


N -body simulation of DGP model

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¹K. C. Chan and R. Scoccimarro, arXiv: 0906.4548 

Some properties of the DGP model

- DGP model is an extra-dimension model, with one extra dimension.
- Gravity is modified in the infrared. It permits a self-accelerating solution without dark energy.
- Gravity is weaker than GR in large scales.
- Vainshtein effect. There are scalar degrees of freedom in this model. But in the small scales the scalar field becomes strongly coupled and frozen. Thus GR is recovered in the small scales.
- Although the self-accelerating DGP has been found to be observationally and theoretically unfavourable, DGP is perhaps the simplest toy model of infrared modification of gravity, we want to understand it well before going to more complicated ones.

Objective

- Most of the studies are limited to linear theory.
- To understand the theory itself and to differentiate the dark energy from modified gravity, we want to extend the study to the nonlinear regime.
- In the nonlinear regime, numerical simulations are vital.
- We carry out N -body simulation for the nonlinear DGP model.

Equations of Motion

- The Friedmann equation (for the accelerating branch) is modified as

$$H^2 = \frac{H}{r_c} + \frac{8\pi G}{3}\rho_m,$$

where r_c is the cross-over scale.

- The perturbation equations are (Scoccimarro 2009)

$$\begin{aligned}\bar{\nabla}^2\phi + \frac{1}{2\eta}\bar{\nabla}^2 C &= \frac{3}{2}\frac{\eta-1}{\eta}\delta, \\ (\bar{\nabla}^2 C)^2 + \alpha\bar{\nabla}^2 C - (\bar{\nabla}_{ij}C)^2 &= \frac{3(\eta-1)}{\eta}\delta,\end{aligned}$$

where $\eta \equiv r_c H$, $\alpha = \frac{3(2\eta^2 - 2\eta + 1)}{\eta(2\eta - 1)}$ and $\bar{\nabla}$ denotes $\frac{\nabla}{aH}$.

$$\text{Solving } (\bar{\nabla}^2 C)^2 + \alpha \bar{\nabla}^2 C - (\bar{\nabla}_{ij} C)^2 = \frac{3(\eta-1)}{\eta} \delta$$

- Linearize the equation in Fourier space to obtain the trial solution $C(\mathbf{k})$.

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- Inverse FFT to get $\bar{\nabla}^2 C(\mathbf{x})$ and $\bar{\nabla}_{ij} C(\mathbf{x})$.

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- Treating the full equation as a quadratic equation in $\bar{\nabla}^2 C$ and the rest as source term, we apply the quadratic formula to solve for $\bar{\nabla}^2 C$.

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- Go back to the second step with the new $\bar{\nabla}^2 C$ to continue the iteration until some tolerance is met.

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- Go back to the second step with the new $\bar{\nabla}^2 C$ to continue the iteration until some tolerance is met.
- Except for the first time step, in step 1, the old solutions are used as the trial solution.

Splitting the nonlinear term

- It turns out that we may not have real solutions in solving the quadratic equation if the trial solution is not good enough.
- If we split the nonlinear term $(\bar{\nabla}^2 C)^2$ into two parts $w(\bar{\nabla}^2 C)^2$ and $(1-w)(\bar{\nabla}^2 C)^2$, where $(1-w)\bar{\nabla}^2 C$ is also treated as a source term:

$$(1-w)(\bar{\nabla}^2 C)^2 + \alpha \bar{\nabla}^2 C - \left[(\bar{\nabla}_{ij} C)^2 - w(\bar{\nabla}^2 C)^2 + \frac{3(\eta-1)}{\eta} \delta \right] = 0.$$

- We find that $w = \frac{1}{3}$, which corresponds to spherical approximation, is good in the sense that it can solve the negative discriminant problem and permit quick convergence.

Since the method requires alternate use of FFT and relaxation, we call it *FFT-relaxation*.

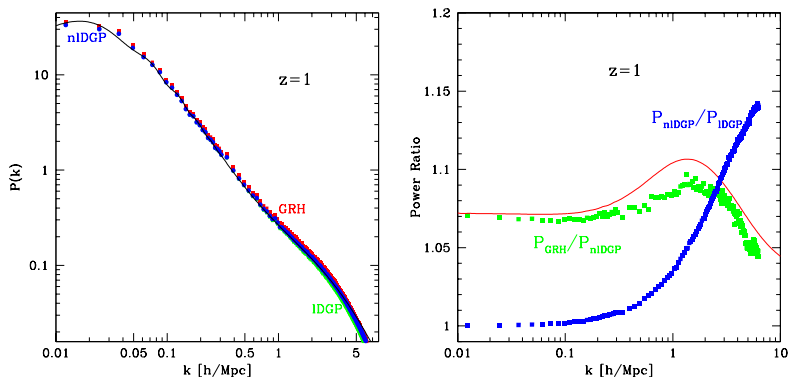
Power spectrum at $z = 1$ 

Figure: Dark matter power spectra from the nonlinear DGP model (nIDGP), linear DGP (IDGP), and GR perturbations with the same expansion history (GRH) at $z = 1$. The solid lines are perturbative calculations (Scoccimarro 2009).

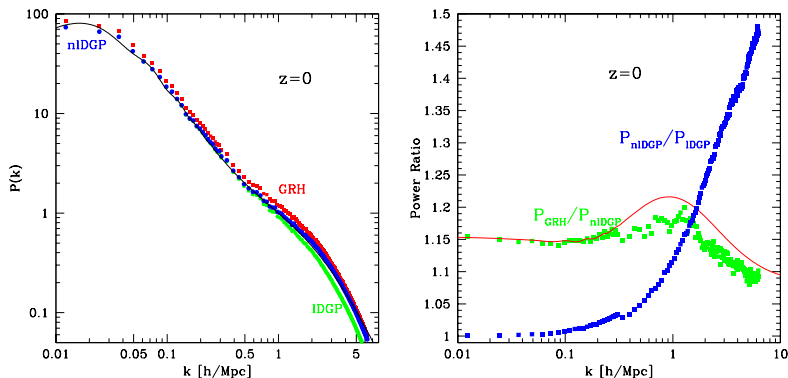
Power spectrum at $z = 0$ 

Figure: Dark matter power spectra from the nonlinear DGP model (nIDGP), linear DGP (IDGP), and GR perturbations with the same expansion history (GRH) at $z = 0$. The solid lines are perturbative calculations (Scoccimarro 2009).

Mass function at $z = 0$

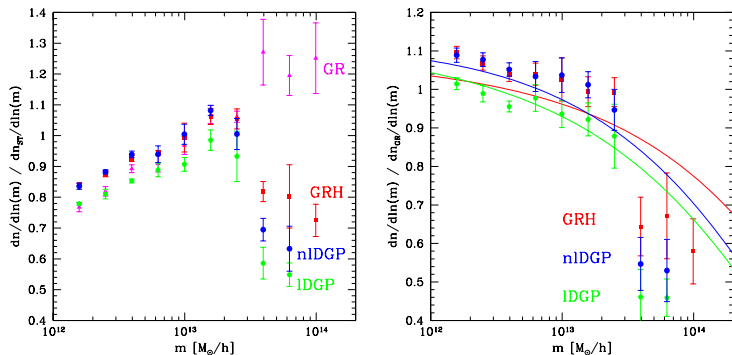


Figure: *Left panel:* mass functions measured in numerical simulations for the different models, as labeled, divided by a reference ST mass function. *Right panel:* the measured mass function for the three non standard models divided by the GR mass function from the simulations. The solid lines are perturbative calculation (Scoccimarro 2009).

Thanks!