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Relativistic Electron Momentum

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Goals

The goal of this experiment is to measure the momentum of relativistic electrons. An adjustable magnetic field is used to direct electrons emitted from a radioactive source along a semi-circular path of known radius; the electron momentum spectrum is determined by measuring the number of electrons detected as a function of magnetic field strength.

Background

A simple version of a semicircular electron spectrograph, shown in Figure 1, is used in this experiment to measure the momentum spectrum of electrons emitted by a radioactive source. The applied magnetic field is oriented perpendicular to the plane of the figure, causing the electrons to travel along circular paths whose radii depend on the electron's momentum and the magnetic field strength.

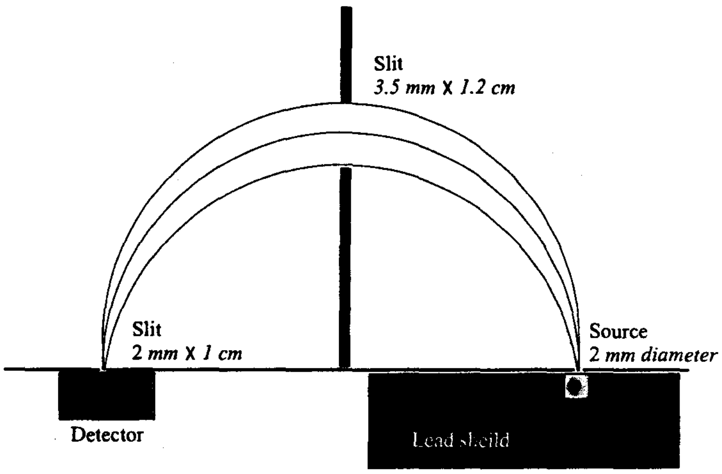

Figure 1: Experimental Configuration

A useful approximation in the relativistic limit is that electrons of momentum p, traveling in a magnetic field B, will follow circular trajectories of radius R, where

$$p = 300BR, \tag{1}$$

with p in units of MeV/c, B in Tesla, and R in meters. However, You will need to arrive at a relativistic relationship between the magnetic field, the electron's momentum, and the radius of the trajectory that is true *far* from the relativistic limit. Be careful to distinguish between the kinetic energy and the total energy for the electron.

Apparatus

Source:

The source in this experiment, ^{207}Bi, emits relativistic electrons, mostly with 1.064 Mev energy by the process of "internal conversion". (The different groups of nearly mono-energetic electrons emitted by sources are referred to as conversion lines.) The conversion line spectrum of ^{207}Bi is relatively simple, consisting of a strong line at 1.064 Mev and a weaker line (by a factor of 6.4) at 0.569 Mev. There are other, considerably weaker lines as well. An energy diagram for ^{207}Bi is shown in the next figure.

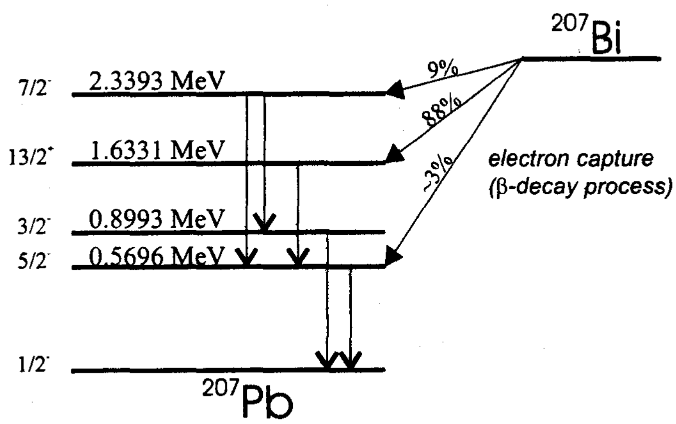

Figure 2: Energy diagram for ^{207}Bi

Detector/Pre-amp:

This experiment makes use of a solid-state detector (Si surface barrier detector) sensitive to charged particles (and high-energy photons.) Each particle detected generates a low-level charge pulse; the pulse is amplified by a charge-sensitive pre-amp. A small reverse bias voltage is applied to the detector (through the pre-amp), in order to reduce noise generated in the detector by thermal electrons. *Note: be sure to turn off the bias voltage whenever you need to unplug the detector.*

Electronics:

Overview

The typical electronics associated with nuclear radiation detectors has the basic function of (a) transforming low level pulses issuing from the detector, through amplification and pulse shaping, into a pulse more suitable for measurement and analysis; (b) analyzing the pulse height distribution; and (c) counting the pulses. The requirements for different detectors are sufficiently similar so that standard "NIM" modules in different combinations can be used.

A block diagram of the arrangement of components is shown in Figure 3:

1. Linear Amplifier
   This unit amplifies and shapes the pulses from the detector. A linear relationship between the input pulse amplitude and the output amplitude exists as long as the gain is set lower than what is needed to produce an output of $\approx$ 12 Volts. The gain should be sufficient to spread out the pulse height distribution over the operating range of the discriminator (0-10 volts).

2 Discriminator
   This unit allows the selection of pulses, from a *positive* pulse height distribution, which exceed a certain height, set by the E_1 control. Thus, for example, low amplitude noise pulses can be eliminated without

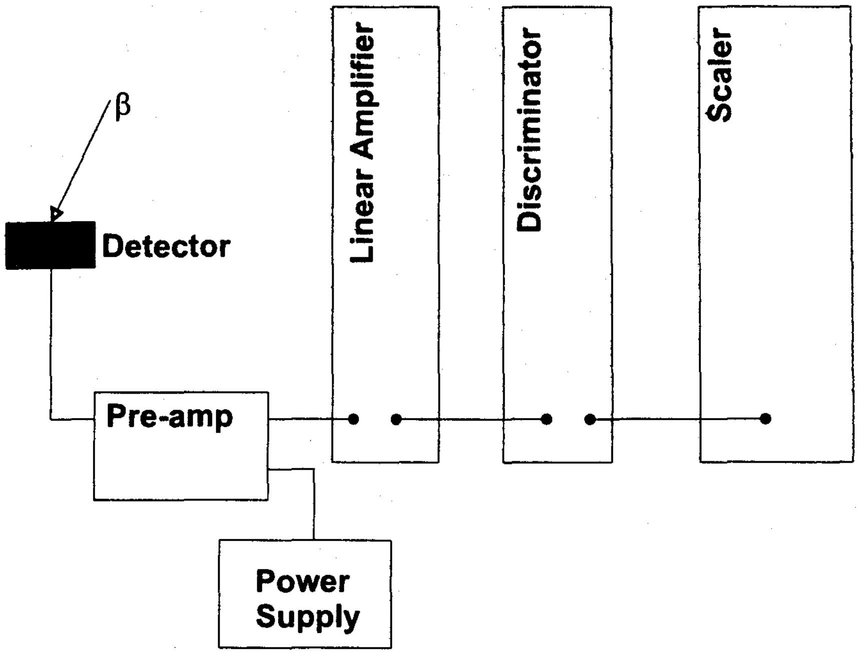

Figure 3: Block diagram of electronic components

affecting the higher amplitude pulses due to β-particles. The output pulses from this unit (E_1 output) all have the same height (square pulses) and are suitable for driving the scaler.

3 Scaler
An electronic counter, capable of counting pulses with a rate as heigh as 10^7 per second.

Adjusting The Electronics:

1. **Check the operation of the detector.**

   Observe the output of the amplifier with an oscilloscope. Place a second Bi source directly in front of the detector during the test to get a high count rate. Note that the polarity switch should be on "inverted" to get positive output pulses. Trigger the oscilloscope *internally* so that the pulse height distribution can be observed. The gain of the amplifier can now be adjusted according to the considerations discussed above.

2. **Check the operation of the discriminator.**

   By triggering the oscilloscope with the output of the discriminator, it is possible to see from the display what part of the pulse height distribution is being selected.

   Set the discriminator lower level such that the count rate is zero when no electrons from the source can reach the detector.

Magnet:

The electro-magnet in this experiment is powered by a low-voltage, high-current DC power supply. The magnetic field strength, which can be measured using a gaussmeter, is adjusted by controlling the current delivered to the magnet. Be very careful using the Hall probe when you measure the magnetic field.

Note: be sure to run the cooling water when operating the magnet.

Use the equation you derived above to estimate the field strength needed to detect 1.064 MeV electrons, and make a "quick" measurement of the count rate around that field strength. Before you record a full conversion line spectrum (i.e., by taking measurements of count rate versus magnetic field), determine *roughly* the necessary integration time, and the size of the increments in magnetic field strength that will be needed to resolve sufficiently (with "reasonable" signal-to-noise ratio) the salient features expected in the momentum spectrum. Steps of roughly 50 gauss in the vicinity of the peak are reasonable.

Miscellaneous:

The Semicircular Focusing Principle Focusing is the key to the proper operation of many devices using charged particles, such as mass spectrometers and particle accelerators. The electron spectrometer used in this experiment relies on the focusing resulting from a simple geometrical property of half-circles, as illustrated in the next figure.

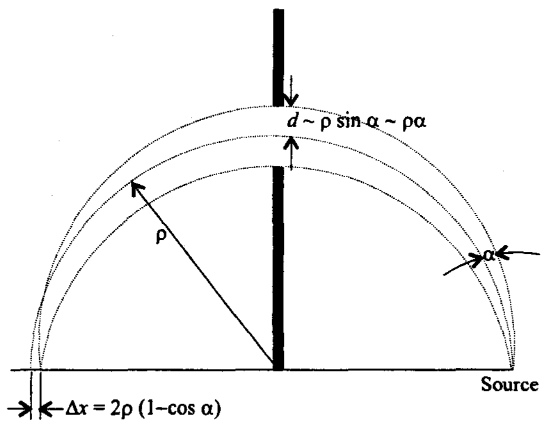

Figure 4: Focusing principle

Poisson Distribution If the expected number of counts in a given counting period is N, then the probability of observing n counts is given by the Poisson distribution:

$$P_N(n) = \frac{N^n e^{-N}}{n!} \tag{2}$$

If the number of counts is not too small, then the expected rate will have a 68% probability (i.e., 1-σ) of falling in the range $n \pm \sqrt{n}$. The percentage error is thus reduced by increasing the number of counts.

The Poisson distribution can be verified for the source used in this experiment by taking many counts with the source directly in front of the detector. This is a useful check on the functioning of the apparatus: a) a distribution much broader than a Poisson distribution may result from an electrical problem such as a bad connection; b) a systematic drift of the count rate, due to some other problem, may be discovered.

Questions

1. Using (exact) relativistic equations, calculate the electron mass (in MeV) from your data, using the known energies of the conversion lines. Also calculate the electron rest mass. Estimate the error in these results due to your estimate of the uncertainty in the magnetic field strength.

2. Calculate the electron mass from your data using non-relativistic equations.

3. Compare the experimental line width (in Gauss, or Tesla) to the calculated line width due to lack of perfect focusing in a 180 deg spectrometer with a finite source and detector size. In particular, *how does the line width depend on the width of the slit d, or on the angle α?*

4. Electron spectrometers are usually evacuated to eliminate electron energy loss and line broadening. What is the energy loss of 1 MeV electrons in traveling from the source to the detector in this experiment? Note that the energy loss dE per distance traveled ds is given by:

$$\frac{dE}{ds} = 4\pi r_0^2 \frac{m_0c^2}{\beta^2} N Z_0 \left\{ \ln \left[\beta \left(\frac{E + m_0c^2}{I} \right) \left(\frac{E}{m_0c^2} \right)^{1/2} \right] - \frac{\beta^2}{2} \right\} \text{(MeV/cm)}, \tag{3}$$

where

$$\begin{aligned} r_0 &= \frac{e^2}{m_0c^2}, \quad 4\pi r_0^2 = 1.0 \times 10^{-24}\text{cm}^2, \quad m_0c^2 = 0.511\text{Mev} \\ \beta &\equiv \frac{v}{c} \text{ for electron velocity } v \\ NZ &= \frac{\text{number of electrons}}{\text{cm}^3} \text{ in the absorber } = 3.88 \times 10^{20}/\text{cm}^3 \text{ for air STP} \\ E &= \text{energy of the electrons} \\ I &= \text{mean ionization and excitation potential of absorber atoms (for air, } I = 86eV). \end{aligned}$$

(This formula for relativistic electrons was obtained by H.A. Bethe, Z. Physik **76**, 293 (1932))

References

[1] K. Siegbahn, "Beta and Gamma-Ray Spectroscopy"

[2] R.D. Evans, "The Atomic Nucleus"

[3] C. Moller, "The Theory of Relativity"