

Ballistic Pendulum Lab 8

Equipment- ballistic pendulum apparatus, 2 meter ruler, 30 cm ruler, blank paper, carbon paper, masking tape, scale

1 Purpose

To measure the speed of a projectile by a kinematic method and also by the use of a ballistic pendulum. The latter method illustrates the use of conservation of energy and momentum, but for different times.

2 Description

A spring gun fires a steel ball horizontally from a bench top. The steel ball is either allowed to fly free and strike the floor, or embeds itself in a pendulum. In the first case the initial velocity of the ball can be obtained by using basic kinematics. In the second case the initial velocity of the ball can be obtained by conservation laws derived from Newton's 2nd law.

The ballistic pendulum was invented in 1742 to measure the speed of bullets. As you can see from this experiment it is not necessary to use a ballistic pendulum to measure the speed of a slowly moving object, but the ballistic pendulum does illustrate the use of several important conservation laws in physics.

3 Kinematic Measurement Of Speed

3.1 Theory

See Fig. 1. Let the vertical distance that the ball drops before hitting the floor be d and the horizontal distance be D . Let the time the ball is in flight be t and the horizontal speed of the ball be v . The vertical and horizontal motions of the ball are uncoupled. There is no horizontal acceleration and $D = vt$. The vertical acceleration is g , and $d = \frac{1}{2}gt^2$. Combining these 2 equations,

$$v = D \left(\frac{g}{2d} \right)^{\frac{1}{2}} \quad (1)$$

D and d are the measured quantities.

3.2 Procedures

Swing the pendulum out of the way so the the latching mechanism holds the pendulum to the side. Put the ball, which has a hole in it, on the rod at the end of the gun. Obtain d by measuring the distance from the bottom of the ball to the floor with a 2 meter ruler.

Push the ball into the gun against the action of the spring. Push far enough so that the spring is latched. Aim the gun across a clear spot in the lab room. Being sure no one is in the way, depress the firing lever on top of the gun and observe where the ball lands on the floor. Tape 2 pieces of white paper together along the $8\frac{1}{2}$ in edge and lay the middle of this double sheet where the ball landed. Orient the long direction of the double sheet to be in the direction of the traveling ball and tape the double sheet to the floor. Place 2 sheets of

carbon paper over the white sheets with the carbon side down. When the ball lands on the carbon paper it will leave a mark on the white paper. Fire the gun a number of times to build up some error statistics for the distance D . Use Eq.(1) to calculate v , the speed of the ball as it leaves the gun.

4 Ballistic Pendulum Measurement Of Speed I

4.1 Theory

See Fig. 2. Assume the pendulum is initially unlatched and hanging vertically at rest. For this section we will assume that the pendulum consists of a mass of small dimensions suspended by a massless cord. Let the mass of the ball be m and the mass of the pendulum be M . Let the horizontal speed of the ball before the collision be v and of the pendulum plus ball right after the collision be V . When the ball strikes the pendulum and sticks it is a completely inelastic collision and energy is not conserved. Define the system as the pendulum mass and the ball. For the system horizontal momentum is conserved for a very short time from just before the collision to just after the collision as for this very short period there are no horizontal forces acting on the system. The system is the pendulum mass and the ball. (When the pendulum has swung to the side a bit the cord of the pendulum exerts a horizontal force on the pendulum mass and horizontal momentum is no long conserved.) The horizontal linear momentum of the system just before the collision is mv and just after the collision is $(M + m)V$. Conservation of horizontal linear momentum gives $mv = (M + m)V$.

Right after the collision the pendulum mass and ball swing up against the force of gravity and eventually come to rest. Assuming no friction, energy is conserved during this part of the motion as the the force of gravity which is conservative and the force of the suspending cord does no work. Let the gravitational potential energy immediately after the collision be zero. At this time the total energy is then kinetic and equal to $\frac{1}{2}(M + m)V^2$. When the pendulum has stopped the kinetic energy is zero but the gravitational potential energy is $(M + m)gh$, where h is the vertical distance that the pendulum mass and ball have gone up. Equating the total energies at these two times gives $\frac{1}{2}(M + m)V^2 = (M + m)gh$. Eliminating V between these equations gives

$$v = \frac{M + m}{m} \sqrt{2gh}. \quad (2)$$

4.2 Procedures

Pull the pendulum to the side, insert the ball into the gun, and compress and latch the gun spring. Release the pendulum so that it hangs vertically. Fire the gun. The pendulum will latch near the highest point of its swing. Measure the height h_2 , the vertical distance from the pendulum platform to the center of the ball when the pendulum is latched to the side. Release the pendulum so that it is vertical and measure h_1 , the distance from the pendulum platform to the center of the ball when the pendulum is vertical. Then $h = h_2 - h_1$. The mass of the pendulum M is written on the pendulum. Measure the mass m of the ball. Use Eq.(2) to calculate the speed of the ball v . Repeat the measurement several times to get some error statistics. Question. What fraction of the kinetic energy is lost in the collision?

Comments. There are a number of approximations that have been made in the analysis for this section. The pendulum used is not a simple pendulum which is a point mass sus-

pended by a weightless string. The pendulum is actually a physical pendulum which has a distribution of mass. Unless the physical pendulum is struck by the ball at one specific spot, the center of percussion, the pendulum support will impart a brief force or impulse during the collision. Horizontal momentum will not strictly be conserved. The kinetic energy immediately after the collision is not that of a moving point mass but of a solid body rotating about an axis. In calculating the change in gravitational potential energy of the pendulum plus ball, the center of mass should be used rather than the position of the ball. The analysis in the following section addresses these difficulties.

5 Ballistic Pendulum Measurement Of Speed II

5.1 Theory

Assume the pendulum is a physical pendulum free to swing in a vertical plane about a horizontal axis. We use the previous definitions for M , m , V , v . We introduce the following symbols.

- a is the vertical distance from the axis to the initial trajectory of the ball.
- b is the distance from the axis to the center of mass of the pendulum *with the ball in it*.
- ω is the angular velocity of the pendulum with ball immediately after the collision.
- I is the moment of inertia of the pendulum plus ball about the axis.
- H is the maximum vertical distance that the center of mass of the pendulum with ball rises after the collision.
- T is the period of the pendulum with the ball in it.

As before, the system is the pendulum and the ball. We invoke conservation of angular momentum about the pendulum axis of the system for the time immediately before the collision to the time immediately after the collision (before the pendulum has swung upward). There are no external torques on the system for this period of time. Any impulse from the pendulum support during the collision contributes zero torque about the pendulum axis as there is no lever arm. The angular momentum of the pendulum does change as the pendulum swings upward due to the torque exerted by gravity about the pivot point, but there will be no torque due to gravity before the pendulum swings away from the vertical. The angular momentum of the system just before the collision is mva , and the angular momentum right after the collision is $I\omega$. Conservation of angular momentum gives $mva = I\omega$.

Let the gravitational potential energy of the system be zero right after the collision. Immediately after the collision the total energy of the system is kinetic and given by $\frac{1}{2}I\omega^2$. When the pendulum has risen to its maximum height the kinetic energy is zero and the potential energy is $(M + m)gH$. Conservation of energy gives $\frac{1}{2}I\omega^2 = (M + m)gH$.

The period of the physical pendulum is given by $T = 2\pi\sqrt{\frac{I}{(M+m)gb}}$. Combining the equations for conservation of angular momentum and energy, and the equation for the period, the initial velocity of the ball becomes

$$v = \frac{(M + m)gT\sqrt{bH}}{\sqrt{2\pi ma}} \quad (3)$$

The quantities needed, in addition to m , M , and g , are a , b , T , and H .

5.2 Procedures

Fire the ball into the pendulum a number of times and measure the values of H , the vertical height that the center of mass of the pendulum plus ball rises during the collisions. Measure the value of a , the vertical distance from the pivot point to the initial line of motion of the ball. The distance b , from the pivot to the center of mass of pendulum plus ball, is 27.0 ± 0.1 cm. This has been measured for you by disassembling the pendulum and balancing it on an edge. With the ball in the pendulum set the pendulum swinging and measure the time for 10 or 20 periods. Divide to get the period T . Use Eq.(3) to calculate values of the initial speed v of the ball.

6 Summing Up

Compare your three different kinds of measurements and associated uncertainties. Are they consistent? What do you think is the most important random error in this experiment? Which method do you think gives the best value? The worst?

7 Finishing Up

Please leave your bench in a ship-shape fashion. Thank you.

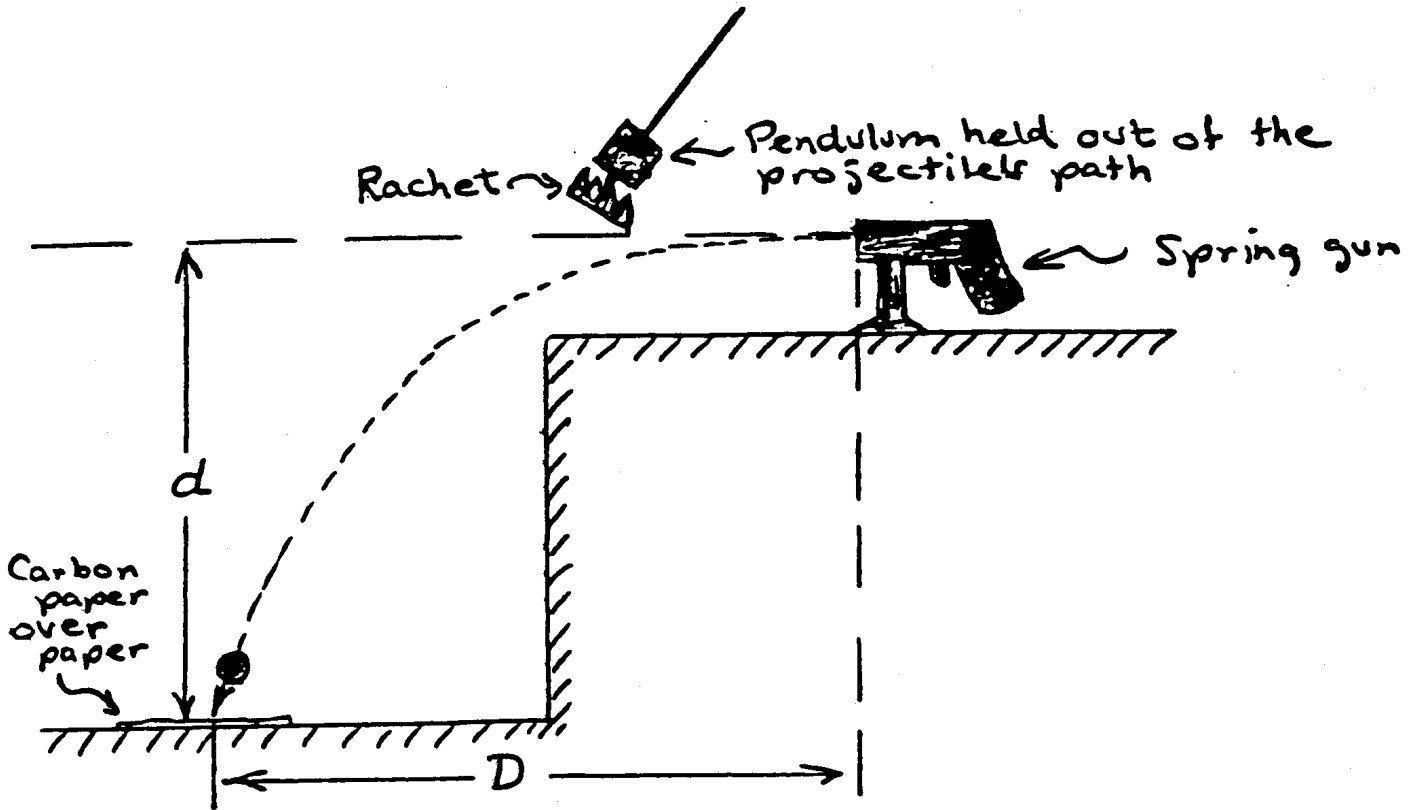


Figure 1. Arrangement for studying a projectile in free fall.

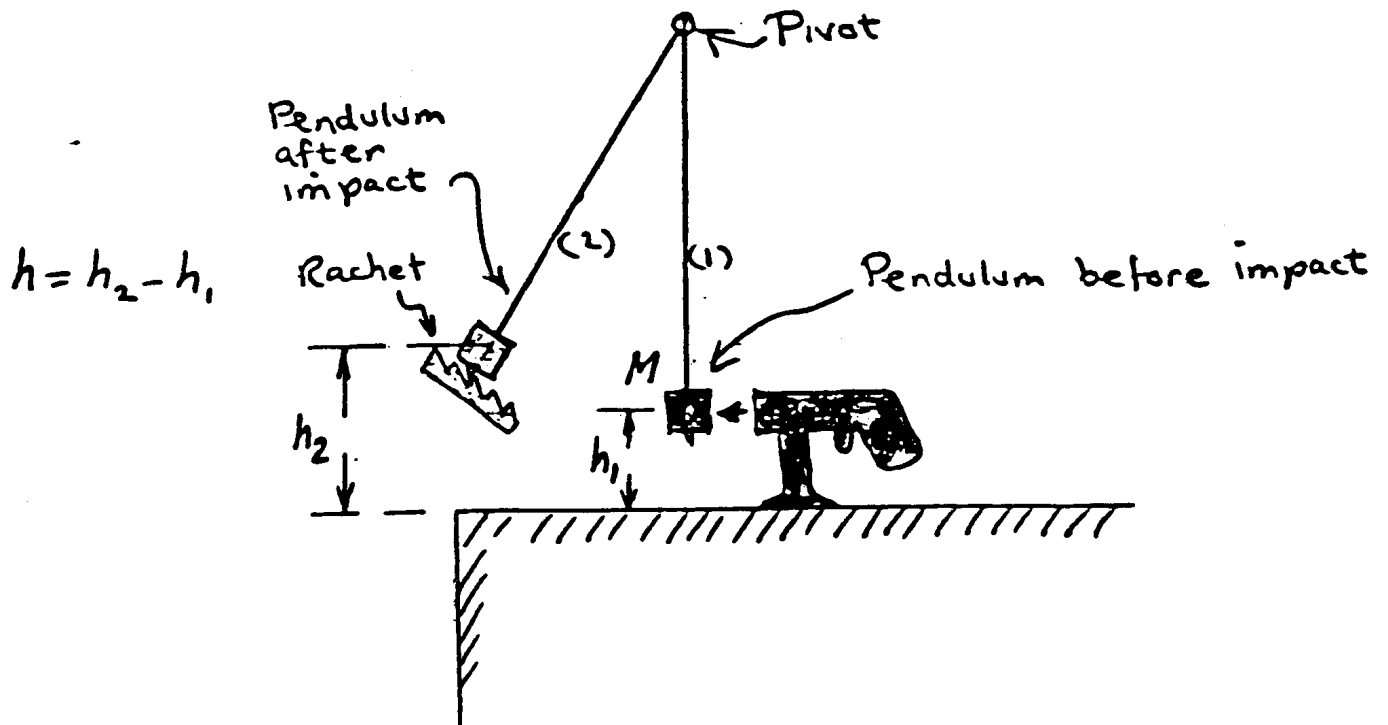


Figure 2. Ballistic pendulum at height h_1 before impact and at height h_2 on reaching its point of maximum swing after impact.