

## The Hydrogen-Deuterium Isotope Shift

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### Goals

*The main goal of this experiment is measure the spectra of hydrogen and deuterium using a grating spectrometer, and from the measured isotope shift determine the proton-deuteron mass difference*

### Background

The spectrum of the one-electron atom has played a very important role in the development of the quantum theory of matter and radiation. The gross features of the line spectrum which are observed in this experiment were first explained by Bohr in 1913, using a theory which was a mixture of classical and quantum physics. A more satisfactory theory which explains the gross spectral features, and which appeared to explain all the small splittings of the lines as well, was developed in 1928 by Dirac. A discrepancy between this theory (i.e., the Dirac equation) and experiment was discovered by Lamb (the Lamb shift). The explanation of the Lamb shift by quantum electrodynamics, which treats the atom and the electromagnetic field as interacting quantum mechanical systems, is one of the greatest triumphs of 20th century theoretical physics.

In this experiment it will be possible to measure the wavelength of some of the visible lines of hydrogen and deuterium with an accuracy of better than 1/10%. It will not be possible to observe the small splittings of the energy levels (fine structure and hyperfine structure) because of line broadening effects and the finite resolution of the grating spectrometer. It will be possible, however, to observe the small difference - predicted correctly by Bohr's theory - between the spectra of hydrogen and deuterium (the isotope shift.) This difference arises from the different reduced masses of the nuclear-electron system in the two cases and can be used to determine the proton-deuteron mass difference.

### Apparatus

#### The Reflection Grating

A diffraction grating is an optical element used to disperse light according to wavelength. The prism, another dispersive optical element, exploits the variation with wavelength in the refractive index of glass (or whatever material the prism is made of), so that different wavelengths are refracted by different amounts. In contrast, a grating disperses light as a result of interference. As such, gratings can be used over a much wider range of wavelengths (i.e., where prisms can no longer be used), and are, in fact, currently used for spectroscopy from the visible to the X-ray region of the spectrum.

Early reflection gratings were made by scratching the surface of a mirror with a diamond, leaving shiny strips separated by irregular dull grooves. In contrast, the grating used in this experiment is *blazed*, i.e., it was ruled by a shaped diamond, resulting in a triangular groove profile, as indicated in Figure 1. (These days, gratings are also made holographically.) The main reason for using a blazed grating is to reduce the amount of light diffracted into zero order, i.e., the specular direction, and thereby increase the amount of light diffracted into the useful non-zero orders.

(The computation of diffraction efficiency, i.e., the determination of the amount of light diffracted into different diffraction orders, is a highly nontrivial problem, dependent on not just the incidence angles and the wavelength, but on a detailed description of the microscopic shape of the grooves and the optical properties of the reflective coating applied to the surface of the grating. Even though the diffraction efficiency of a grating is determined by the classical laws of electricity and magnetism, which have been around since the

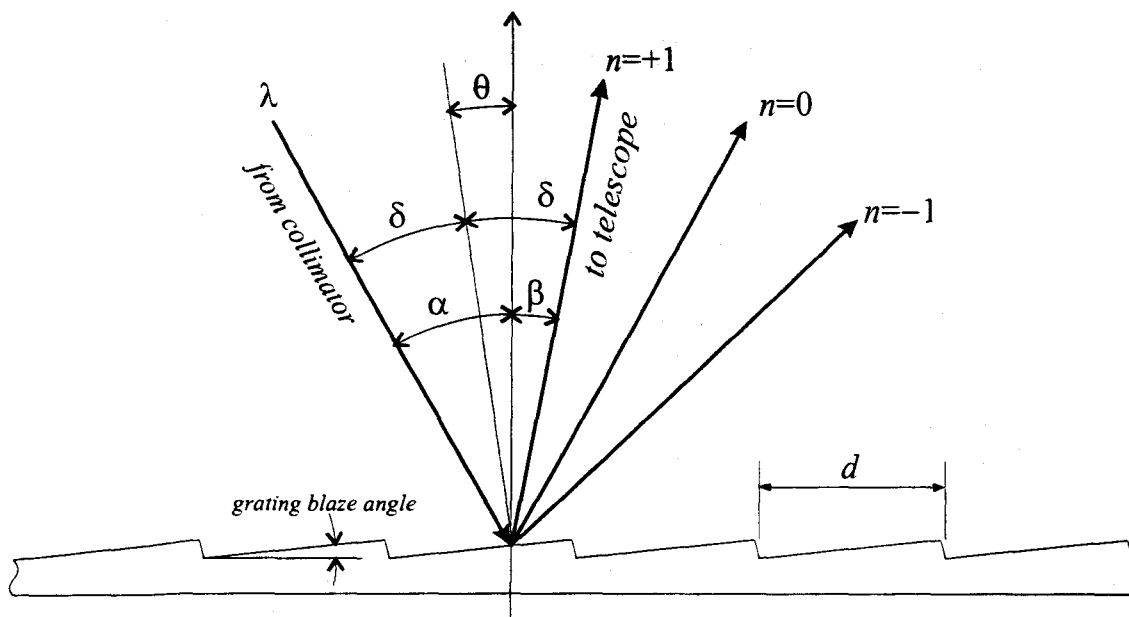


Figure 1: Diffraction from a reflection grating.

1800's, this particular problem has not yet been solved analytically, and so numerical methods - which are approximate at best, in this case - must be used.)

The grating equation gives the positions of the principle maxima in the dispersed spectrum, i.e., the positions of constructive interference, and is given by

$$n\lambda = d(\sin \alpha + \sin \beta), \quad (1)$$

where (see Fig. 1)  $\alpha$  and  $\beta$  are the incidence and exitance angles, respectively,  $d$  is the grating period (i.e., the inverse of the ruling density, in grooves/mm, for example),  $n$  is an integer called the diffraction order, and  $\lambda$  is the wavelength of light. Note that  $\alpha$  and  $\beta$  must be taken to have opposite signs if the incident and diffracted rays are on opposite sides of the normal to the grating. [e.g., consider the zero-order ( $n=0$ ) condition.]

One common method for measuring wavelengths is to keep fixed the angle between the collimator and the telescope,  $2\delta$ , and to rotate the grating. Defining  $\theta$  as the angle the grating is rotated from the position which centers the undispersed slit image (zero order, identifiable if white light is used), then for constructive interference

$$n\lambda = 2d \sin \theta \cos \delta. \quad (2)$$

For the recommended method of measuring the separation of closely spaced spectral lines, refer to questions 3, 4, and 5 in the next section.

### Advance Preparation

As you will see below [Equation (4)], the proton-deuteron mass difference will be determined from the difference in the measured values of the H and D spectral lines,  $(\lambda_H - \lambda_D) \equiv \Delta\lambda/\lambda_D \ll 1$ . The following questions should lead you to a configuration appropriate for the most accurate determination of  $\Delta\lambda$ .

Assume for the moment that we wish to use a reflection grating spectrometer (i.e., as in Figure 1) to measure the value of  $\Delta\lambda$  of the sodium D lines (589.0 nm and 589.95 nm.) using a grating with a ruling density of 600 grooves/mm. Assume that for mechanical reasons the telescope and collimator cannot be less than  $30^\circ$  apart.

1. What is the maximum order that can be observed? At what angle  $\theta$ ?

- Suppose the collimator axis is further from the grating normal than the telescope axis, as in Figure 1. (a) Find an expression for  $d\theta/d\lambda$ , when the angle  $2\delta$  between the telescope and the collimator is kept fixed. What is the angular separation in 4<sup>th</sup> order (in degrees of rotation of the grating) between the sodium D lines? (b) Find an expression for the dispersion,  $d\beta/d\lambda$ , when the incidence angle  $\alpha$  is kept fixed. What is the angular separation in 4<sup>th</sup> order between the sodium D lines for this configuration?
- Now suppose that the collimator and telescope are reversed from the configuration in question 2. (a) Is the maximum accessible order affected? (b) What is  $d\theta/d\lambda$  (with fixed  $2\delta$ ), and what is the angular separation in 4<sup>th</sup> order between the sodium D lines? (c) What is  $d\beta/d\lambda$  (with fixed  $\alpha$ ), and what is the angular separation in 4<sup>th</sup> order between the sodium D lines for this configuration?
- Even if the values of  $\Delta\beta$  calculated in (2) and (3) are large compared to the smallest  $\Delta\beta$  that can be measured with a given telescope, cross hair, and eyeball arrangement, the precise measurement of  $\Delta\beta$  may still be impossible (or at least difficult) if the lines produced by the grating are too wide. For a perfectly ruled grating, the half width of the principal maximum is given by  $\Delta\phi = \frac{\lambda}{W \cos \beta}$ , where  $W$  is the width of the grating. ( $W = 25.4$  mm in this experiment.) (a) Compare  $\Delta\beta$  and  $\Delta\phi$  for the arrangement described in (3). (b) What is the minimum wavelength difference  $\Delta\lambda_{\min}$  that can be resolved, i.e. when  $\Delta\beta = \Delta\phi$ ? Find an expression for the resolving power  $\lambda/\lambda_{\min}$  for a perfectly ruled grating.
- Use the numbers computed above to show why it is better, in the observation of the H-D separation, for example, to observe spectra in a configuration which places the telescope, rather than the collimator, at the largest angle with respect to the grating normal, and why it is better to use a small  $\delta$ .
- Denoting by  $\lambda_M$  the wavelength of an emission line from an atom with nuclear mass  $M$ , and  $\lambda_\infty$  the wavelength we would calculate assuming an infinitely massive nucleus, use the relationship

$$\lambda_M = \lambda_\infty \left( 1 + \frac{m_e}{M} \right) \quad (3)$$

to show that the difference in the H and D nuclear masses,  $\Delta M$ , is given by

$$M_D - M_H \equiv \Delta M = \frac{M_H + m_e}{\frac{m_e \lambda_D}{M_H \Delta \lambda} - 1} \quad (4)$$

where  $\Delta\lambda = \lambda_H - \lambda_D$ , and  $m_e$  is the mass of the electron.

## Alligning the Spectrometer

The spectrometer used in this experiment is an optical instrument consisting of (see Figure 2):

- a **collimator**, which is a telescope-like device used to convert the divergent rays from the source into a beam of parallel rays which are then incident on the grating;
- an adjustable **mounting table**, used to support the dispersive optical device being utilized, in our case, a reflection grating;
- a **telescope**, which refocuses the (collimated) diffracted light from the grating, in order to observe the features of the spectrum under study.

Any high precision optical instrument, such as a spectrometer, is useful only to the extent that it is carefully aligned; alignment is the key to precision. As one might expect, there is at least one alignment procedure needed for each of the three spectrometer components listed above.

### Telescope Adjustment

- First, the eyepiece must be removed and adjusted so that the cross hairs appear as sharp as possible to the relaxed eye. Relax your eye by focusing it on a distant object, and then look through the eyepiece and adjust for sharp cross hairs. Once the parallax between the cross hairs and the distant object is eliminated, the eyepiece will be in focus and should not be adjusted any further.

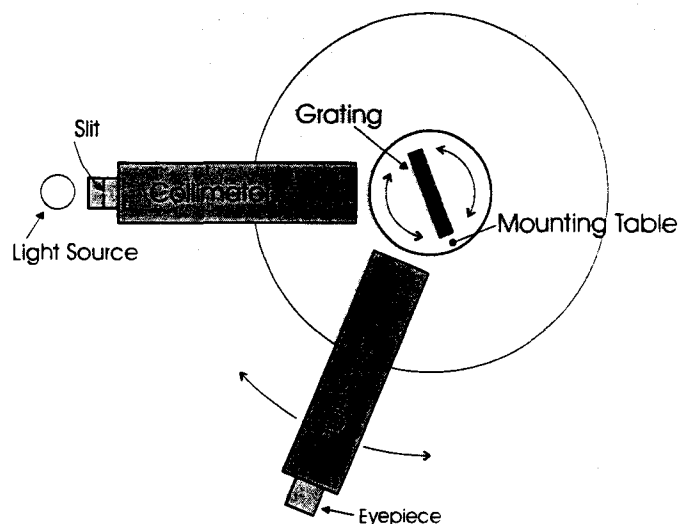


Figure 2: The Spectrometer.

2. Next you will focus the telescope, making use of the so-called *autocollimation* technique:
  - a. First, set a 2-sided mirror on the mounting table, as parallel to the axis of rotation as possible (within reason). Note that the mirror does not need to be coincident with the axis of rotation of the table. See Figure 3.

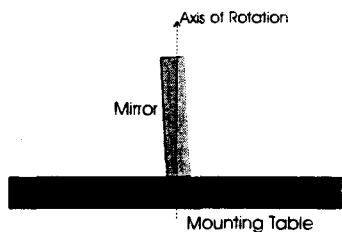


Figure 3: Mounting the mirror.

- b. Next, direct light into the side aperture of the eyepiece. (See Figure 4.) The light will illuminate the cross hairs, and due to the beamsplitter, will traverse the telescope tube. Rotate the mounting table so that the mirror reflects the light back down the telescope tube.
  - c. Looking into the eyepiece, you should see the illuminated cross hairs (the object), as well as the image of the cross hairs reflected from the mirror. Make the reflected image of the cross hairs as sharp as possible by adjusting the rack and pinion (*don't adjust the eyepiece!*). Once the parallax between the image and object is removed, the telescope will be focused.
3. Finally, you will set the optical axis of the telescope perpendicular to the axis of rotation of the table. When you focus the telescope in the previous step using the autocollimation technique, it will be apparent that the reflected image of the cross hairs is displaced in the field of view relative to the actual cross hairs. This is due to the fact that the two-sided mirror is not parallel to the axis of rotation of the mounting table. Rotate the table through  $180^\circ$ , and the image will appear again, but in a different area of the field of view. Adjust the leveling screws of the table until rotation of the table through  $180^\circ$  does not change the image position in the field of view. When this is accomplished, the telescope inclination may be adjusted so that the image and object coincide. The axis of the telescope will now be perpendicular to the axis of rotation, and the mirror will be parallel to the latter.

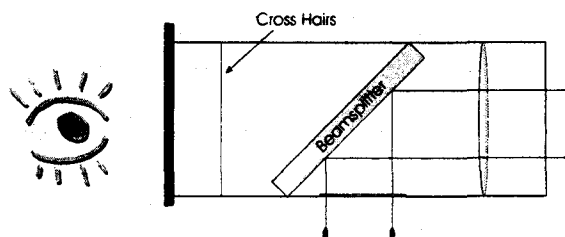


Figure 4: The eyepiece.

### Collimator Adjustment

1. To focus the collimator, rotate the telescope until its optical axis is roughly collinear with the optical axis of the collimator. Illuminate the entrance slit of the collimator with light from an ordinary bulb, and adjust the distance of the slit from the objective lens until the image of the slit as seen through the telescope is as sharp as possible, and parallax between image and cross hairs is minimized.
2. The inclination of the collimator should then be adjusted, so that the cross hairs appear halfway up the image of the slit. When this is accomplished, the axes of the collimator and the telescope are collinear, and perpendicular to the axis of rotation of the table.

### Grating Adjustment

The final alignment step is to align the grating so that the face of the grating is perpendicular to the axis of the telescope, and so the grooves are parallel to the rotation axis of the spectrometer.

1. Mount the grating on the mounting table. (Don't touch the optical surface!) The face of the grating should bisect approximately the mounting table and be perpendicular to the line joining two of the leveling screws.
2. Adjust one of the leveling screws on the mounting table until the face of the grating is perpendicular to the axis of the telescope. Use the grating as a mirror, and employ the same procedure used to align the mirror described above.
3. Finally, orient the grating so that the grooves are parallel to the rotation axis of the spectrometer. To do this, observe a continuous spectrum with the slit so narrow that the spectrum shows dark horizontal bands, and adjust the appropriate mounting table leveling screws so that these bands stay at constant height as the grating is rotated about the axis (from one end of the spectrum to the other). (These bands are caused by dust on the jaws of the slit.)

## Questions

Record angles for as many hydrogen lines as you can see. The tube has mixed hydrogen and deuterium, so that if these are unresolved, you will be observing an average angle. Record angles for some of the lines in several orders on both sides of the grating normal. Calculate the wavelengths assuming that the grating has exactly 600 lines/mm. A systematic deviation from accepted values should be discussed. (Note that the exact grating ruling density normally doesn't concern the manufacturer, as it is easily calibrated.)

In your report, calculate the wavelengths expected from the Bohr formula. How does an error in  $\theta$  affect an error in  $\lambda$ ? Estimate how well you can expect to measure  $\lambda$  knowing the angular measurement error.

Are all of the lines you see hydrogen or deuterium lines? You may wish to replace the hydrogen-deuterium lamp with one containing only hydrogen, to verify that the observed doubling of the lines is indeed due to the presence of two isotopes.

The wavelength differences between the lines of hydrogen and deuterium can be read in high order from the spectrometer circle. A higher precision can be obtained by using the micrometer eyepiece. Changing eyepieces will necessitate refocusing the telescope, but it is permissible to refocus in on the lines, if the collimator focus is undisturbed. Calibrate the micrometer eyepiece in angle units against the spectrometer circle by moving the telescope about a degree, using a sharp spectral line for reference. This measurement is fairly difficult and must be done carefully if you want to get a meaningful measure of  $\delta\lambda$  (and therefore of the H-D nuclear mass difference.) You can use the splitting of the sodium D lines to check your method. (A sodium arc source is available.)

Given that the mass of the proton is 1836.12 times the mass of the electron, compute the mass of the neutron or rather, the difference between the mass of the deuteron and the proton. The neutron mass can be obtained from  $\Delta M$  by observing that the binding energy of the deuteron is 2.23 MeV.

Optional: Compare the relatively simple spectrum of hydrogen to that of molecules and other atoms (other discharge tubes are available).

## References

- [1] Jenkins and White, *Fundamentals of Optics*, Chapter 17 and Section 12.10.
- [2] Leighton, *Principles of Modern Physics*, Chapters 2 and 5.