NYU Physics 1—small variations

1 You have a cubic block of ice, 1.000 m on a side. By what fractional amount does its mass decrease if you shave a millimeter off of it in each dimension (ie, so that it becomes a cubic block of ice, 0.999 m on a side)?

2 Explain why your answer to the above was so close to 3×10^{-3}. You have two explanations to give, *(a)* one from the point of view of the three operations you did (shave one face, then the next, then the next), and *(b)* one from the point of view of small variations (write the formula for the volume V in terms of the side length ℓ and differentiate with respect to ℓ).

3 Give a general argument—using calculus—that

$$\lim_{\epsilon \to 0} (1+\epsilon)^n = 1 + n\,\epsilon \quad . \tag{1}$$

4 With a calculator, compute the sine of the angle $\theta = 0.1\,\mathrm{rad}$. Now compute the error you make if you use $\sin\theta = \theta$. Now compare that error to the quantity $\theta^2/2$ and $\theta^3/6$. Is it close to either?

5 Now compute the error you make if you use

$$\lim_{\theta \to 0} \sin\theta = \theta - \frac{1}{6}\,\theta^3 \quad . \tag{2}$$

Do you think the *next* term in the Taylor series for sine will be negative or positive on this basis?

6 Inside of what angle is the approximation $\sin\theta = \theta$ good to one percent? What is this angle in degrees?

7 Now square the expression in Equation (2) and compare it to the second-order expression for cosine:

$$\lim_{\theta \to 0} \cos\theta = 1 - \frac{1}{2}\,\theta^2 \quad . \tag{3}$$

Show that this is consistent with the trigonometric identity

$$\sin^2\theta = \frac{1 - \cos(2\,\theta)}{2} \quad . \tag{4}$$