy theorem is tested for a system of two masses. The theorem is applied to the two masses separately and to the entire system.

A horizontal string is attached to a glider on a horizontal air track. The string goes over a smart pulley and has a mass on the end. The glider is let go and the velocity of the glider is measured as a function of distance. The velocity of the glider is measured at two points and the change of KE calculated for each mass (glider and hanging mass) and for both together. The work done on each mass and for the system is calculated.

4.2 Theory

The hanging mass is denoted M_1 and the glider mass by M_2 . From the equations given in the Lab Newton's 2nd Law the tension T in the string can be calculated as

$$T = \frac{M_1 M_2 g}{M_1 + M_2}$$

This is a constant and the work done by T simplifies to $T \times (\text{distance})$ with an appropriate sign.

The work done on M_1 is $(M_1 g - T)(x_f - x_i)$. Explain the minus sign before the T . The work done on M_2 is $T(x_f - x_i)$.

The work done on the system is $M_1g(x_f - x_i)$. Explain clearly why T does not appear in this expression. (Due to Newton's 3rd law, internal forces cancel out. Internal work does not always cancel out.)

4.3 Programming

Program SWS for the smart pulley (linear). Open the graph display, choosing velocity. Click the horizontal input menu button and choose position from the digital menu. Your graph will have velocity on the vertical axis and position on the horizontal axis.

4.4 Taking Data

Move the air track and the smart pulley so that a string with weight hanging from the smart pulley will not hit the force sensor. Level the air track. Hook the string to the glider and pass the string over the smart pulley. Hang a 50 g mass from the string. Pull the glider back until the 50 g mass is close to the smart pulley, Click the Rec button, and let the glider go. Click Stop just before the glider hits the floor.

4.5 Analysis

Pick suitable x_i and x_f and determine the velocities at those points. For M_1 , M_2 and the system, calculate the work done and the change in KE. Compare the work done with the change in KE for these three items.

Is the velocity vs distance curve a straight line? If not, what do you think it is?