Beams of light with helical wavefronts focus to rings, rather than points, and also carry orbital angular momentum [1–3] that they can transfer to illuminated objects [3–6]. When focused strongly enough, such helical modes form toroidal optical traps known as optical vortices [4, 7, 8], whose properties present novel opportunities for scientific research and technological applications. For example, optical vortices should be ideal actuators for microelectromechanical systems (MEMS) [9], and arrays of optical vortices [10] have shown a promising ability to assemble colloidal particles into mesoscopic pumps for microfluidic systems. All such applications will require a comprehensive understanding of the intensity distribution and angular momentum flux within optical vortices. This Letter describes measurements of the structure of optical vortices created with the dynamic holographic optical tweezer technique [10], and of their ability to exert torques on trapped materials. These measurements reveal qualitative discrepancies with predicted behavior, which we explain on the basis of scalar diffraction theory.

A helical mode ψ(⃗r) is distinguished by a phase factor proportional to the polar angle θ around the beam’s axis,

\[
\psi(⃗r) = u(r, z) e^{-ikz} e^{iℓθ}.
\]

Here, \( \vec{k} \) = \( k \hat{z} \) is the beam’s wavevector, \( u(r, z) \) is the field’s radial profile at position \( z \), and \( ℓ \) is an integral winding number known as the topological charge. All phases appear along the beam’s axis, \( r = 0 \), and the resulting destructive interference cancels the axial intensity. Similarly, each ray in such a beam has an out-of-phase counterpart with which it destructively interferes when the beam is brought to a focus. Constructive interference at a radius \( R_ℓ \) from the optical axis yields a bright ring whose width is comparable to \( \lambda \), the wavelength of light. The semi-classical approximation further suggests that each photon in a helical mode carries \( ℓ \) optical angular momentum [2], so that the beam can exert a torque proportional to its intensity.

Conventional beams of light can be converted into helical modes with a variety of mode converters [11]. Most implementations yield topological charges in the range \( 1 \leq ℓ \leq 8 \) [2, 6]. By contrast, dynamic holographic optical tweezers [10] can create helical modes up to \( ℓ = 200 \), and so are ideal for studying how optical vortices’ properties vary with \( ℓ \).

Our system, depicted in Fig. 1, uses a Hamamatsu X7550 parallel-aligned nematic liquid crystal spatial light modulator (SLM) [12] to imprint computer-generated patterns of phase shifts onto the wavefront of a linearly polarized TEM\(_{00}\) beam at \( \lambda = 532\text{ nm} \) from a frequency-doubled Nd:YVO\(_4\) laser (Coherent Verdi). The modu-

\begin{figure}
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\includegraphics[width=\textwidth]{fig1.png}
\caption{(a) Schematic diagram of dynamic holographic optical tweezers creating an optical vortex. The SLM imposes the phase \( ϕ(⃗r) = \theta \odot (2\pi r) \) on the incident TEM\(_{00}\) beam, converting it into a helical beam that is focused into an optical vortex. The inset phase mask encodes an \( ℓ = 40 \) optical vortex. (b) Image of the resulting optical vortex obtained by placing a mirror in the focal plane. The central spot is the diffraction-limited focus of the incident TEM\(_{00}\) beam at \( λ = 532\text{ nm} \) from a frequency-doubled Nd:YVO\(_4\) laser (Coherent Verdi). (c) Time-lapse image of a single colloidal sphere illuminated by the vortex.}
\end{figure}
lated wavefront is transferred by a telescope to the back aperture of a 100 × NA 1.4 oil immersion objective lens mounted in a Zeiss Axiovert S100TV inverted optical microscope. The objective lens focuses the light into optical traps, in this case a single optical vortex. The same lens also forms images of trapped particles that are relayed to an attached video camera through a dichroic mirror.

The SLM can shift the light’s phase to any of 150 distinct levels in the range 0 ≤ φ ≤ 2π radians at each 40 μm wide pixel in a 480 × 480 square array. Imprinting a discrete approximation to the phase modulation ϕ(ℓ) = ℓθ mod 2π onto the incident beam, yields a helical mode with 70 percent efficiency, independent of laser power over the range studied. Conversion efficiency is reduced for ℓ > 100 by the SLM’s limited spatial resolution. Fig. 1(b) is a digital image of an ℓ = 40 optical vortex reflected by a mirror placed in the objective’s focal plane. The unmodified portion of the TEM00 beam travels along the optical axis and comes to a focus in the focal plane. The unmodified portion of the TEM00 beam reflects by a mirror placed in the objective’s focal plane and comes to a focus in the center of the field of view. This conventional beam does not overlap with the optical vortex in the focal plane and so does not affect our observations. Fig. 1(c) shows a time-lapse multiple exposure of a single 800 nm diameter colloidal polystyrene sphere trapped on the optical vortex’s circumference in an 85 μm thick layer of water between a coverslip and a microscope slide. Angular momentum transferred from the optical vortex drives the sphere once around the circumference in a little under 2 sec at an applied power of 500 mW. The image shows 11 stages in its transit at 1/6 sec intervals. We studied the same particle’s motions at different topological charges and applied powers to establish how helicity influences optical vortices’ intensity distribution and local angular momentum flux.

Observing that a single particle translates around the circumference of a linearly polarized optical vortex distinguishes the angular momentum carried by a helical beam of light from that carried by circularly polarized light. The latter causes an absorbing particle to spin on its own axis. Observing that the particle instead translates around the optical axis demonstrates that the angular momentum density in a helical beam results from its own axis. Observing that the particle instead translates around the optical axis distinguishes the angular momentum carried by a helical beam of light from that carried by circularly polarized light. The latter causes an absorbing particle to spin on its own axis.

Figs. 2 reveal qualitatively different behavior. We obtain Rℓ from digitized images such as Fig. 1(b) by averaging over angles and locating the radius of peak intensity. Projecting different values of ℓ reveals that Rℓ scales linearly with the topological charge, and not as √ℓ.

This substantial discrepancy can be explained by considering how the phase-modulated beam propagates through the optical train. The field in the focal plane of a lens of focal length f is related in scalar diffraction theory to the field at the input aperture (and thus at the face of the SLM) through a Fourier transform [14]. Transforming the helical beam first over angles yields

\[ u_\ell(r, 0) = \int_0^\Sigma r' u(r', -f) J_\ell \left( k f r' \right) dr', \]  

where \( J_\ell(x) \) is the \( \ell \)-th order Bessel function of the first kind, and \( \Sigma \) is the radius of the optical train’s effective aperture. For our system, \( \Sigma = 1.7 \) mm. Setting \( u(r, -f) = u_0 \) for a uniform illumination yields

\[ \psi(r, \theta, 0) = e^{i \ell \theta} \sum_{n=0}^{\infty} (-1)^{(n+\frac{\ell}{2})} \left[ \Sigma k r / (2f) \right]^{2(n+\ell)} / \left( 1 + \frac{\ell}{2} + n \right) (\ell + n)! n! \]  

The radius \( R_\ell \) of the principal maximum in |\( \psi_\ell(r, \theta, 0) \)| is approximated very well by

\[ R_\ell = a f / (\pi \Sigma) \left( 1 + \frac{\ell}{\ell_0} \right), \]  

with \( a = 2.585 \) and \( \ell_0 = 9.80 \). This also agrees quantitatively with the data in Fig. 2.
Even if the mode converter created a pure \( p = 0 \) LG mode, the optical trapping system’s limited aperture, \( \Sigma \), still would yield a superposition of radial eigenmodes at the focal plane [2], and a comparable linear dependence of \( R_\ell \) on \( \ell \). The superposition of higher-\( p \) modes in our system is evident in the hierarchy of diffraction fringes surrounding the principal maximum in Fig. 1(b).

This linear dependence on \( \ell \) leads to scaling predictions for optical vortices’ optomechanical properties with which we can probe the nature of the angular momentum carried by helical modes. In particular, a wavelength-scale particle trapped on the circumference of an optical vortex is illuminated with an intensity \( I_\ell \propto P/(2\pi \lambda R_\ell) \), where \( P \) is the power of the input beam, assuming that the photon flux is spread uniformly around the vortex’s circumference in a band roughly \( \lambda \) thick (see Fig. 2). If we assume that each scattered photon transfers an angular momentum proportional to \( \hbar \), then the particle’s tangential speed should be proportional to \( \ell P/R_\ell^2 \), and the time required to make one circuit of the optical vortex should scale as

\[
T_\ell (P) \propto \frac{R_\ell^2}{(\ell P)}. \tag{6}
\]

If the radius had scaled as \( R_\ell \propto \sqrt{\ell} \), then the particle’s speed would have been independent of \( \ell \), and the period would have scaled as \( \sqrt{\ell}/P \). Instead, for \( \ell > \ell_0 \), we expect \( T_\ell (P) \propto \ell^2/P \).

The data in Fig. 3(a) show that \( T_\ell (P) \) does indeed scale according to Eqs. (5) and (6) for larger values of \( \ell \). For \( \ell < 40 \), however, the period is systematically larger than predicted. Similarly, \( T_\ell (P) \) scales with \( P \) as predicted for lower powers, but increases as \( P \) increases. In other words, the particle moves slower the harder it is pushed. This unexpected effect can be ascribed to the detailed structure of optical vortices created with pixellated diffractive optical elements; the mechanism presents new opportunities for studying Brownian transport in modulated potentials.

When projected onto our objective lens’ input pupil, each of our SLM’s effective phase pixels spans roughly \( 10 \lambda \). Numerically transforming such an apodized beam reveals a pattern of \( 2\ell \) intensity corrugations, as shown in Fig. 3(a). These establish a nearly sinusoidal potential through which the particle is driven by the local angular momentum flux. We model the intensity’s dependence on arclength \( s \) around the ring as

\[
I_\ell (s) = \frac{P}{2\pi \lambda R_\ell} (1 + \cos qs), \tag{7}
\]

where \( q \) is the depth of the modulation, and \( q = 2\ell/R_\ell \) is its wavenumber. For \( \ell > \ell_0 \approx 10 \), \( q \) is approximately independent of \( \ell \).

This modulated intensity exerts two tangential forces on the trapped sphere. One is due to the transferred angular momentum,

\[
F_\ell (s) = A_0 \frac{P}{R_\ell} (1 + \cos qs), \tag{8}
\]

where we assume a local angular momentum flux of \( \hbar \) per photon. The prefactor \( A_0 \) includes such geometric factors as the particle’s scattering cross-section. The other is an optical gradient force due to the polarizable particle’s response to local intensity gradients:

\[
F_q (s) = -\epsilon A_0 \frac{2\pi \lambda}{q} \frac{\partial I_\ell (s)}{\partial s} = \epsilon A_0 \frac{P}{R_\ell} \alpha \sin qs, \tag{9}
\]

where \( \epsilon \) sets the relative strength of the gradient force. Combining Eqs. (8) and (9) yields Eqs. (8) and (9) yields

\[
F(s) = A_0 \frac{P}{R_\ell} (1 - \eta \cos qs), \tag{10}
\]

where we have omitted an irrelevant phase angle, and where \( \eta = \alpha (1 + \epsilon^2)^{1/2} \). Even if \( \alpha \) is much smaller than unity, both \( \epsilon \) and \( \eta \) can be much larger. In that case, reducing \( \ell \) at fixed power increases the depth of the modulation relative to the thermal energy scale \( k_B T \), and the particle can become hung up in the local potential minima. The modulated potential thus increases the effective drag.

More formally, a particle’s motion along an inclined sinusoidal potential with strong viscous damping is described by the Langevin equation

\[
\gamma \frac{ds}{dt} = F(s) + \Gamma(t), \tag{11}
\]
$\gamma \mu = 1 + 2 \text{Im}\left\{ \frac{\frac{1}{2} \eta^2}{\ell_T} + i + \frac{\frac{1}{2} \eta^2}{2 \ell_T} + i + \ldots \right\}$, \hspace{1cm} (12)

where $\ell_T = A_0 P_\eta/(4\pi k_B T)$ is the topological charge at which the modulation reaches $k_B T$ at power $P$. Given this result, the transit time for one cycle should be

$$T_1(P) = T_1 \frac{P_1}{P} \frac{\ell^2}{\gamma \mu},$$ \hspace{1cm} (13)

where $T_1 = 2\pi \gamma R_1^2/(P_1 A_0)$ is the expected period for $\ell = 1$ at $P = P_1$ in the absence of modulation. The solid curve in Fig. 3(a) is a fit to Eqs. (12) and (13) for $T_1$, $\ell_T$ and $\eta$. The results, $T_1 P_1/P = 1$ msec, $\eta P_1 = 32$, and $\eta = 19$, are consistent with the strongly modulated potential shown in the inset to Fig. 3(b). Rather than smoothly processing around the optical vortex, the particle instead makes thermally activated hops between potential wells in a direction biased by the optical vortex’s torque.

Replacing $\ell/\ell_T$ with $P_T/P$ in Eq. (12) yields an analogous result for the period’s dependence on applied power for fixed $\ell$, as shown in Fig. 3(b). Here, $P_T = 4\pi \ell_T k_B T/A_0$ is the power at which the modulation reaches $k_B T$. Using $\eta$ and $P_T T_1$ obtained from Fig. 3(a), we find that the sphere’s motions above $P = 1.5$ W are slower even than our model predicts. The period’s divergence at high power is due to a localized “hot spot” on the $\ell = 19$ optical vortex resulting from aberrations in our optical train. Such hot spots have confounded previous attempts to study single-particle dynamics in helical beams [3]. Because hot spots also deepen with increasing power, they retain particles with exponentially increasing residence times. The total transit time becomes [15]

$$T(P) = T_1(P) + T_H \exp(P/P_H).$$ \hspace{1cm} (14)

The data in Fig. 3(b) are consistent with $T_H = 5$ msec and $P_H = 270$ mW. Localization in hot spots becomes comparable to corrugation-induced drag only for powers above $P = 1$ W and so does not affect the data in Fig. 3(a). Consequently, these data offer insights into the nature of the helical beam’s angular momentum density.

In particular, the simple scaling relation, Eq. (6) is remarkably successful at describing a single particle’s motions around an optical vortex over a wide range of topological charges. This success strongly supports the contention that each photon contributes $\hbar$ to the local angular momentum flux of a helical beam of light, and not only to the beam’s overall angular momentum density. It hinges on our observation that the radius of a practical optical vortex scales linearly with its topological charge. The corrugations in apodized optical vortices broaden this system’s interest. Not only do they provide a realization of the impossible staircase in M. C. Escher’s lithograph “Ascending and Descending”, but they also offer a unique opportunity to study overdamped transport on tilted sinusoidal potentials. As a practical Brownian ratchet, this system promises insights germane to such related phenomena as transport by molecular motors, voltage noise in Josephson junction arrays, and flux flow in type-II superconductors. Preliminary observations of multiple particles on an optical vortex also suggest opportunities to study transitions from jamming to cooperativity with increasing occupation.

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