N-body simulation of DGP model

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Introduction

DGP (Dvali-Gabadadze-Poratti) model is an extra dimension model with one extra dimension. The gravity is modified in the infrared. The scale that the transition of gravity from 3D to 4D is characterized by the cross-over scale $r_{\rm c}$. DGP model allows self-accelerating solution and so it can explain the cosmic acceleration without invoking the dark energy. In large scacle, gravity is weaker than general relativity (GR). This model can be approximated as spin two gravity plus some extra scalar degrees of freedom. In small scales, thanks to the peculiar derivative self-interations, the extra scalar degrees of freedom become strongly coupled and frozen. Thus GR is recovered via the Vainshtein effect.

Most of the studies so far are limited to the linear theory, and we would like to study the grow of perturbations in the nonlinear regime. This is important for understanding the theory itself and to differentiate modified gravity from dark energy model. The Vainshtein mechanism is important in this kind of model and is expected to kick in in the nonlinear regime. In the nonlinear regime, numerical simulations are important. Thus we carry out N-body simulations to study the large scale structure formation for the DGP model.

Solving the Field Equations

N-body simulation for DGP gravity is largely similar to the standard gravity except the equations of motion are replaced by the DGP counterparts. The background modified Friedmann equation reads

$$H^2 = \frac{H}{r_{\rm c}} + \frac{8\pi G}{3}\rho_{\rm m}. \qquad (1)$$

The Poisson equation is replaced by a pair of equations (Scoccimarro 2009)

$$\bar\nabla^2\phi + \frac{1}{2\eta}\bar\nabla^2 C = \frac{3}{2}\frac{\eta-1}{\eta}\delta, \qquad (2)$$

$$(\bar\nabla^2 C)^2 + \alpha\bar\nabla^2 C - (\bar\nabla_{ij}C)^2 = \frac{3(\eta-1)}{\eta}\delta, \qquad (3)$$

where $\eta \equiv r_{\rm c}H$ and $\alpha \equiv= \frac{3(2\eta^2-2\eta+1)}{\eta(2\eta-1)}$, and $\bar\nabla$ denotes $\frac{\nabla}{aH}$. At each time step in the simulation, we need to solve Eq. 2 and 3 given δ. Eq. 2 can be easily solved when C is obtained from Eq. 3. The nonlinear derivative operators make Eq. 3 hard to solve.

We solve Eq. 3 as follows:

1. Linearize Eq. 3 in Fourier space to obtain the trial solution $C(\mathbf{k})$.
2. From $C(\mathbf{k})$, we get $\bar\nabla_{ij}C(\mathbf{k}) = -\bar k_i \bar k_j C(\mathbf{k})$.
3. Inverse FFT to get $\bar\nabla_{ij}C(\mathbf{x})$ and $\bar\nabla^2 C$.
4. Treating Eq. 3 as a quadratic equation in $\bar\nabla^2 C$ and the rest as source term, we apply the quadratic formula to solve for $\bar\nabla^2 C$.
5. Go back to the second step with the new $\bar\nabla^2 C$ to continue the iteration.

Except for the first time step, in step 1 the old solution is used. It turns out that we may not have real solutions in solving the quadratic equation if the trial solution is not good enough. However, the problem is solved if we split the nonlinear term $\bar\nabla^2 C$ into two part $w\bar\nabla^2 C$ and $(1-w)\bar\nabla^2 C$, where $(1-w)\bar\nabla^2 C$ is also treated as a source term. We find that $w = \frac{1}{3}$, which corresponds to spherical approximation, is good in the sense that it can solve the negative discriminant problem and permit quick convergence.

Since this method require alternate use of FFT and relaxation, we call it *FFT-relaxation method*.

Numerical Simulation Results

Due to limited resouces and the fact that FFT-relaxation takes, relative to GR simulation, more than 10 times more CPU time and memory we can only simulate 256^3 particles. The code we use is of the particle-mesh type.

On the right hand side, the first and second rows are the power spectrum output at $z = 1$ and 0 respectively. The labels "nlDGP", "lDGP" and "GRH" represents the full nonlinear DGP model, the linear DGP model, and the model with GR Poisson equation but with DGP Friedmann equation respectively. For clarity, we show on the right panels the ratio of the power spectrum. In the linear regime, nlDGP reduces to lDGP, while in large k limit, nlDHP approaches GRH because of the Vainshtein effect.

In the third row we show the mass functions for various models at $z = 0$. The left panels we divid the numerical results by the Sheth-Tormen (ST) function, while in the right panel by the GR mass function. In the small mass region, the differences among the models are small. Large deviations appear in the relatively large mass $m \sim 10^{14} M_\odot h^{-1}$ range. In increasing order of σ_8, the order is lDGP, nlDGP, GRH, and GR. The ordering of the models in the figure agrees with the expectation from the ordering of σ_8.

The solid lines in these plots are the perturbative calculations in Scoccimarro 2009.