Electromagnetic Induction

Equipment: Capstone, RLC circuit board, box with 2 coils, iron rod, and cylindrical magnet, 2 voltage sensors (no alligator clips), 2 leads (35 in.), Fluke multimeter with leads

1 Introduction

The phenomenon of electromagnetic induction was discovered by Joseph Henry in New York in 1830 and by Michael Faraday in England in 1831. Faraday’s name is commonly associated with the phenomenon (Faraday induction), although in this century Henry’s contribution has been recognized by assigning his name to the unit for inductance. In the 1820’s it was known that an electric current produces a magnetic field, and Henry and Faraday were trying to reverse the process and produce a current with a magnetic field. The result is elusive however, since a steady magnetic field will not cause a current to flow in a circuit. It is a time changing magnetic field that “induces” a current to flow. The changing magnetic field produces an electric field which is not conservative. The line integral of the electric field around a loop or circuit is not zero and is called an electromotive force or EMF. The EMF can drive a current in a circuit. The unit of EMF is the volt (V).

In this experiment an emf will be induced in one coil by

• Moving a permanent magnet into and out of the coil,
• Moving a second coil with a current in it near the first coil, and
• Changing the current in the second coil.

Some of your results will be qualitative. You should describe and explain what you observe. Bear in mind that the strength of the induced emf depends on the rate of change of the magnetic field. When an emf is induced in a coil the current that results will depend on the resistance of the circuit, which the coil is part of. A voltage will also appear across the coil and can be measured by standard means. Because the voltage across the coil used in this experiment will be measured by a high impedance (resistance) device, the amount of current that flows will be small.

2 Theory

The flux (Φ) of the magnetic field is defined in a similar way to the flux of the electric field. See Fig. 1. A magnetic field \( \vec{B} \) passes through a differential area \( d\vec{A} \). The differential element
of magnetic flux associated with \( d\vec{A} \) is \( d\Phi = \vec{B} \cdot d\vec{A} \). It is only the component of \( \vec{B} \) that is perpendicular to the surface area that contributes to the flux. For a finite area the magnetic flux is \( \Phi = \int \vec{B} \cdot d\vec{A} \). The line integral of the electric field around the boundary of the area is the EMF. The positive directions of \( d\vec{A} \) and the line integral must be consistent with the right hand rule where the thumb points in the direction of \( d\vec{A} \). Faraday’s law says that for a single circuit around the boundary of the area the EMF is given by \( \text{EMF} = -d\Phi/dt \). The non-conservative electric field giving rise to the emf exists whether there is a wire around the boundary or not. But if there is a wire a current will flow if the circuit is “complete.” The current will be limited by the resistance of the circuit. If the loop of wire has two ends a voltage of \( V = +d\Phi/dt \) will appear between the two ends of the wire. This voltage is due to a conservative electric field that opposes the non-conservative electric field producing the EMF. If the wire has N complete turns it is called a coil. The Voltage across the N turn coil is \( V = +N d\Phi/dt \). The magnetic field is said to “link” the coil. It has been assumed that the cross section of the coil is small and that the same magnetic flux goes through each turn of the coil.

The changing magnetic flux through the coil can be produced by any means you can think of. This includes relative motion between a magnet and the coil, relative motion between the coil and a separate circuit with steady currents, changing currents in a separate circuit (mutual inductance), and a changing current in the coil itself (self inductance).

### 3 EMF Induced By A Permanent Magnet

In this section an EMF is induced in a coil by moving a magnet in and out of the coil. The emf will be detected by measuring the voltage across the coil. The voltage across the coil as a function of time will be displayed on a graph.

- Plug the voltage sensor into one of the analog plugs, and connect it directly across the coil on the RLC circuit board.
- In the hardware set-up window, click on the 850 interface analog channel that has the voltage sensor.
• Choose the voltage sensor from the drop down menu.

• Setup a graph by dragging its icon from the Displays window to the center of the white screen. Note: If needed, you can adjust each axis of the graph by clicking on and scrolling across each axis.

• Set up the voltage sensor in the graphs display by clicking on the y-axis in Select Measurements and then select Voltage. The x-axis should then be the default value of time.

• In the bottom center of Capstone, adjust the sampling rate to 100 Hz. The following illustration displays functions you need to be aware of when using the graph.

In what follows, keep track of which end of the rod magnet you are using. The magnetization of the magnet is along its center axis. Align the axis of the rod and the axis of the coil. Click Record. Keeping the axes of the coil and magnet collinear, stick the magnet through the center of the coil and rest one of the ends on the lab bench. Wait a second, keeping the magnet motionless, then pull the magnet out of the coil. Click Stop. You should see two peaks of induced voltage in the graph. Note the polarities of the peaks. Try slower motions and faster motions. Try not moving the magnet at all. Turn the magnet around and repeat your motions, noting the voltage polarities. Turn the axis of the magnet perpendicular to the axis of the coil and bring the magnet down onto the coil. Is there an induced voltage?

Repeat the above experiments by keeping the magnet stationary and moving the coil.

Lift the RLC circuit board above the bench. Align the axes of coil and magnet, hold the magnet about one inch above the coil. Click Record and drop the magnet through the coil. Which voltage peak is higher? Why?

4 EMF Induced By Moving A Coil With A Steady Current

The experiments of section 3 will be repeated with a current carrying coil instead of a magnet. A coil with a current has a magnetic field similar to that of a bar magnet. If this coil is moved with respect to another coil the changing magnetic field will induce an EMF.

At this point you will restart Capstone. Keep the voltage sensor plugged into the interface and connected across the inductor on the RLC circuit board.

• Re-program Capstone for the voltage sensor.
Connect the output of the 850 interface to one of the coils from the wooden box.

In the Tools column click on the signal generator icon and the signal generator window will pop open.

Click on 850 Output 1. In waveform select DC and program the signal generator for 7 V DC after click on Auto.

Open a graph by dragging its icon from the Displays window to the center of the white screen.

Set up the graph display as you have done in section 3.

The induced voltages will be smaller so in the graph display, change the vertical axes to read from approximately -0.02 to +0.02 V. Double click the voltage sensor gear icon and change the gain to (100X).

Click Record and quickly move the 2nd coil in a coaxial fashion with the coil on the RLC circuit board. The recorder trace will be noisy, but an effect should be observable. Reverse the current to the coil by switching the leads. (In theory, the coil could be turned over, but the connectors to the coil then make it difficult to move the 2 coils together.) Do you see a change in the direction of the induced voltage? Explain why.

Comment: There is a great deal of similarity between moving a magnet and moving a coil with a current. The magnet also has currents, but the currents are not produced by conduction electrons but by electron orbits and spins in the magnetized material from which the magnet is made.

5 EMF induced by Changing The Current In A Coil

In the previous experiment, coil 1, with a steady current, was moved with respect to coil 2 in such a way that the magnetic flux changed through coil 2, inducing an EMF and voltage in coil 2. Now assume the two coils are fixed and that there is a time varying current through coil 1 that produces a time varying magnetic flux through coil 2. A time varying EMF will be produced in coil 2. If there are no magnetic materials around the magnetic field produced by coil 1, then the current \( i_1 \) will be proportional to the magnetic field of coil 1. Likewise the magnetic flux through coil 2 produced by coil 1 will also be proportional to \( i_1 \). It is customary to write the magnitude of this EMF in coil 2 as \( M \frac{di_1}{dt} \). The quantity \( M \) is the mutual inductance between the two coils and depends on the geometry of the coils and the number of turns of each coil. \( M \) does not depend on the current. It is possible to orient the two coils so that \( M \) is zero. In this experiment the EMF induced in one coil by a changing
current in another coil will be investigated. If the coils are moved with respect to each other the mutual inductance $M$ will change as will the induced voltage.

- **Again restart Capstone.**
- As in the previous experiment, program the voltage sensor in Capstone.
- Next, in the experiment setup window click on the output of the 850 interface and select **Output Voltage - Current Sensor**.
- In the displays column drag the scope icon to the center of the white data window.
- Now you have to program the oscilloscope to read two signals. To do so, go to the top of the scope and click on **add new y-axis**.
- Program channel one to read the voltage output of the 850 interface, and channel two to read the voltage sensor channel. Look at the illustration below for guidance.

Next, click on the signal generator and program it for a 3 V sinusoidal wave with a frequency of 500 Hz and click on auto.
The last software item you need to change is the Continuous Mode to Fast Monitor Mode.

Click Monitor and observe the voltage output and the induced voltage on the 2 scope traces. (Remember you might have to use the trigger option on the scope to obtain a stable signal. The trigger button is located on the top left of the scope.) How are the 2 scope traces related? What happens if you reverse the two leads from the 850 Output to the coil? What happens if you move the coils apart? What is happening to the mutual inductance when you do this?

Lay the wired coil on top of the circuit board coil and do not move the coils. Now see what happens when you use frequencies of 500, 1000, 2000, and 4000 Hz (do not change the voltage of the signal generator). The voltage across the wired coil should not change. You should also find that the induced voltage across the circuit board coil does not change very much! As an increase in frequency will result in a more rapidly changing magnetic flux in the circuit board coil, this implies that the current in the wired coil goes down as the frequency goes up. This will be examined in the next section.

Return to a frequency of 1000 Hz. Insert the iron rod from the box partway into the wired coil. What happens to the induced voltage in the circuit board coil? Insert the iron rod fully so that it goes through both coils. (Iron is a magnetic material. If a modest magnetic field is applied to iron, the iron will magnetize and greatly increase the magnetic field.) What happens to the induced voltage? When the iron is inserted partway into the wired coil the current in the wired coil is reduced, reducing the flux in the circuit board coil. When the iron in completely inserted, the iron produces more than enough extra magnetic flux in the circuit board coil so as to make up for the decrease of current in the wired coil. The voltage increases. Note. In the above analysis the EMF produced in coil 2 by the current in coil 2 has been neglected. This is an excellent assumption as the only thing connected across coil 2 is a voltage sensor which has a very high resistance. The current in coil 2 is negligible.

Comment: Two coils magnetically coupled by iron are called a transformer. A sinusoidal voltage is applied to the primary coil and the sinusoidal output voltage appears at the secondary coil. If the secondary coil has more turns than the primary coil the voltage is stepped up. If the secondary coil has fewer turns, the voltage is stepped down. Neglecting losses, power is conserved, making the input voltage times the input current equal to the output voltage times the output current. The existence of transformers is why the power distribution system is AC, not DC. Tesla had it right and Edison had it wrong.
6 Self Inductance

Current in a coil produces a magnetic flux in the same coil. This magnetic flux, if it varies with time, produces a self EMF in the coil. As the magnetic flux in the coil is proportional to the current \( i \) in the coil, the EMF in the coil can be written \( \text{EMF} = -L \frac{di}{dt} \). \( L \) is called the self inductance of the coil and depends on the coil geometry and the number of turns of wire but not on the current. If a time varying voltage is applied to a coil the electric field drives a time varying current through the coil. This current creates an induced counter electric field that limits the current. The EMF across the coil is \( -L \frac{di}{dt} \) and the voltage across the coil is \( +L \frac{di}{dt} \). This is most easily explored by applying a sinusoidal voltage \( v = v(t) = V_P \cos \omega t \) across the coil. \( V_P \) is the peak value of the sinusoidal voltage. In what follows it is assumed that the coil has no ohmic resistance. This is an assumption and that for some parameters this is not completely valid. We have \( v = V_P \cos \omega t = L \frac{di}{dt} \). Integrating this expression the current becomes \( i = \frac{V_P}{\omega L} \sin \omega t \). The voltage across the coil is said to lead the current through the coil by \( \pi/2 \) rad. If \( V_P \) is kept constant the current decreases as \( 1/\omega \) as the frequency increases. The quantity \( \omega L \) is called the inductive reactance \( X_L \). \( X_L \) is somewhat akin to resistance and has the units of ohms. In the same way that resistance determines the amount of current in a resistor, the inductive reactance determines the amount of current in an inductor. There is no energy dissipation in a coil that has no resistance. Energy is continually exchanged between the magnetic field of the coil and the voltage source, and it averages to zero over one cycle. There is of course energy dissipation in a resistor. For a sinusoidal voltage in a resistor, the current and voltage are in phase. In an inductor, the voltage leads the current by \( \pi/2 \) rad.

6.1 Setting up the experiment

- Restart Capstone and plug the voltage sensors into channels A and B.
- In the Capstone Hardware set-up window program channel A and B for the voltage sensors.
- Click on Output 1 of the 850 interface and select Output Voltage - Current Sensor.
- In the displays column, drag the scope icon to the center of the white data window.
- Now you have to program the oscilloscope to read three signals. To do so, go to the top of the scope and click on **add new y-axis twice**.

In our theory it was assumed that the coil had no resistance. **Use the Fluke multimeter to measure the resistance of the coil.** Set the Fluke meter dial to \( \Omega \) and place the Fluke meter across the coil to measure the resistance.

Choose Output Voltage for channel 1. Choose voltage sensor A for channel 2 of the scope and voltage sensor B for channel 3 of the scope. Hook up the circuit shown in Fig. 2. The 10 ohm resistor is included in the circuit for current measurement.
• Click on the **Signal Generator** icon and then on the 850 Output 1.

• Next, select **Auto**. Before you start the scope, you need to switch the program to run in **Fast Monitor Mode** as you have done in Section 5.

### 6.1.1 Running the experiment

Start with a frequency of 1000 Hz and a peak voltage of 1.5 V across the coil. You need to measure the voltage across the resistor and calculate the current. **Also note the phase difference between the current through and voltage across the coil.** Repeat these measurements for frequencies of 500, 2000, and 4000 Hz, for each case adjusting the voltage across the coil to be 1.5 V. In light of the theory given above, discuss your results.

Calculate the inductive reactance of coil at 1000 Hz and compare it to the resistance of the coil. The resistance of the coil is not enough to drastically change our principle results.

The derivative of the current through an ideal coil is proportional to the voltage across the coil. The voltage across a resistor is proportional to the current through the resistor. At a frequency of 1000 Hz, look at the waveforms on the scope for the square wave and triangular wave. **Do the shapes of the voltage across the coil and resistor make sense?** The shapes of the waveforms may not be what you expect, but they will be close. For example, note that the applied square wave has a very finite rise time.

Returning to the 1000 Hz sine wave, thrust an iron rod into the coil. **What happens to the current?** Why?

**Problem:** The power into a circuit element is the product of the voltage across the element and the current through the element. Assuming a voltage \( v(t) = V_p \cos(\omega t) \) across an inductor \( L \), integrate the power over one cycle and show that the net energy into the inductor is zero.
7 Finishing Up

Please leave the bench as you found it. Cheers and thank you!