

# Large Scale Nonlocal Bias from Gravitational Evolution

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## Introduction and Conclusion

After galaxies form, the bias parameters are expected to evolve over time. The gravitational evolution will induce nonlocality in the bias parameter even if the initial biasing prescription is local. Using the fluid description, we solve the continuity equation and Euler equation of the matter and galaxies together. This enables us to include not only the density bias but also the velocity bias. Velocity bias, like the density bias, relaxes to 1, but at a faster pace. We find that a quadrupole is induced in the second order coupling when there is no velocity bias and the evolution due to the dark matter is subtracted. When there is velocity bias, a dipole term is also induced. We have also checked that exactly the same results for the case without velocity bias can be recovered from the Lagrangian approach, provided that the initial conditions are handled carefully. We go on to show that in the presence of a generic local galaxy source, the results are rather similar to the conserved model and we still have nonlocal quadrupole term induced. This result is independent of the specific form of the source. Therefore we have shown that gravitational evolution inevitably induce large scale nonlocality in the bias parameter.

## Evolution of Density and Velocity Biases

We adopt the fluid description for the matter and galaxies and solve the system of equations

$$\frac{\partial \delta}{\partial \tau} + \theta = - \int d^3 k_1 d^3 k_2 \quad (1)$$

$$\times \delta_D(\mathbf{k} - \mathbf{k}_{12}) \alpha(\mathbf{k}_1, \mathbf{k}_2) \theta(\mathbf{k}_1) \delta(\mathbf{k}_2),$$

$$\frac{\partial \theta}{\partial \tau} + \mathcal{H} \theta + \frac{3}{2} \mathcal{H}^2 \Omega_m (\delta + \epsilon \delta_g) = - \int d^3 k_1 d^3 k_2 \quad (2)$$

$$\times \delta_D(\mathbf{k} - \mathbf{k}_{12}) \beta(\mathbf{k}_1, \mathbf{k}_2) \theta(\mathbf{k}_1) \theta(\mathbf{k}_2),$$

$$\frac{\partial \delta_g}{\partial \tau} + \theta_g = - \int d^3 k_1 d^3 k_2 \quad (3)$$

$$\times \delta_D(\mathbf{k} - \mathbf{k}_{12}) \alpha(\mathbf{k}_1, \mathbf{k}_2) \theta_g(\mathbf{k}_1) \delta_g(\mathbf{k}_2),$$

$$\frac{\partial \theta_g}{\partial \tau} + \mathcal{H} \theta_g + \frac{3}{2} \mathcal{H}^2 \Omega_m (\delta + \epsilon \delta_g) = - \int d^3 k_1 d^3 k_2 \quad (4)$$

$\times \delta_D(\mathbf{k} - \mathbf{k}_{12}) \beta(\mathbf{k}_1, \mathbf{k}_2) \theta_g(\mathbf{k}_1) \theta_g(\mathbf{k}_2),$

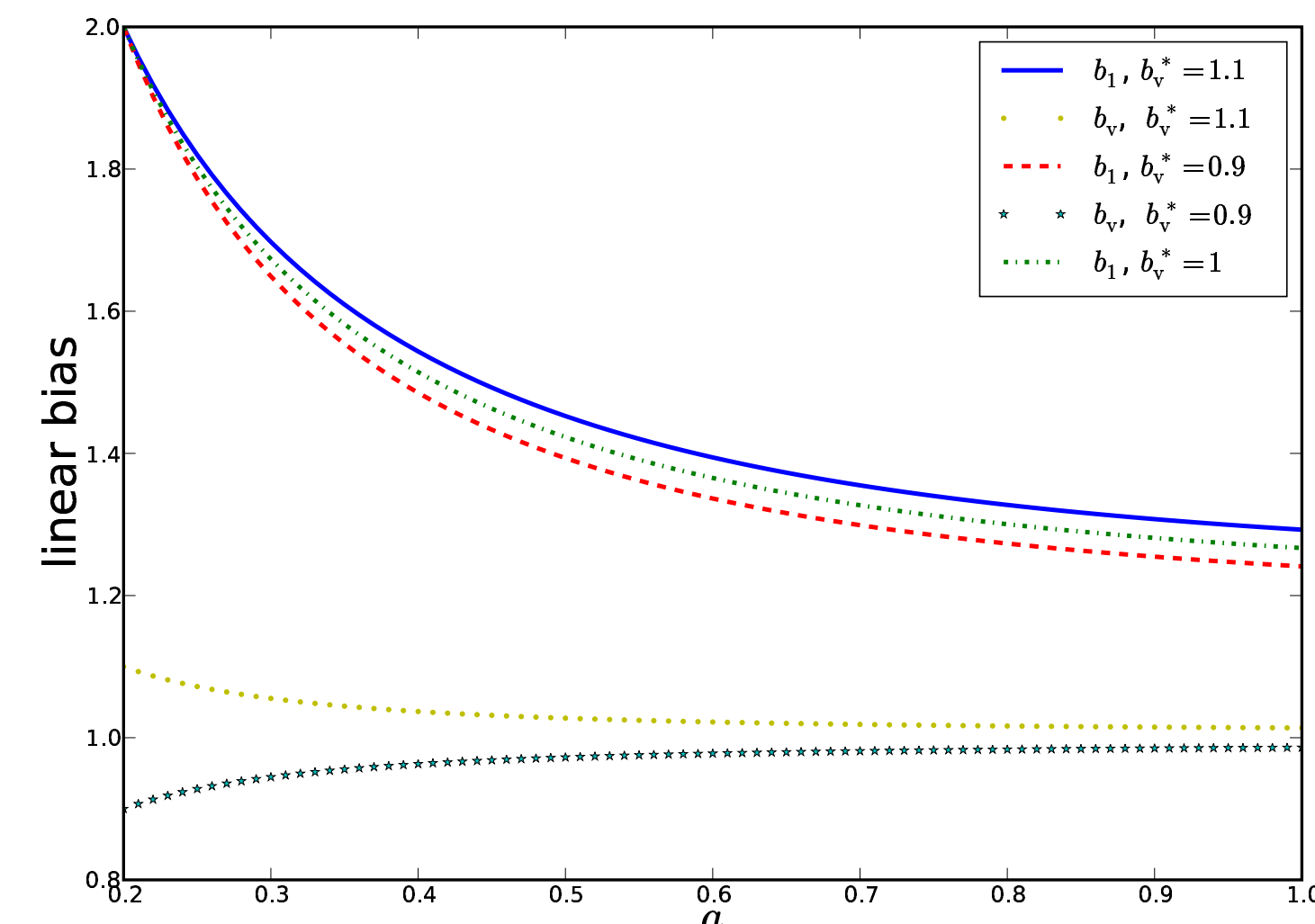
where  $\delta$  and  $\theta$  are the density contrast and velocity divergence of the dark matter, and analogous quantities are defined for galaxies. This means that we assume that the densities and momentum of dark matter and galaxy separately conserved. To first order, the evolution of the density bias  $b_1$  and velocity bias  $b_v$  are given by

$$b_1(a) = 1 + (b_1^* + 2b_v^* - 3)D^{-1} - 2(b_v^* - 1)D^{-3/2}, \quad (5)$$

$$b_v(a) = 1 + (b_v^* - 1)D^{-3/2}, \quad (6)$$

where  $b_1^*$  and  $b_v^*$  are the density and velocity biases at the formation time, and  $D$  is the linear growth factor.

Both the density and velocity biases relax towards to 1.



To second order, we consider the second order coupling kernel  $\mathcal{K}_g$ , which is defined via  $\delta_g(\mathbf{k}, t) = \int d^3 q_1 d^3 q_2 \delta_D(\mathbf{k} - \mathbf{q}_1 - \mathbf{q}_2) \mathcal{K}_g(\mathbf{q}_1, \mathbf{q}_2, t) \delta_m(\mathbf{q}_1) \delta_m(\mathbf{q}_2)$ . To study the nonlocal bias, we shall subtract the part due to dark matter evolution, which can be described by a local  $b_1$  (Eq. 5), *i.e.* we consider  $\mathcal{K}_g^{b_2^{\text{eff}}} \equiv \mathcal{K}_g - b_1 \mathcal{K}_m$ , where  $\mathcal{K}_m$  is defined analogous to  $\mathcal{K}_g$  but for the dark matter. We also decompose  $\mathcal{K}_g^{b_2^{\text{eff}}}$  into Legendre polynomial of  $l=2, 1$  and  $0$ . Without velocity bias ( $b_v^* = 1$ ) they are

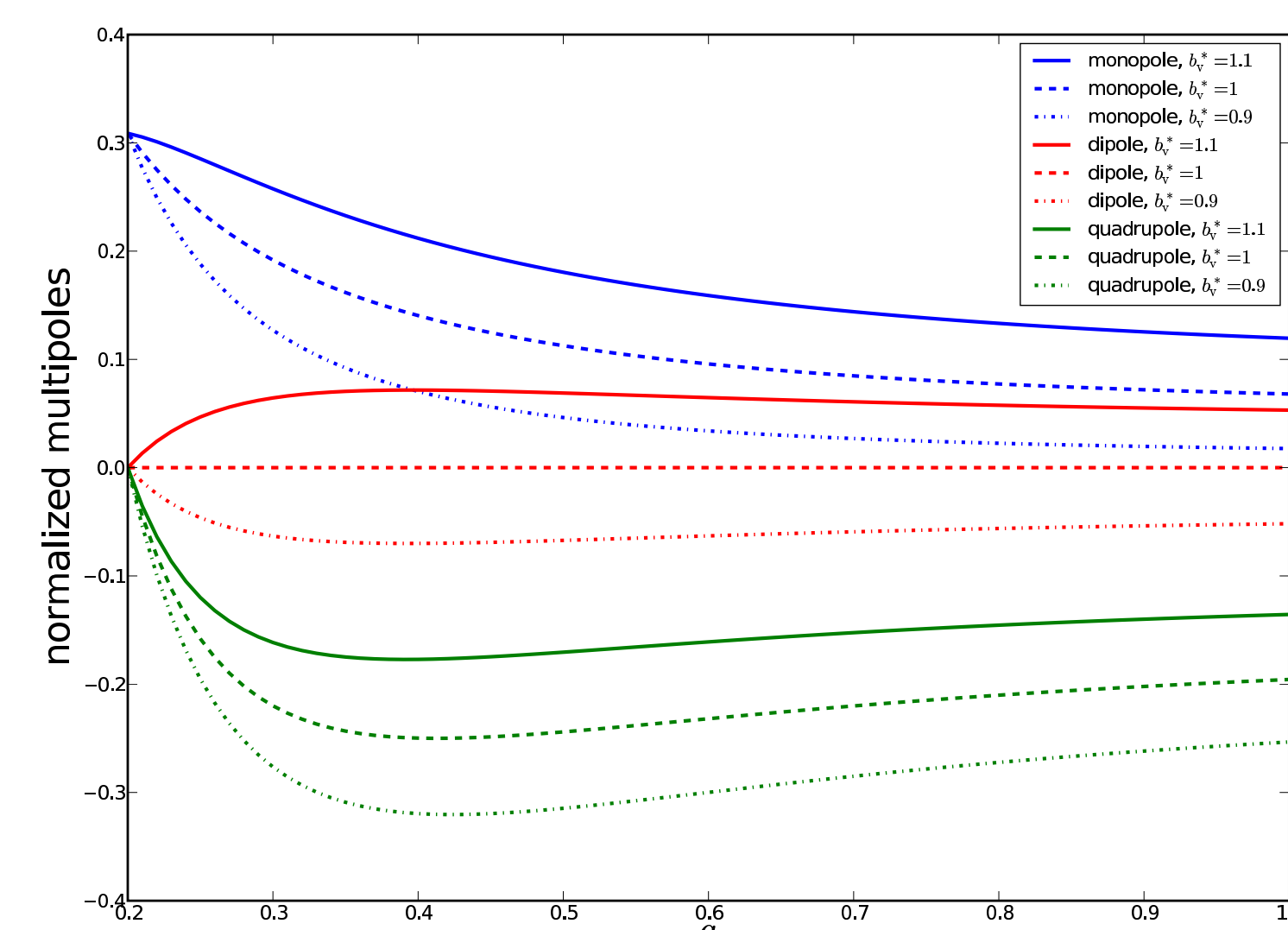
$$\mathcal{K}_{g,l=2}^{b_2^{\text{eff}}}(a) = -\frac{4}{21}(b_1^* - 1)(D - 1), \quad (7)$$

$$\mathcal{K}_{g,l=1}^{b_2^{\text{eff}}}(a) = 0, \quad (8)$$

$$\mathcal{K}_{g,l=0}^{b_2^{\text{eff}}}(a) = \frac{b_2^*}{2} + \frac{4}{21}(b_1^* - 1)(D - 1). \quad (9)$$

We note that the dipole term is exactly zero when there is no velocity bias, and this is not true when there is velocity bias. The dipole and quadrupole are manifestation of the nonlocal bias, and they don't appear if spherical collapse is assumed.

Evolution of the normalized Legendre coefficient of the effective second order coupling kernel  $\mathcal{K}_g^{b_2^{\text{eff}}}$ .



We can recover exactly the same results for the case without velocity bias from the Lagrangian approach using the relation

$$\frac{1 + \delta_g(\mathbf{x})}{1 + \delta(\mathbf{x})} = \frac{1 + \delta_g(\mathbf{q})}{1 + \delta(\mathbf{q})}, \quad (10)$$

and Zel'dovich approximation, provided that we take the initial conditions into account carefully.

## Bias Evolution with Galaxy Source

We study the evolution of bias by including an effective galaxy formation/merging source in the galaxy continuity equation

$$\frac{\partial n_g^{(c)}}{\partial \ln a} + \frac{1}{aH} \nabla \cdot (n_g^{(c)} \mathbf{u}) = A^{(c)} j(\rho), \quad (11)$$

where  $n_g^{(c)}$  is the comoving galaxy number density, and  $\mathbf{u}$  is the dark matter peculiar velocity. Note that we assume there is no velocity bias here. We adopt a simple model of galaxy source, assuming the period of the galaxy formation can be parametrized by some profile  $A^{(c)}(a)$ , and the dependence on the local dark matter density  $\rho$  is encoded in the function  $j(\rho)$ . But our results are independent of the detailed form of the source. In this model, the instantaneous formation bias are given by

$$b_1^*(t) \equiv \frac{j'(\bar{\rho}) \bar{\rho}}{j(\bar{\rho})}, \quad b_2^*(t) \equiv \frac{j''(\bar{\rho}) \bar{\rho}^2}{j(\bar{\rho})}, \quad (12)$$

where  $\bar{\rho}$  is the average dark matter density. For the sake of concreteness, we consider the model

$$A^{(c)}(t) = e^{-\frac{(\ln a - \ln a_0)^2}{2\sigma_0^2}}, \quad (13)$$

where  $a_0$  and  $\sigma_0$  are free parameters, and a simple quadratic form for  $j(\rho)$

$$j(\rho) = \alpha_1 \frac{\rho}{\rho_0} + \alpha_2 \left( \frac{\rho}{\rho_0} \right)^2, \quad (14)$$

where  $\alpha_1$  and  $\alpha_2$  are model parameters, and  $\rho_0$  is the average matter density today.

We solve Eq. 11 by expanding  $n_g^{(c)} = \bar{n}_g^{(c)} (1 + \delta_g^{(1)} + \delta_g^{(2)})$ . The background solution is

$$\bar{n}_g^{(c)} = \int_{\ln a_{\text{ini}}}^{\ln a} d(\ln a) A^{(c)} j(\bar{\rho}). \quad (15)$$

Writing  $\delta_g^{(1)} = D_g \delta_0$  and defining the effective linear bias  $b_1^{\text{eff}}(t)$  as  $D_g \equiv b_1^{\text{eff}}(t) D$ , we get

$$b_1^{\text{eff}}(t) = 1 + \frac{1}{\bar{n}_g^{(c)} D} \int_0^{\bar{n}_g^{(c)}} d\bar{n}_g^{(c)} D (-1 + b_1^*). \quad (16)$$

We can see the effect of galaxy formation on the evolution of bias by considering the evolution equation of  $b_1^{\text{eff}} - 1$

$$\frac{d}{d \ln a} (b_1^{\text{eff}} - 1) = -(b_1^{\text{eff}} - 1) f - \frac{d \ln \bar{n}_g^{(c)}}{d \ln a} (b_1^{\text{eff}} - b_1^*). \quad (17)$$

The first term on the RHS drives the relaxation of bias to 1 due to growth of large scale structure, while the second term may speed up the decay of bias if the newly formed galaxies are less biased than the existing one ( $b_1^{\text{eff}} - b_1^* < 0$ ), and vice versa.

For the second order, we again consider the effective second order kernel and they are given by

$$\mathcal{K}_{g,l=2}^{b_2^{\text{eff}}} = \frac{4}{21 \bar{n}_g^{(c)}} \left[ D \int_0^{\bar{n}_g^{(c)}} d\bar{n}_g^{(c)} D (1 - b_1^*) \right. \\ \left. - \int_0^{\bar{n}_g^{(c)}} d\bar{n}_g^{(c)} D^2 (1 - b_1^*) \right], \quad (18)$$

$$\mathcal{K}_{g,l=1}^{b_2^{\text{eff}}} = 0, \quad (19)$$

$$\mathcal{K}_{g,l=0}^{b_2^{\text{eff}}} = -\mathcal{K}_{g,l=2}^{b_2^{\text{eff}}} + \frac{1}{2 \bar{n}_g^{(c)}} \int_0^{\bar{n}_g^{(c)}} d\bar{n}_g^{(c)} D^2 b_2^* \quad (20)$$

It is easy to show from Eq. 18 that the nonlocal term  $\mathcal{K}_{g,l=2}^{b_2^{\text{eff}}}$  is present when the galaxy is biased, and this result is independent of the detailed form of the source.

The parameter  $\sigma_0$  in profile  $A$ , is taken to be 0.2, while the characteristic galaxy formation time  $a_0$  in  $A$  is chosen to be 0.3 (blue), 0.5 (red) and 0.7 (green) respectively. We have chosen three sets of values for  $\alpha_1$  and  $\alpha_2$ , they are 4 and 1 (solid), 1 and 1 (dashed), and 1 and 4 (dashed-dotted). For  $\bar{n}_g^{(c)}$  we have normalized it by the value at  $a=1$ . For  $D_2^{b_2^{\text{eff}}}$  and  $D_0^{b_2^{\text{eff}}}$ , we have normalized them by the corresponding dark matter multipole coefficients  $4D^2/21$  and  $17D^2/21$  respectively.

