Centripetal Force Lab 9

1 Introduction

In classical mechanics, the dynamics of a point particle are described by Newton's 2nd law, $\vec{F} = m\vec{a}$, where $\vec{F}$ is the net force, m is the mass, and $\vec{a}$ is the acceleration. This equation guarantees that $\vec{F}$ and $\vec{a}$ are parallel to each other. If $\vec{v}$ is the velocity, by definition $\vec{v}$ is always parallel to path of the particle. For an arbitrary motion, it is not necessary for the directions of $\vec{v}$ and $\vec{a}$ to have any particular direction with respect to each other.

In the *special* case of a particle moving in a circle with constant speed, $\vec{F}$ and $\vec{a}$ are always perpendicular to $\vec{v}$ and point from the particle's position toward the center of the circle. The force $\vec{F}$ is called the centripetal force (centripetal means "center seeking"). In this experiment a mass m is rotated in a circle and the force on the mass is compared to the theoretical value.

A related term which is often misused is "centrifugal force." This term is only relevant in a rotating frame of reference. If you want to use $\vec{F} = m\vec{a}$ in a rotating frame of reference you must add this radially outward force. If the particle is moving in the rotating frame, you must also add the velocity dependent "Coriolis" force. These two "forces" arise because the frame of reference is not inertial.

2 Theory

2.1 Angles In Radians

Mathematicians and Physicists often express angles in radians (rad). In Fig. 1, a circle of radius r has been drawn with its center at the vertex of an angle θ. Let the length of arc intercepted by the angle θ be $\widehat{ab}$. The angle in radians is defined by

$$\theta = \frac{\widehat{ab}}{r}.$$

This result is independent of the size of r. A full circle has 2π rad, and $360\,\text{deg} = 2\pi$ rad.

2.2 Angular Velocity $\vec{\omega}$, Frequency f, and Period T

How fast an object is rotating or moving in a circle is often expressed in terms of the angular velocity $\vec{\omega}$, with ω expressed in rad/s. Refer to Fig. 2 and let $\vec{r}$ be a rotating position vector with constant magnitude such that the tip of $\vec{r}$ describes a circle. Suppose that $\vec{r}$ sweeps out a small angle $\Delta\theta$ in the small time Δt. The magnitude of $\vec{\omega}$ is defined by

$$\omega = \frac{d\theta}{dt} = \lim_{\Delta\theta \to 0} \frac{\Delta\theta}{\Delta t}.$$

The direction of $\vec{\omega}$ is perpendicular to the plane in which $\vec{r}$ is rotating and is given by the right hand rule. With the fingers pointing in the direction in which $\vec{r}$ rotates, the extended thumb points in the direction of $\vec{\omega}$. In general, ω does not have to be a constant, but in the case of a particle rotating in a circle at constant speed, ω is a constant. Rotation rates are also expressed in frequency (f or ν), which is the number of cycles or rotations per second. One cycle per second is called a hertz (Hz). The relationship between f and ω is $\omega = 2\pi f$.

with ω in rad/s and f in Hz. The period T is the time for one revolution. It is given by T = (1/f).

2.3 Velocity, Acceleration, and Force

The velocity $\vec{v}$ and acceleration $\vec{a}$ are defined by

$$\vec{v} = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \; and,$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}.$$

In the case where a particle moves in a circle with constant speed, the magnitudes of $\vec{r}$ and $\vec{v}$ are constant but their directions change. This leads to the following equations for v and a:

$$v = r\omega, \; and \; a = r\omega^2.$$

As usual, Δ denotes a small change in the quantity following it. Letting F_c be the centripetal force on a particle moving in a circle with constant speed, and substituting the expression for the acceleration a into Newton's 2nd law, we have

$$F_c = mr\omega^2. \tag{1}$$

This is the equation that will be examined experimentally.

3 Apparatus

Refer to Fig. 3. On a horizontal base, a vertical shaft is supported by a bearing that allows the shaft to rotate. At the top of the shaft is a horizontal arm which supports a movable slider and a counter weight. A mass M_1 is suspended by string from the arm. Mass M_1 has a vertical pointer at its bottom, and is attached to the shaft by a spring whose tension can be varied. There is a vertical movable pointer on the base that projects upward toward the pointer on M_1. The shaft can be rotated with the fingers of one hand by twirling the knurling near the bottom of the shaft. The shaft is twirled so that the pointers on the base and M_1 line up. The centripetal force is supplied by the spring, and by the horizontal component of the tension in the string supporting M_1. To measure the centripetal force, the apparatus is brought to rest and a string attached to the side of M_1. This string goes horizontally to a pulley and then vertically to the suspended mass M_2. The value of M_2 is adjusted so that the pointers again line up. The centripetal force is M_2g, where g is the acceleration of gravity.

4 Procedures

1. Level the base by using the thumb screws in the base and utilizing the small bubble level on the base.
2. Check the length of the string supporting mass M_1. The two pointers should come within a few mm of each other. When the string attached to M_1 and M_2 is in place, the part of the string nearest to M_1 should be approximately horizontal.

3. Adjust the pointer in the base so that it is about midway between its two extreme radial positions. Measure the distance from the pointer to the axis of rotation.

4. With the apparatus at rest, the positions on the arm of the slider and the counter weight, and the position of the arm with respect to the shaft, should be adjusted so that

   (a) the two pointers lie on the same vertical line, and

   (b) when the thumbscrew holding the arm is loosened, the arm is approximately balanced.

   You will probably not be able to fulfill these conditions exactly.

5. Adjust the tension in the spring so that the pointers line up for a "reasonable" rate of rotation. While rotating the shaft and looking at the pointers, **watch your head!** Don't get knocked by M_1. The rate of rotation should not be so slow that the centripetal force will be hard to measure. It should not be so fast that it is dangerous, or that the apparatus does not stay quietly on the bench.

6. Take data. With the string from M_2 not attached to M_1, rotate the shaft so that the pointers line up and measure the time for something like 10 or 20 revolutions. Calculate the period T, which is the time for one revolution or cycle. Repeat a few more times to build up some statistics. With the apparatus at rest, attach the string from M_2 to M_1 and add weights to the weight holder so that the the pointers line up. Don't forget the mass of the weight holder. Repeat the above procedures with the base pointer as far out as it will go, and as far in as it will go. Measure the rotation radii for these two positions.

5 Analysis

Convert the periods you have measured to angular velocity, using

$$\omega = \frac{2\pi}{T}.$$

Then referring to Eq. (1), compare your values of $M_2 g$ with the corresponding values of $M_1 r\omega^2$.

6 Questions

1. What are the most significant errors in this experiment?
2. Are these errors random or systematic?
3. Does any of your data enable to to estimate any of these errors?
4. Do you think your data supports the use of Newton's 2nd law when it is applied to a mass going in a circular orbit at constant speed?

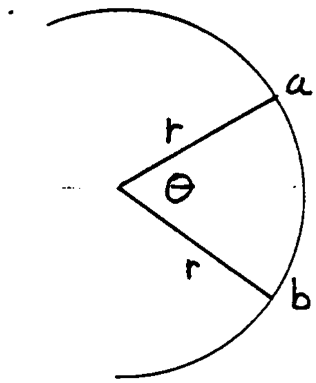

Fig. 1

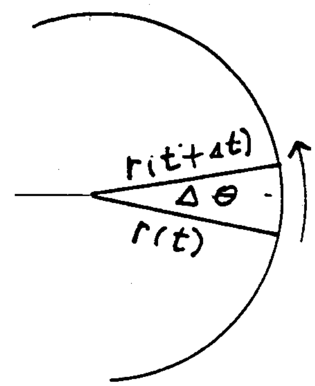

Fig. 2

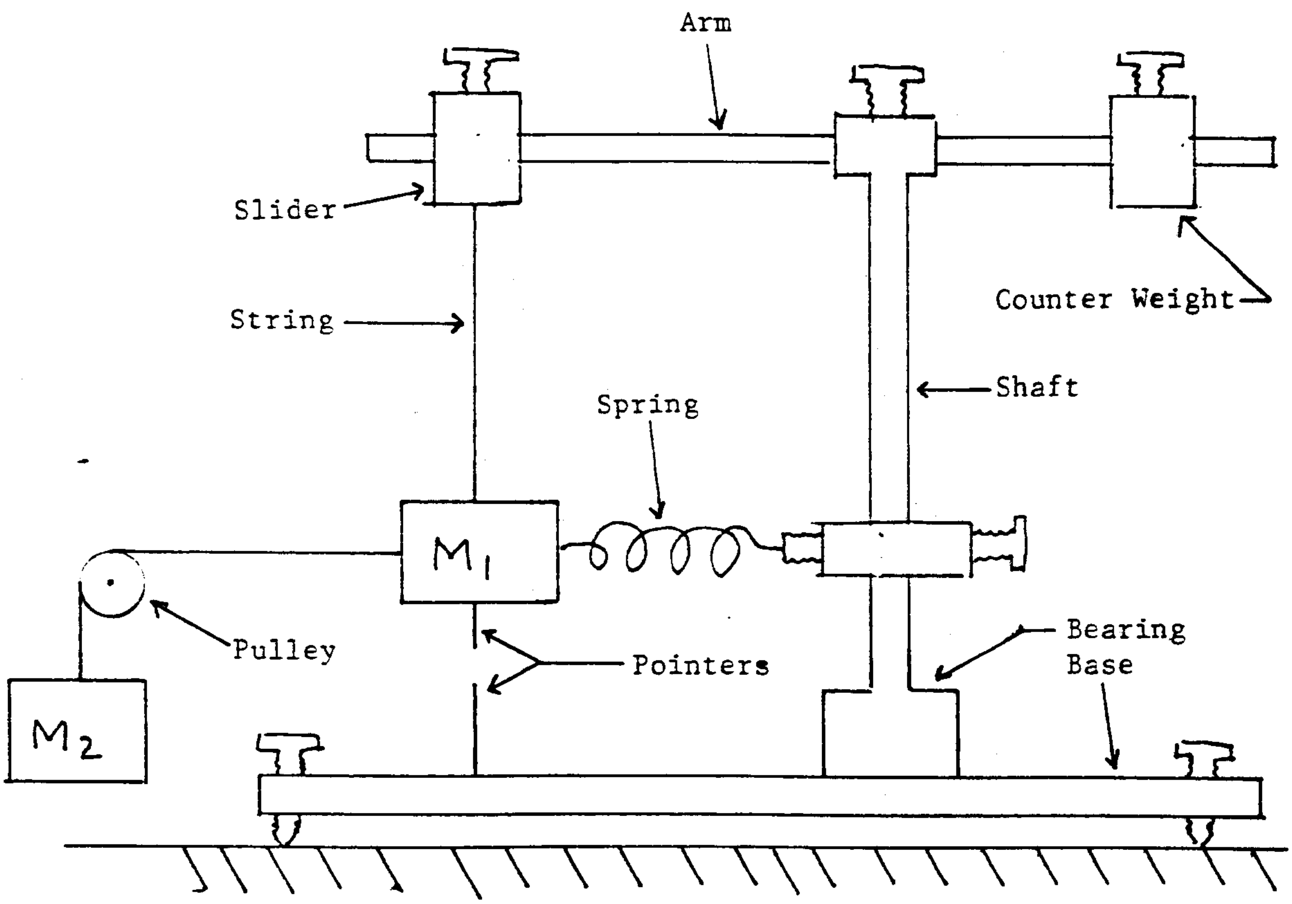

Fig. 3