NYU Physics 1—Problem set 12

Due Tuesday 2009 December 15 at the beginning of lecture.

Problem 1: Compare the angular momentum of the Earth's *spin* to the angular momentum in the Earth's *orbit* around the Sun. Which is larger, and why? Why is it physically impossible that the Earth spin so fast that it have as much angular momentum in its spin as in its orbit? I want to see an argument that does *not* refer to relativity. Even in the absence of relativistic effects, why would this be impossible? *Hint:* Think about gravitational forces.

Problem 2: *(a)* A figure skater spins in place on frictionless ice at angular speed ω_i with her hands outstretched. She has a total moment of inertia I_i. As the skater draws her hands into her body, her moment of inertia decreases to $I_f = I_i/2$. Does her kinetic energy K increase, decrease, or stay the same? If it increases, where does the energy come from? If it decreases, where does the energy go to? *Explain all your answers concisely but clearly: What is conserved?*

(b) Now estimate the moments of inertia: I_i of an ice skater with her hands outstretched, and I_f of an ice skater with her hands drawn in. Is the factor of 2 used in part *(a)* reasonable?

Problem 3: *(a)* Immediately after being hit, at $t = 0$, a cue ball of mass M and radius R slides along the felt at speed v_i, not rotating at all. As time goes on, the ball slows down (because of friction) and, at the same time, starts to spin. Draw a free-body diagram for the cue ball. At what time t_r does the ball get to the situation of "rolling without slipping"? Assume that there is a coefficient μ of sliding friction.

(b) Plot $v(t)$ and $R\,\omega(t)$ vs t on a single plot. *Note that the two things I have asked you to plot have the same dimensions.* Clearly label t_r on your diagram.

(c) The area between the two curves has dimensions of length. What is the meaning of this length? It is the distance of what?

Extra Problem (will not be graded for credit): A pool ball of mass m rolls without slipping at speed v towards a bumper on the pool table.

The bumper contacts the ball above the midline, that is, at a height $h > R$ above the surface of the table. When the ball hits the bumper, the impulse reverses the direction of the ball, *and,* if the bumper is properly designed, the direction of *rotation* of the ball.

(a) What is the optimal height h for the bumper to contact the ball? Give your answer in terms of the ball radius R. *Hint:* Treat the point of contact as providing a large force over a short time, in the exactly horizontal direction to reverse the motion; now work out the angular impulse from that contact.

(b) Now go measure the height of the contact point of a bumper and the diameter of a pool ball in your dorm or a nearby pool hall. Is it high or low? Be careful to measure the point of *contact* of the bumper. If you get an answer that is high or low, does the bumper seem to work okay nonetheless? Also, you might take the time to shoot a few games and think about last week's problem set.

(c) How would your answer to part (a) change if pool balls were thin-walled hollow spheres instead of solid spheres?

(d) What are you assuming when you assume that the force is purely horizontal? Warning, it is not trivial.