The Muon Lifetime

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Goals

The goal of this experiment is to measure the lifetime of the muon. Positively charged muons incident as cosmic rays are detected and stopped in a slab of aluminum. The positrons that result from the decay of these muons are also detected. The lifetime of the muon is determined from the distribution of delay times between muon and positron detection.

Background

Positively charged muons ($\mu^+$) are created in the upper atmosphere of Earth from the decay of pions (or $\pi$ mesons). Although the lifetime of the $\mu^+$ particle is only around 2 $\mu$sec, due to relativistic time dilation many are present near sea level. (The muon flux at sea level is roughly $10^{-2}$/cm$^2$/ster/sec.) The $\mu^+$ decays according to

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$  \hspace{1cm} (1)

with an exponential decay rate given by

$$N(t) = N_0 e^{-t/\tau},$$  \hspace{1cm} (2)

where $N_0$ is the initial number of particles, $N(t)$ is the number of particles at time $t$, and $\tau$ is the $\mu^+$ lifetime.

In this experiment, you will determine the value of $\tau$ by measuring a distribution of decay events $N(t)$, using coincidence techniques to measure the time delay between the detection of $\mu^+$ and $e^+$ particles. The $\mu^+$ particles will be stopped in a slab of aluminum, where they will decay according to Equation (1). Scintillation detectors will be used to detect both the muons and the positrons; coincidence detection will filter out only the $\mu^+$ decay events. A Time-to-Amplitude Converter (TAC) will then be used to convert the time delay between $\mu^+$ and $e^+$ detections into a voltage that is then sent to a multichannel analyzer (MCA). The multichannel analyzer forms a histogram of the voltages that it receives (voltage on the horizontal axes and frequency of occurrence on the vertical scale).

From the distribution of these voltages, which corresponds to the distribution of time intervals between the $\mu^+$ and $e^+$ detections, you will deduce the muon lifetime.

Apparatus

A schematic diagram of the experimental apparatus is shown in Figure 1. A charged particle passing through a scintillation detector is detected by the small flash of light produced in the plastic sheet; this light flash is converted to a pulse of electrons by the photomultiplier tube. The output from the photomultiplier is fed to the input of an amplifier/discriminator, which is used to filter out the noise inherent in the detector as well as to amplify the incoming charge pulse. The output from the amplifier/discriminator is then fed into the coincidence circuit.

To measure only the muons and positrons associated with the muon decay, coincidence detection - a combination of logic modules and delay lines - is used as follows. We wish to keep track of the timing between two events: (a) the detection of an incoming muon that will subsequently decay in the aluminum slab, and (b) the detection of the positron that results from the decay of the muon we detected in (a). We’ll use these two events to ‘start’ and ‘stop’ a clock; it is the distribution of delay times between the ‘start’ and ‘stop’ events that we want to use to determine the muon lifetime. The muons in (a) are the particles that
Figure 1: Schematic diagram of the apparatus.

are detected by both the A and B detectors (see Figure 1), but not the C detector. The positrons in (b) are
the particles that are detected by either the A or the C detectors, but not by both. In other words:

\[
\text{‘start’ signal: } (A \text{ AND } B) \text{ AND } \overline{C} \tag{3}
\]

\[
\text{‘stop’ signal: } B \text{ XOR } C. \tag{4}
\]

One coincidence circuit that can be used to satisfy equations (3) and (4) is shown in Figure 2. One

thing to think about here is how important it is in this experiment to have \(B \text{ XOR } C\) for the ‘stop’ signal
as opposed to just \(B \text{ OR } C\) (which is slightly easier to implement). What events is the logic with the XOR
eliminating that the logic with just OR includes? What is the rate that these events would occur with just
the OR circuit, and on what factors does this rate depend?

In addition to the logic modules shown, delay lines (i.e., calibrated lengths of cable) must be used between
the various elements in the circuit to ensure that each of the three detectors are in coincidence with each
other - so that a (hypothetical) particle detected “simultaneously” by all three detectors produces pulses that are all synchronized - as well as to ensure that the start and stop signals are timed for proper operation of the Time-to-Amplitude Converter (TAC). The TAC is used to convert the time between the ‘start’ and ‘stop’ signals into a voltage. The multichannel analyzer then produces a histogram of these voltages in the following way. Each time a pulse enters the MCA

**Procedure**

1. **Adjust the detectors**

In order to optimize the signal-to-noise ratio (SNR) of the photomultipliers, you’ll need to adjust the high voltage applied to each of these detectors. As shown schematically in Figure 3, the noise level from a photomultiplier will increase monotonically with voltage, while the signal output will reach a maximum value and then plateau. Thus, the highest SNR is achieved when the voltage is set close to the start of the ‘plateau region’.

![Photomultiplier Response Diagram](image)

**Figure 3:** Photomultiplier response versus applied voltage.

Use an electron source (supplied) to plateau the detectors: with the threshold level on the discriminator set to 25 mV, the minimum, measure the counting rate as a function of voltage applied. Start at a value of -1000 V, and increase the voltage in 50 V increments up to a value of -1500 V.

**WARNING: DO NOT EXCEED -1500 V ON ANY OF THE PHOTOMULTIPLIER TUBES**

From these measurements you can determine where the start of the plateau region occurs, and thus where to set the HV.

Note that in this experiment, you will be detecting events by observing **coincidences** between detector outputs. Even if there is a large rate of counts due to noise from each detector, if you are looking at
coincidences between two detectors (due to, say a muon going through both detectors), the accidental coincidence rate between the output of the two detectors (due to noise) will be much less than the singles rate due to noise from each detector. You can take advantage of this fact to find the optimum signal to noise ratio for coincidence events.

2. Adjust the coincidence circuit

You'll first need to verify the operation of all the logic units, and insert delay lines as appropriate to ensure that all three detectors are in coincidence. Using the dual-beam oscilloscope, observe the coincident arrival of test pulses in detectors A and B. Measure the A+B coincidence rate using one of the logic units and a scaler/timer. Then ‘and’ the output of detector C with the A and B signals, and measure the three-fold coincidence rate; you'll need to add delay lines (after the discriminator outputs) in order to maximize this rate. A good rule of thumb is that one foot of RG59 cable delays the pulse by about one nanosecond.

a. Muon detection: To detect muon decay events, wire one of the logic modules as per equation (3). You'll need to connect the output from the C detector threshold circuit to the veto input of the logic module. For maximum efficiency, adjust the width of the veto input signal (C) so that it arrives 10 ns earlier, and lasts 10 ns longer, than the A and B signals.

b. Positron detection circuit: Now connect another logic module according to equation (4). Pay close attention to the lengths of all the cables you use, in order to avoid a systematic error in the lifetime measurement. Remember that each NIM module introduces approximately 10 nanoseconds of delay.

3. Calibrate the TAC and A/D

The output of the Time-to-Amplitude Converter (TAC) is an electrical pulse whose amplitude (voltage) is proportional to the length of time between the ‘start’ and ‘stop’ input signals. Set the TAC so that a time interval of 16 µs corresponds to a full scale output of ??? volts. (Thus, any pulse pairs separated by more than 16 µs will be rejected.) Use the oscilloscope and the Systron Donner 100A pulse generator (which can output two pulses separated by a known delay time) to verify and calibrate the operation of the TAC and the MCA. (Be sure to use negative-going test pulses as inputs to the TAC.) Note that the ‘start’ pulse of
the TAC must be delayed relative to the ‘stop’ pulse, so that the TAC is not stopped immediately. This can easily be done by using different length cables to the start and stop inputs of the TAC.

Questions

You will need to acquire data for at least 24 hours in order to accumulate enough counts to determine the muon lifetime with ‘reasonable’ precision. In your report, be sure to include an estimate of the experimental uncertainty in the determination of the value for the lifetime $\tau$, as well as a discussion of the source of errors, and how these errors might be minimized or eliminated.

Pre-Lab questions

This lab involves counting statistics, which involves the Poisson distribution. Read about the Poisson distribution and answer the following questions:

1. Consider an experiment that counts the number events in a given amount of time (where an “event” can be, for example, the detection of a particle). Suppose that after many repetitions of the experiment, you find that the average number of events in the given amount of time is $\nu$. What is the standard deviation of the results in your experiment?

2. Suppose the count rate (events/sec) from two detectors in the muon experiment is $R_1$ and $R_2$. What is the rate of accidental coincidences if the width of the coincidence window (measured in sec) is $W$?

3. How can the result of Question 2 allow you to improve your count rate without significantly increasing the noise in your experiment?

4. Assume events occur at a particular rate $R$ (events/s). If you start looking for events at some time (say $t = 0$), what is the probability that the first event you detect occurs between times $T$ and $T + \Delta T$. Assume that $\Delta t$ is a small interval of time so that $R\Delta T \ll 1$. Hint: First calculate the probability that zero events occur in time interval $T$.

In your first attempt you will fit the data to an exponential in order to measure $\tau$. You will note that the fit is not especially good. The primary reason for this is that the cosmic ray muon flux consists of both positive and negative muons. Whereas the positive nuons can only decay, the negative muons can disappear via decay and via capture on a proton. The latter process depends on the proton number to the fourth power. For aluminum the capture rate is about 1.5 times the decay rate.

Therefore, you should try to fit the data with two exponentials. See the book by Rossi, “Cosmic Ray Particles” for data on positive and negative muon fluxes at sea level.

Compare your measured total flux rate per square cm with Rossi’s. To what do you attribute any discrepancies?

References

