

## Magnetic Field of a Circular Coil

## Lab 12

Equipment- coil apparatus, BK Precision 2120B oscilloscope, Fluke multimeter, Wavetek FG3C function generator, 3 leads

Comment The computer monitor should be off to avoid picking up an interference signal with the search coil.

Reading Electrical Safety at the beginning of this manual.

## 1 Introduction

A wire coil that is carrying a current produces a magnetic field  $\vec{B}(\vec{r})$ , where  $\vec{r}$  is the distance from the center of the coil to the field point. The strength of the field  $\vec{B}$  is proportional to the current  $I$  in the coil. The strength and direction of the field depend on  $\vec{r}$ . For large distances from the coil ( $r \gg a$ , where  $a$  is the radius of the coil), the shape of the magnetic field of a coil is identical to the electric field produced by a point electric dipole. For large distances both fields fall off as  $1/r^3$ . In this experiment you will measure the magnetic field of a circular coil at distances that are fairly close to the coil. The large distance approximation is not valid.

A constant magnetic field can be measured in many ways. You can use a compass, a Hall Probe, a rotating coil of wire, or nuclear magnetic resonance. In this experiment the magnetic field will not be constant but will vary sinusoidally with time. Such a time varying magnetic field will induce a time varying voltage in a small coil which will be called the “search coil.” The search coil will be used to measure the magnetic field produced by a larger coil called the “field” coil. The current in the field coil will be varied sinusoidally with time and produce a sinusoidally varying magnetic field.

## 2 Electromagnetic Induction

A magnetic field can be described at each point in space and time by a vector  $\vec{B}$  whose direction coincides with the direction of the field and whose length is proportional to the magnitude  $B$  of the field. A magnetic field that changes with time produces a *non-conservative* electric field. This phenomenon, called electromagnetic induction, was discovered by Faraday, Henry, and others. The non-conservative electric field will produce a current and voltage in the search coil. By measuring this voltage for different positions and orientations of the small “search” coil the time varying magnetic field produced by the large field coil can be mapped out. The frequency of the sinusoidal current will be low enough so that the magnetic field mapped out by the search coil will be essentially identical to the magnetic field produced by a field coil carrying a constant current.

## 3 The Magnetic Field

A wire carrying a current generates a magnetic field  $\vec{B}$  whose magnitude and direction at each point in space depend on the length and shape of the wire, the current flowing through the wire, and the location of the point at which the field is determined. A convenient way to depict the pattern of the magnetic field is to draw a lines such that each line is always parallel to the magnetic field  $\vec{B}$ . The pattern of lines shows the direction of the magnetic

field everywhere in space. The intensity of the field is indicated by having the density of the lines show the strength of the field. In certain cases it is easy from the symmetry of the situation to deduce the nature of such a field pattern. For instance, the magnetic field pattern around a long straight current-carrying wire must describe circles centered on the wire, as shown in Fig. 1. The direction of the field is given by the “right hand rule.” The strength of the field decreases with increasing distance from the wire. This is shown by drawing the lines further apart from each other where the field is weaker.

Suppose that the straight wire is bent into the shape of a thin circular coil, with many closely spaced turns of wire. The field at any position is the sum of contributions from the many short elements of wire composing the entire length of wire. You can deduce the general nature of the field pattern of this coil from the magnetic field produced by short length of wire. As shown in Fig. 2, the field contributions of all the short elements add together near the center of the coil to produce a field whose strength is much greater than that of any one element. Furthermore, from the symmetry, you may deduce that the direction of the field is along the axis of the coil. The direction of the axial field is given by the right hand rule where the fingers curl around the coil in the direction of the current and the extended thumb points in the direction of the field. The field in the central region is comparatively strong and uniform. Just outside the coil, very near the wire, the field is due predominantly to the closest portion of the wire, with the current from the far side of the coil contributing relatively little owing to the greater distance of the source. The field has the opposite direction to the field in the center of the coil. Farther out to the side of the coil, the distances from the near portion of the wire and far portion are not so very different, so the fields contributed from each have nearly the same strength but opposite directions. The field from the far side nearly cancels the field from the near side, and the strength of the field must decrease much more rapidly with distance from the coil than if we had a single length of wire. For a long straight wire the field varies as the inverse distance from the wire, and for large distances from a coil it varies as the inverse cube of the distance.

## 4 Theory

Except along the axis, the magnetic field of a circular coil cannot be expressed in closed form. Along the coil axis, if the origin of the coordinates is taken at the center of the coil and if the  $z$  axis is taken along the coil axis, the magnitude of the magnetic field  $B$ , which points in the  $z$  direction, is given by

$$B = \frac{\mu_0 N a^2 I}{2 (a^2 + z^2)^{3/2}}, \quad (1)$$

where  $B$  is in tesla if

- $\mu_0 = 4\pi \times 10^{-7}$  is the vacuum permeability,
- $N$  is the number of turns of the field coil,
- $I$  is the current in the wire, in amperes,
- $a$  is the radius of the coil in meters, and
- $z$  is the axial distance in meters from the center of the coil.

## 5 The Experiment

Summarizing, the sinusoidally varying current in the field coil produces a magnetic field that varies sinusoidally with time. The part of the magnetic field that threads through the search coil produces a sinusoidally varying voltage in the search coil. This voltage will be measured on an oscilloscope and will be used to determine the magnetic field.

The voltage generated in the search coil is due to electromagnetic induction. Assume that the search coil is small and that the magnetic field at a given instant of time is approximately uniform over the area of the search coil. For this situation the flux  $\Phi$  of the vector  $\vec{B}$  through the search coil is defined as the product of the area  $A$  of the coil times the component of  $\vec{B}$  normal to the plane of the search coil. Let the angle between  $\vec{B}$  and the normal to the plane of the search coil be  $\alpha$ . See Fig. 3. The flux through the search coil is then  $\Phi = AB \cos \alpha$ . Faraday's law of induction then gives for the voltage induced in one turn of the search coil as  $-\frac{d\Phi}{dt}$ . If  $N$  is the number of turns in the search coil, the voltage  $V$  induced in the search coil is

$$V = -N \frac{d\Phi}{dt} = -NA \cos \alpha \frac{dB}{dt}. \quad (2)$$

Due to the current induced in the search coil the search coil produces its own magnetic field. The minus sign in Eq.(2) means that this magnetic field produced by the search coil opposes the change in the magnetic field produced by the field coil. This is an application of Lenz's law. (The voltage in the search coil is produced by a non-conservative electric field whose line integral is called an electromotive force, or *emf*. The unit of *emf* is the volt (V), which is work per unit charge.)

Eq.(2) uses SI units. The voltage is expressed in volts, the flux in webers, the magnetic field in tesla, the area in square meters, and the time in seconds.

The current  $I$  through the field coil varies sinusoidally with time so the magnetic field  $\vec{B}$  will also vary sinusoidally with time. The magnitude of the magnetic field  $B(\vec{r}, t)$  produced by the field coil can be written as

$$B = \frac{1}{2} B_{pp} \cos \omega t, \quad (3)$$

where  $B_{pp}$  is the peak to peak value of the magnetic field and  $\omega$  is the angular frequency of the alternating current.  $B_{pp} = B_{pp}(\vec{r})$  depends on position but not on the time. Combining Eq.(2) and Eq.(3), the voltage across the search coil becomes

$$V = \frac{1}{2} \omega N A B_{pp} \cos \alpha \sin \omega t. \quad (4)$$

Let  $V_{pp}$  be the peak to peak value of the measured voltage for  $\alpha = 0$  or  $\pi$ . Then from Eq.(4)

$$V_{pp} = \omega N A B_{pp}. \quad (5)$$

The voltage  $V_{pp}$  will be measured on the oscilloscope. To measure  $V_{pp}$ , rotate the search coil so that maximum amplitude of the signal is seen on the scope screen. Use the most sensitive VOLT/CM scope setting that will keep the pattern wholly on the screen and use the grid on the scope screen to determine  $V_{pp}$ . Recall that the 2 position knobs on the scope, which move the trace up-down and left-right, are useful in moving the trace to an appropriate place on the scope grid. Be sure the variable gain knob of the scope is fully clockwise in the calibrated

position. To determine the direction of the magnetic field, it is more accurate to rotate the coil so that a zero or minimum signal is seen rather than to look for a maximum. As you minimize the signal on the scope you can increase the gain of the scope. Add or subtract 90 deg to the orientation of the search coil to find the direction of the field.

In this experiment the peak to peak current  $I_{pp}$  through the field coil is held constant. For a constant  $I_{pp}$  the magnetic field  $B_{pp}$  is proportional to  $V_{pp}$ , so that we can write  $B_{pp} = KV_{pp}$ , where  $K$  is a constant. To determine  $K$ , measure  $V_{pp}$  at a point on the axis of the coil 15 cm from the center and then use Eq.(1) to calculate the field at that point. The field coil current  $I_{pp}$  is measured with a Fluke multimeter.

## 6 Apparatus

A schematic drawing of the apparatus is shown in Fig. 4. The search coil is free to slide along a Lucite arm, its distance from the center of the field coil being indicated by a scale along the arm. The angular orientation of the search coil relative to the arm is indicated by a protractor. The Lucite arm is free to turn about a point at the center of the field coil. Another protractor indicates the angular orientation of the Lucite arm.

The protractor for the arm reads 90 deg when the arm is along the axis of the field coil. (It would be preferable if the reading was 0 deg.) When you record the data, record what the protractor says and make a note as to what it means. You also have the option of using the top scale or the bottom scale of the protractor. Again, choose one and make a note of your choice. Similar remarks apply to the protractor that measures the angle between the axis of the search coil and the arm. Be sure you know what the angles you write down mean. Is the search coil axis tilted toward the axis or toward the plane of the field coil?

## 7 Preparation and Calibration

1. Connect the Wavetek oscillator to a series combination of the field coil and the Fluke multimeter. On the multimeter, use the COM and 300 mA receptacles. Turn on the multimeter by setting the dial at  $\tilde{A}$ , which stands for ac amperes. Set the oscillator's frequency to 2,000 Hz.
2. Connect the search coil to the vertical input of the oscilloscope, connecting the shield wire to ground. (Note the tab with GND on the double banana plug.)
3. Turn on the scope and oscillator. Set the amplitude knob on the oscillator not quite fully clockwise. This will enable you to adjust the current in the field coil both up and down if necessary. Turn the variable gain knob on the scope fully clockwise to the calibrated position and leave it there for the whole experiment.
4. Measure the average diameter ( $2a$ ) of the field coil and record the number of turns ( $N$ ) of the field coil.
5. Record the current through the coil by using the Fluke. This is an rms (root mean square) value,  $I_{rms}$ . Convert it to a peak to peak value  $I_{pp}$  by multiplying by  $2\sqrt{2}$ . Check the current from time to time, and if you find that it has drifted from the initial value, reset it by using the amplitude knob on the oscillator.

6. Calculate the magnetic field  $B_{pp}$  of the coil for a point on its axis at a distance of 15 cm from its center for a current of  $I_{pp}$ . See Eq.(1).
7. Set the coil at the calibration position (on the coil axis and 15 cm from the center) and measure  $V_{pp}$ , the peak to peak search coil voltage, on the scope. A reminder. Rotate the search coil to obtain the maximum signal. Determine the constant  $K$  in the equation  $B_{pp} = KV_{pp}$ .

## 8 Response of Search Coil

At the calibration position on the axis of the field coil, determine how the voltage  $V_{pp}$  varies as a function of the orientation of the search coil relative to the field. Take readings at about every 10 degrees by rotating the search coil without moving it. You should observe the measured voltage varying from a maximum to a minimum (ideally zero) as the search coil's orientation  $\alpha$  changes by 90 degrees. Plot your data. Does it follow a cosine curve as suggested by Eq.(2). Is the direction of the magnetic field what you expect?

## 9 Determining the Field

The coordinates and geometry are shown in Fig. 5. The center of the field coil is taken as the origin of polar coordinates  $(r, \theta, \phi)$ . The axis of the field coil coincides with the  $z$  axis. The vector  $\vec{r}$ , not shown in Fig. 5, is assumed to lie along the Lucite arm with the tip of  $\vec{r}$  at the center of the search coil. The axis of the search coil is shown and is assumed to be along the direction of the magnetic field  $\vec{B}$ . The angle between the magnetic field and the Lucite arm is  $\beta$ .

It is clear from the symmetry of the field coil that the magnetic field is axially symmetric and does not depend on the angle  $\phi$  shown in Fig. 5. You need only record field readings for various values of the radial ( $r$ ) and angular ( $\theta$ ) coordinates. The angle  $\beta$  should be between -90 deg and +90 deg. When  $\beta = 0$  the field  $\vec{B}$  is parallel to the Lucite arm. Use the convention that for  $\beta$  positive the field tilts toward the plane of the field coil and for  $\beta$  negative the field tilts toward the axis of the field coil.

At a given position of the search coil's center, rotate the search coil to produce the maximum scope signal,  $V_{pp}$ . Record  $V_{pp}$ ,  $r$ , and the arm's protractor readings. Calculate  $B_{pp}$  and  $\theta$ . Now rotate the search coil for minimum signal and record the search coil's protractor reading. Calculate  $\beta$ .

1. Make a few measurements of the field at a given  $r$  and  $\theta$  but for different values of  $\phi$  to convince yourself that the field does not depend on  $\phi$ .
2. To get a "feel" for what the field looks like, make a few measurements of  $\vec{B}_{pp}$  scattered over the complete range of  $r$  and  $\theta$  available to you. Estimate the number of measurements you can make in the time available and how these measurements should be distributed in space. It might be convenient to make your measurements at a fixed number of values of the coordinate  $r$ , but to make more measurements for the larger  $r$ 's. When you have a reasonable plan, make your measurements.
3. Plot your data on a piece of paper as shown in Fig. 6 using appropriate distance scales. This rough sketch shows only a very few points. Your Fig. should have more. Make

your plot as large as feasible. Draw a vector of appropriate magnitude and direction at each measuring position. The tails of the arrows should be at the field points. Make a few copies of your Fig. Then using a colored pencil or pen draw field lines on the copies to indicate the general aspects of the field pattern revealed by your measurements. Submit your best drawing with your report.

4. Comment on the accuracy of your measurements. What factor or factors limit the accuracy the most?
5. How rapidly does the field intensity decrease with distance from the center of the coil (a) along the coil's axis and (b) to the side of the coil?
6. Why do you think it is more accurate to measure the direction of the field by looking for a minimum and not a maximum in  $V_{pp}$ ?
7. How big is the earth's magnetic field, and how does its magnitude compare to the fields produced in this experiment? Why doesn't the earth's field interfere with your measurements? Do you think the technique used in this experiment that separates out the earth's field and the coil's field could be used in other experiments?

## 10 Comment

The voltage and *emf* induced in the search coil by the field coil is described by a parameter called the "mutual" inductance ( $M$ ).  $M$  depends on the shape and number of turns of the two coils, their separation, and their relative orientation. If  $i_F$  is the current in the field coil, the voltage induced in the search coil is  $M \frac{di_F}{dt}$ . If  $i_S$  is the current in the search coil, the voltage induced in the field coil is  $M \frac{di_S}{dt}$ . In this experiment, due to the large input resistance of the oscilloscope, the current in the search coil is so low that the voltage induced in the field coil by the search coil is negligible.

Both the field coil and the search coil have "self" inductance. A coil's self inductance is due to its *own* magnetic field threading through the coil.  $L$  depends on the shape and number of turns of the coil. If  $L$  is the self inductance of the coil and  $i$  is the current through the coil, the voltage induced in the coil is  $L \frac{di}{dt}$ . In this experiment the voltage induced in the search coil by its self inductance is negligible.

## 11 Finishing Up

Please leave the bench as you found it. Thank you.

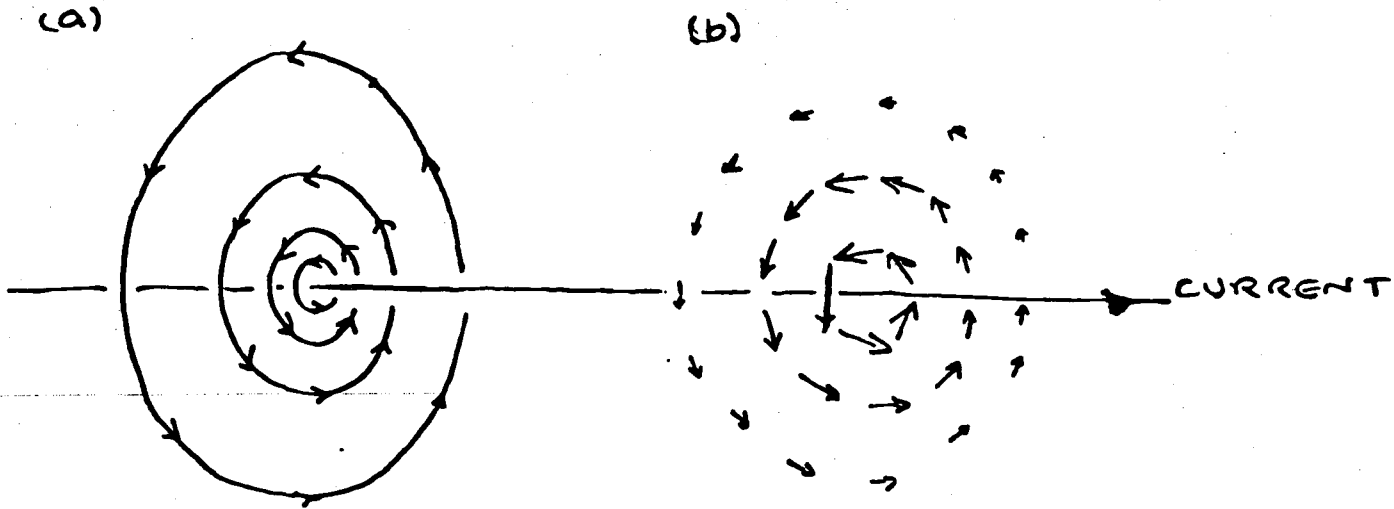


Figure 1. Two ways to describe the magnetic field of a straight current-carrying wire: (a) field lines, whose closeness to each other indicates the field intensity; and (b) vector picture, where the direction and magnitude of each vector indicates the direction and field intensity at that point. At each point the field line has the same direction as the field vector.

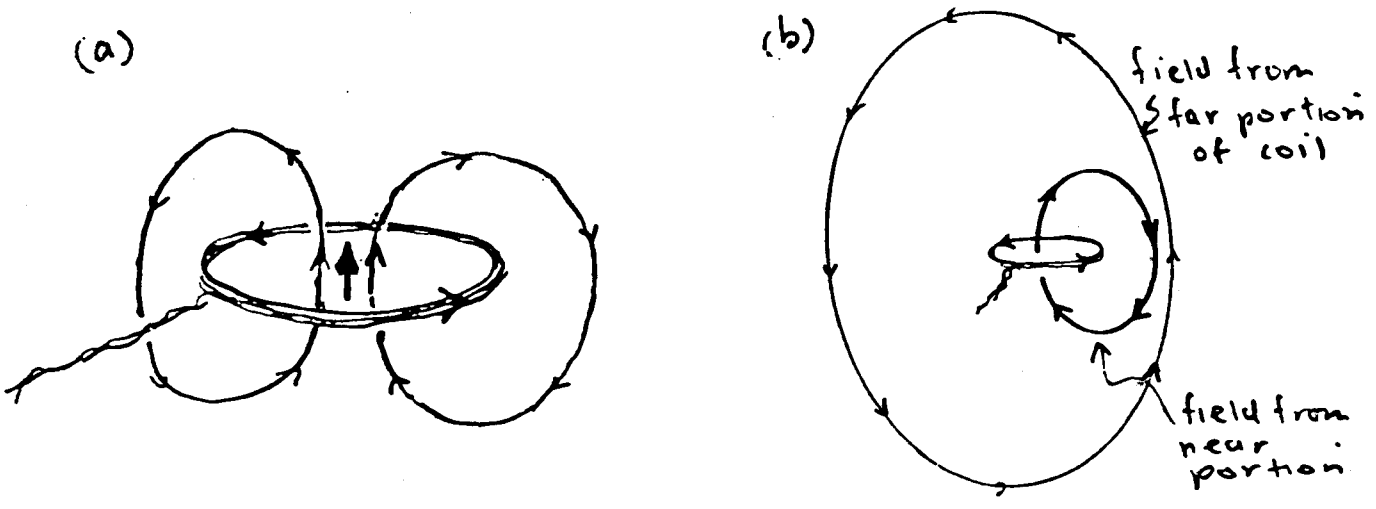


Figure 2. (a) Fields from the far portion and near portion of the coil of wire reinforce at the center of the coil to produce a strong field. (b) Viewed from a greater distance, the field from the far portion of the coil opposes the field from the near portion off to the side of the coil, thus partially cancelling.

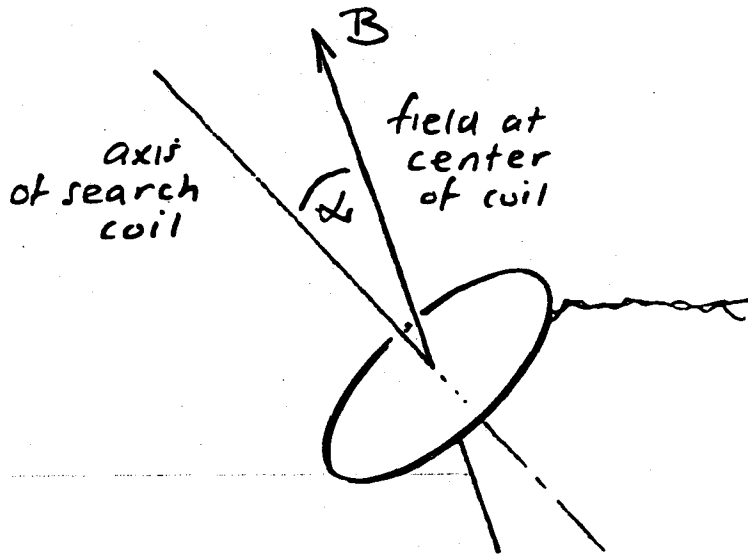


Figure 3. Search coil oriented at an angle  $\alpha$  with respect to the magnetic field  $B$ .

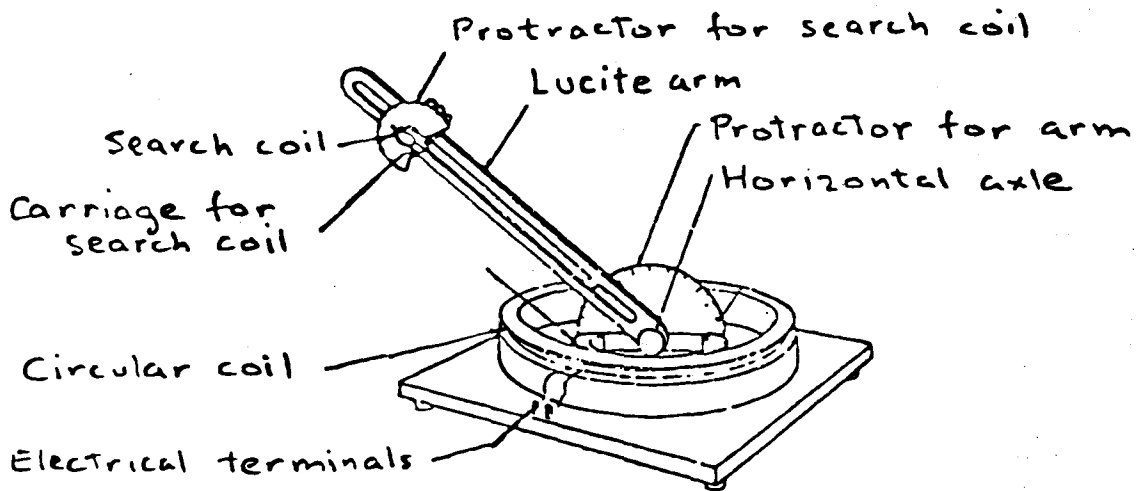


Figure 4. Circular field coil oriented with its axis vertical. At its center is mounted a lucite arm to support the search coil, which can be moved to various positions along the arm and rotated with respect to a protractor.

# Magnetic Field of a Circular Coil

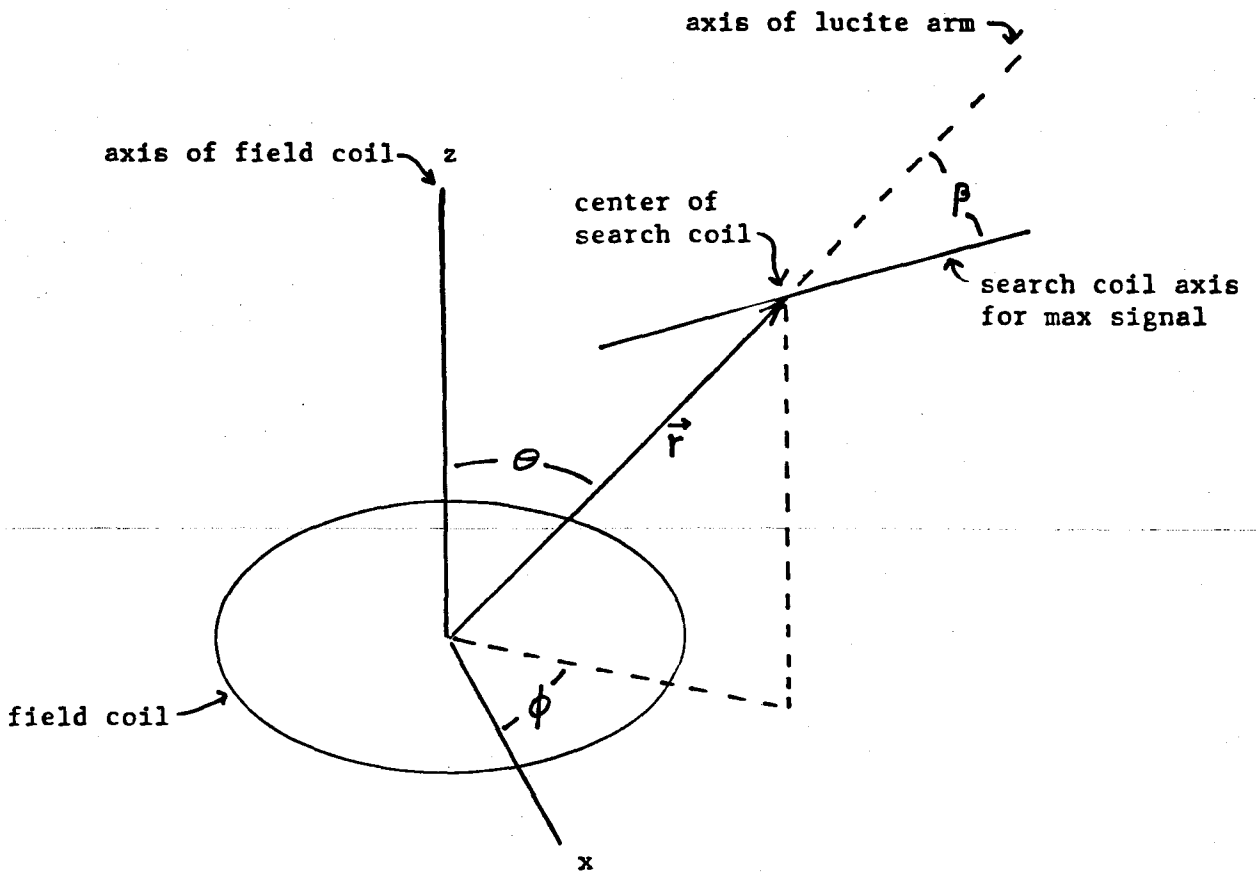


Fig. 5

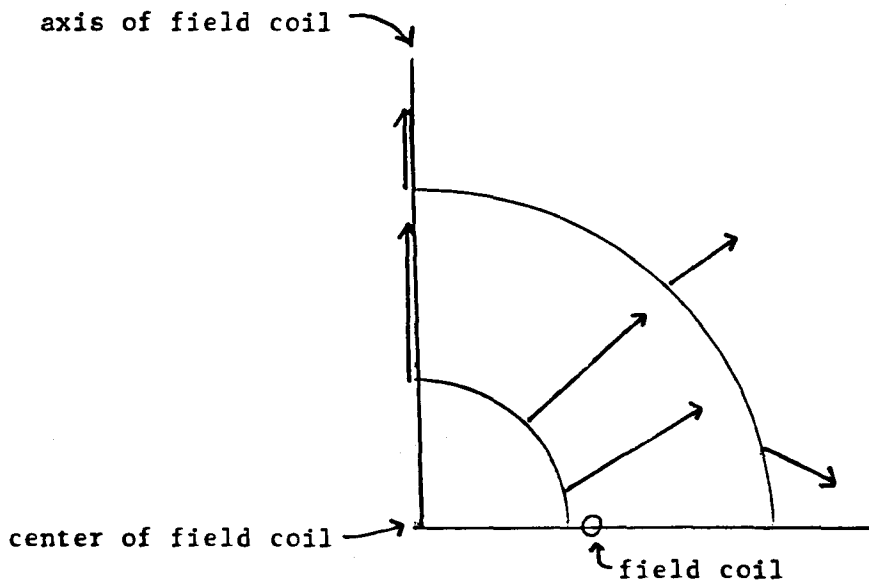


Fig. 6