

Rotational Motion Lab 10

Equipment SWS, rotary motion sensor mounted on 80 cm rod and heavy duty bench clamp (PASCO ME-9472), string with loop at one end and small white bead at the other end (125 cm bead to end of loop), rod with 2 brass weights, disk, cylinder, set of weights with hooks, vernier calipers, meter stick

Rotary Motion Sensor A 2 plug SWS digital sensor. Some pictures are on the last page of this write-up. It is similar to the smart pulley in that it measures the angular rotation of a pulley, but differs in that the rotary motion sensor can also detect the direction of the rotation. There are two angular resolutions: 360 divisions per rotation (1 degree) and 1440 divisions per rotation (1/4 degree). The plugs should be inserted into 2 adjacent digital channels. Reversing the order of the plugs will change the sign of the sensor output.

1 Purpose

To investigate the relationship between torque, moment of inertia, and angular acceleration for a rigid body rotating about a fixed axis. Conservation of angular momentum will also be examined.

2 Theory

2.1 Equation of Motion for a Rotating Rigid Body

The equation of motion for a particle moving in a circle may be derived as follows. Consider a point mass m constrained to move on a circle of radius r . Due to a net force \vec{F} acting on it m may speed up or slow down in its circular motion. The dynamics of this are given by Newton's 2nd Law $\vec{F} = m\vec{a}$, where \vec{a} is the acceleration. At any instant of time it is convenient to write the component of this equation which is in the direction of the velocity, or tangential to the circle at the point where the mass is. Let the component of the force in that direction be F_{tan} , where "tan" stands for tangential and denotes the component of the force that is tangential to the velocity of the mass. For circular motion we also have that the component of the acceleration tangential to the circle is $r\alpha$, where r is the radius of the circle on which the particle moves and α is the angular acceleration in rad/s^2 . The relevant component of Newton's 2nd Law becomes $F_{tan} = mr\alpha$. Multiplying both sides of this equation by r gives $rF_{tan} = mr^2\alpha$. The quantity rF_{tan} is called the torque τ about the axis of the circle. One could also say that this is the component of the torque along the axis of rotation. The quantity mr^2 is called the moment of inertia I of the mass about the axis of the circle. (The axis of the circle is a line that runs through the center of the circle and is perpendicular to the plane of the circle.) The equation of motion now has the form $\tau = I\alpha$.

This analysis is easily extended to a rigid body rotating about an axis fixed in space. Suppose the rigid body to be constructed of point masses m_i each moving in a circle of radius $r_{i\perp}$ and acted upon by a force \vec{F}_i whose component in the direction of the velocity is $F_{i,tan}$. All the point masses will have the same angular acceleration α . Note that the $r_{i\perp}$ do not have a common origin but that each represents the perpendicular distance from the axis of rotation to m_i . The equation of motion for each mass is $r_{i\perp}F_{i,tan} = m_i r_{i\perp}^2 \alpha$. Summing

this over all the point masses of the rigid body we have

$$\sum_i r_{i\perp} F_{i,tan} = \sum_i m_i r_{i\perp}^2 \alpha.$$

The forces acting on any mass m_i can be divided into internal and external. Internal forces are those exerted on m_i by the other masses of the rigid body. External forces are all other forces that originate outside of the system of masses. It can be shown that due to Newton's 3rd Law the torques due to the internal forces of the rigid body will cancel. Without changing notation, we assume the torque is due solely to external forces. Extending our definitions of torque and moment of inertia to $\tau = \sum_i r_{i\perp} F_{i,tan}$ and $I = \sum_i m_i r_{i\perp}^2$, the equation of motion for a rigid body rotating about a fixed axis is $\tau = I\alpha$, where τ is the external torque. In words, for a rigid body rotating about a fixed axis, the sum of the external torques computed about the axis is equal to the moment of inertia computed about the same axis times the angular acceleration.

If the rigid body is described by a continuous mass distribution with a mass per unit volume of $\rho(\vec{r})$, the moment of inertia I is given by $\int \rho(\vec{r}) r_{\perp}^2 dv$, where dv is a differential volume element and r_{\perp} is the perpendicular distance from the axis to dv .

2.2 Torque and Angular Momentum

For the motion of particles which are not a rigid body, the torque is defined about a point. If \vec{r} is the vector from that chosen point to the point of application of a force \vec{F} , the torque is defined as $\vec{\tau} = \vec{r} \times \vec{F}$. Applying this definition to our rigid body rotation, if the chosen point is taken anywhere along the axis of rotation, the torque as it was defined is the component of torque along the axis of rotation. Angular momentum \vec{L} about the point for a particle with linear momentum \vec{p} is defined as $\vec{L} = \vec{r} \times \vec{p}$. For a system of particles, if τ represents the sum of the external torques about a point, and \vec{L} represents the sum of the angular momenta for the particles about the same point, it is easily shown that an equation of motion is $\vec{\tau} = \frac{d}{dt} \vec{L}$. If any component of $\vec{\tau}$ is zero, that component of the angular momentum is a constant.

2.3 Angular Momentum and Rigid Body Rotation

For a single rigid body rotating about an axis, the angular momentum along the axis is given by $L = I\omega$. The equation of motion becomes $\tau = \frac{d}{dt}(I\omega) = I\alpha$, as I is a constant. In this case, if τ is equal to zero, then ω is a constant. It is interesting to consider a body that is rigid, then deforms itself, and then becomes rigid again. In the deformation, assume that the moment of inertia is changed but that there are no external torques. The angular momentum $I\omega$ must stay the same, so that for example if I is decreased then ω must increase. An ice-skater pulls in his/her arms to increase the rate of spin.

Another case is that of two rigid bodies, one of which is spinning about an axis and has angular momentum $I\omega$ and the other is not spinning and has no angular momentum. Assume no external torques. If one of these objects is dropped on the other and they stick, the angular momentum must stay the same. We have $I\omega = I_f \omega_f$, where I_f is the final moment of inertia of the composite body and ω_f is the final angular velocity.

2.4 Rotational Kinetic Energy

The kinetic energy of a rigid body rotating about an axis is $\frac{1}{2}I\omega^2$.

2.5 Moments of Inertia

In these experiments various rigid bodies will be rotated about an axis. Some of these rigid bodies will be made of components. The moment of inertia of each component about the chosen axis can be calculated and then the total moment of inertia obtained by summing the moments of inertia of the components. We list the moments of inertia of various components each of mass M and assumed uniform.

1. A small mass a distance R from the axis: $I = MR^2$.
2. A thin rod of length L about an axis through the center: $I = \frac{1}{12}ML^2$.
3. A solid cylinder (or disk) of radius R about its axis: $I = \frac{1}{2}MR^2$.
4. A tube of outer radius R_2 and inner radius R_1 about its axis: $I = \frac{1}{2}M(R_1^2 + R_2^2)$.
5. A tube of outer radius R_2 and inner radius R_1 and length L about an axis through the center of the tube and perpendicular to the tube axis: $\frac{1}{12}ML^2 + \frac{1}{4}M(R_2^2 + R_1^2)$.

2.6 Equation for the Experiments

A horizontal 3-step pulley is mounted on a vertical axis. See figures at the end of this write-up. Let the radius of the pulley used be R_p . A rigid body whose moment of inertia about the pulley axis is I is mounted on the pulley. A string is wound around the pulley, goes horizontally to a "super" pulley, and then goes straight down to a hanging mass m . Let the tension in the string be T . The equations of motion for the rotating rigid body and the hanging mass are $\tau = R_p T = I\alpha$ and $mg - T = ma = mR_p\alpha$, where a is the linear acceleration of m , α is the angular acceleration of the rigid body, and g is the acceleration of gravity. The tension T can be eliminated to give

$$\alpha = \frac{mR_p g}{mR_p^2 + I}$$

Use this equation in your analyses. In this analysis, the moments of inertia of the two pulleys is taken as zero.

3 Description and Procedures

Refer to the figures. The rotary motion sensor (RMS) is mounted on a vertical support rod. A 3-step pulley is mounted on the axis of the RMS. When you enter the lab a rod with 2 brass weights is mounted on the 3-step pulley as shown in the figures. A loop in one end of the string can be attached to a hanging mass. The other end of the string has a small bead on it. This end of the string can be attached to one of the three grooves in the pulley by catching the bead in one of the notches in the rims of the pulleys. The height of the super pulley should be adjusted so that the string between the two pulleys is horizontal. The angle of the super pulley should be adjusted so that the string enters the super pulley parallel to the groove of the super pulley. See the diagrams, and note that the super pulley can be easily moved by loosening the two screws on the bracket of the super pulley. The string can be wound onto one step of the 3-step pulley by rotating the rod until the hanging mass is just below the super pulley. Let go the rod or disk and click REC. Just before the mass hits

the floor click STOP and bring the rotation to rest by holding the shaft on the bottom side of the RMS with your fingers.

A 2nd series experiments use a disk mounted on the 3-step pulley instead of a rod. Remove the rod by rotating the screw holding the rod to the 3-step pulley. Do not rotate this screw so much that you remove the screw from the rod. The screw is "captured" and is designed to stay on the rod. Remove the 3-step pulley by pulling it off. Observe that the hole in the pulley has a projection that fits into a slot in the shaft of the RMS. Turn the pulley over so the small pulley is on top and put it back onto the axle, aligning the projection in the hole of the pulley with the slot in the RMS shaft. The end of the RMS pulley that now shows is square. Mount the disk so that the square end of the pulley goes into the square hole in the disk. Use the screw in the dish on the bench to attach the disk to the pulley. For another experiment a hollow cylinder is mounted on the disk. Note that the cylinder has 2 projections that fit into holes on the disk. When you remove the disk from the pulley put the screw back into the dish.

4 Programming

Program for the digital rotary motion sensor. In the sensor setup window, choose 360 divisions/rotation. Program for the graph display window, choosing angular velocity for the vertical axis. The slope of the angular velocity vs time line is the angular acceleration α . Determine the slope by fitting a straight line through an appropriate portion of your data.

5 Experiments

5.1 Rod + 2 Brass Cylinders

Remove the rod and 3-step pulley from the RMS and measure the mass of the rod, the masses of the 2 brass cylinders, and the diameter of the large and medium grooves of the pulley. Remount the 3-step pulley, large pulley on top, and then the rod on the pulley laying the rod between the ridges on the face of the large pulley and using the captured screw. Put the 2 cylinders on the very ends of the rod. Measure the length of the rod and the distance from the axis to the middle of each cylinder. Determine α experimentally using a 20 g mass. Take several runs and then repeat for a 30 g or 40 g mass. Compare your measurement to the theoretical value of α . You may treat the cylinders as point masses. Move the cylinders in so that they are outside the super pulley but just clear it when the rod is rotated. Repeat your measurements.

After doing the other experiments, if you have time repeat some of these experiments using the medium pulley for the string.

5.2 Disk and Cylinder

Measure and weigh the aluminum disk and the large cylinder. Remove the rod and the 3-step pulley. Remount the 3-step pulley with the large pulley on top, and then mount the disk on the 3-step pulley and secure it with a screw. Using a 20 g mass determine an experimental α . Use this data to find an experimental value of I for the disk. Compare to the theoretical value. Mount the large cylinder on the disk using the projections on the end of the cylinder

and the holes in the disk. Determine experimentally the I for the disk + cylinder. Determine I for the cylinder (rotation axis is cylinder axis) and compare to the theoretical value.

Now mount the cylinder on the disk with the axis of the cylinder horizontal. A hole in the side of the cylinder allows the cylinder and disk to be attached to the disk with the long screw. Determine the moment of inertia of the cylinder about an axis of rotation through the center of the cylinder but perpendicular to the axis of the cylinder. Compare to the moment of inertia when the axis of rotation is the same as the axis of the cylinder.

5.3 Conservation of Angular Momentum

Remove the string from the pulley. Remove the cylinder from the disk, click REC, and set the disk spinning. This is best done by twirling the part of the axle of the RMS that is underneath the RMs between thumb and index finger. With the cylinder projections up (cylinder axis vertical), gently drop the cylinder as near the center of the disk as you can and press STOP. Determine how well angular momentum is conserved. It will be difficult to drop the cylinder exactly onto the center of the disk. Measure the eccentricity of the dropped cylinder and use the parallel axis theorem to get the moment of inertia of the slightly eccentric cylinder. Is rotational kinetic energy conserved in this process?

6 Finishing Up

Please leave the bench as you found it. Be sure the 2nd screw is in the dish. Thank you.

Acknowledgement Measuring the moment of inertia of the cylinder about an axis perpendicular to the cylinder axis was suggested by Alexei Tonyushkin, a graduate student and teaching assistant.

Setup

- ① Attach a mass on each end of the rod (part of the Mini-Rotational Accessory) equidistant from the rod center. You may choose any radius you wish.
- ② Tie one end of the string to the Mass Hanger and the other end to one of the levels of the 3-step Pulley on the RMS.
- ③ Mount the thin rod to the pulley on the Rotary Motion Sensor. Please note the orientation of the 3-step Pulley.
- ④ Mount the RMS to a support rod and connect it to a computer. Make sure that the support rod does not interfere with the rotation of the accessory rod. See Figure 1.1.
- ⑤ Mount the clamp-on Super Pulley to the Rotary Motion Sensor.
- ⑥ Drape the string over the Super Pulley such that the string is in the groove of the pulley and the Mass Hanger hangs freely (see Figure 1.1).

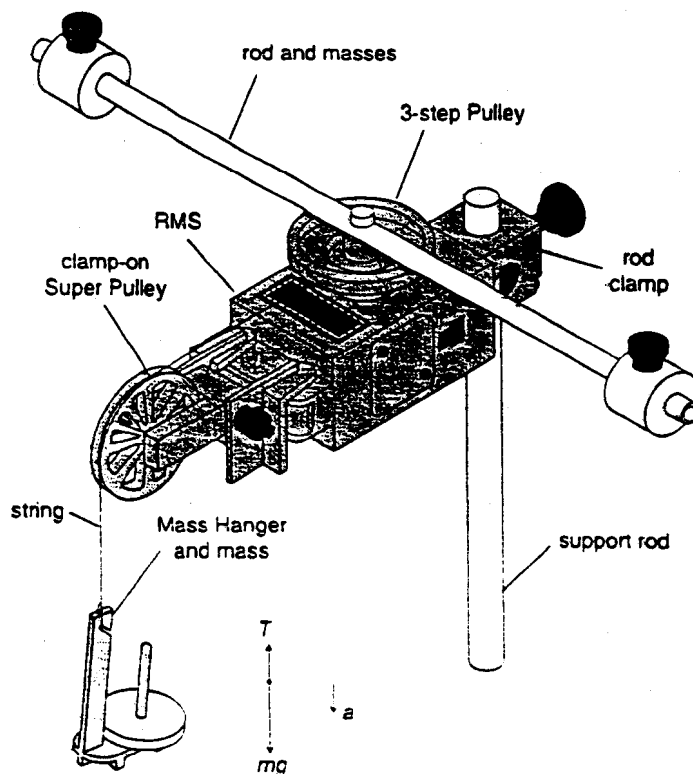
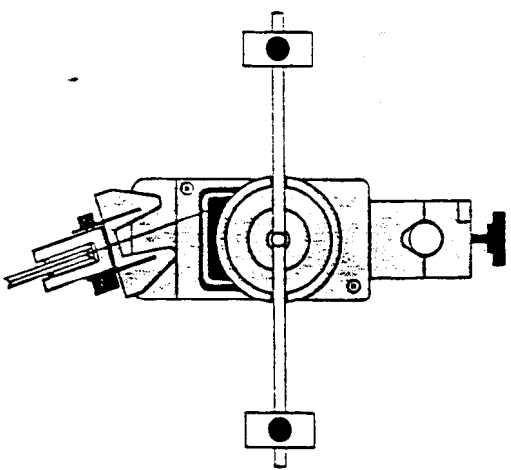


Figure 1.1: Rotary Motion Sensor and Free-Body Diagram

- **NOTE:** The clamp-on Super Pulley must be adjusted at an angle so the thread runs in a line tangent to the point where it leaves the 3-step Pulley and straight down the middle of the groove on the clamp-on Super Pulley.



- ⑦ Adjust the Super Pulley height so the thread is level with the 3-step Pulley.