Robustness of Holographic Optical Traps Against Phase Scaling Errors

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Complex three-dimensional patterns of multifunctional optical traps can be encoded in phase-only computer-generated holograms and projected with the holographic optical trapping technique. The trap-forming holograms, in turn, are implemented as diffractive optical elements whose phase transfer functions generally do not faithfully reproduce the design. We demonstrate that phase encoding errors reduce the overall intensities of the projected traps but, remarkably, do not affect their positions, relative intensities or mode structure. We exploit this robust performance to implement dual-color holographic optical tweezers with a single hologram.

Holographic optical traps use phase-only holograms to form large arrays of optical traps from a single input laser beam. By combining the beam-splitting and wavefront-shaping capabilities of computer generated holograms, holographic traps can be arranged in arbitrary three-dimensional configurations, with each trap having independently specified characteristics, including relative intensity and mode structure. The unsurpassed control over the microscopic world afforded by this technique has been widely adopted for fundamental research in soft-matter systems and for biomedical and industrial applications [1].

In principle, holographic trapping patterns can be projected with absolute fidelity to design and near-ideal efficiency. Practical diffractive optical elements (DOEs), however, seldom offer the requisite continuously varying phase profiles, and almost never provide precisely the phase pattern required for in a given design. This has been recognized as a central problem for holographic projection systems since the introduction of the kinoform [2]. Imperfectly imprinting the designed phase pattern onto the input beams wavefronts degrades the projected intensity patterns. To quantify this, we introduce an expansion of the projected field into generalized conjugates of the designed field. This analysis demonstrates that the performance of optimized [3] holographic trapping systems is remarkably robust against phase defects, and further suggests useful generalizations of the technique.

A standard holographic optical trapping system [4–8] is powered by a collimated laser beam, which is relayed to the input pupil of a high-numerical-aperture lens such as a microscope objective lens. This lens focuses the beam to a diffraction-limited spot at a location determined by the beam’s angle of incidence and degree of collimation at the lens’ input pupil. Such a focused spot acts as a single-beam optical gradient force trap known as an optical tweezer [9] and is capable of capturing and holding mesoscopic objects in three dimensions. Placing a wavefront-shaping hologram in a plane conjugate to the input pupil transforms the single optical tweezer into a pattern of holographic optical traps whose number, three-dimensional configuration, relative and absolute intensities, and mode structure are all encoded in the hologram. Deficiencies in the hologram’s implementation might reasonably be expected to degrade all of these characteristics. An analysis based on scalar diffraction theory shows otherwise.

The complex field $E(r)$, in a plane at distance $z$ from the focal plane of a lens of focal length $f$ is related to the field in the lens’ input plane by the Fresnel diffraction integral [10],

$$E(r) = \frac{1}{k f} \int_{\Omega} u_0(\rho) \exp(i \varphi_0(\rho)) \exp(i \Phi(\rho)) \exp\left(-i \frac{k \rho^2}{2 f z} \right) \exp\left(-i \frac{k \mathbf{r} \cdot \rho}{f} \right) d^2 \rho, \quad (1)$$

where we have suppressed overall phase factors and assumed $z \ll f$. Here, $u_0(\rho)$ and $\varphi_0(\rho)$ are the real-valued amplitude and phase profiles, respectively, of the input beam at position $\rho$ in the input pupil $\Omega$, as shown in Fig. 1, and $k = 2\pi/\lambda$ is the wavenumber of light of wavelength $\lambda$. A diffractive optical element (DOE) imposes the additional phase profile $\Phi(\rho)$, which ideally would correspond to the computer-generated hologram $\varphi(\rho)$ encoding a desired pattern of traps. In practice, $\Phi(\rho)$ differs from the $\varphi(\rho)$, so that $I(r) = |E(r)|^2$ differs from the planned trapping pattern in the plane of best focus.

Particularly when considering DOEs that encode three-dimensional trapping configurations, identifying the plane of best focus is the first step in assessing performance. If a simple beam-splitting DOE is illuminated with a collimated beam, we may take $\varphi_0(\rho) = 0$, and the resulting pattern of traps comes to sharpest focus when the remaining $\rho^2$-dependent phase term in the integrand of Eq. (1) vanishes, which occurs in the plane $z = 0$. The optimized holographic trapping technique [3] instead uses a slightly converging beam with

$$\varphi_0(\rho) = -\frac{k \rho^2}{2 f^2} z_0, \quad (2)$$

which shifts the plane of best focus to $z = -z_0$, for $z_0 \ll f$. Artifacts due to imperfect phase modulation need not focus in the same plane, as we will see, and the resulting axial displacement can minimize their influence.
Our implementation of optimized holographic optical trapping is described in detail in Ref. [3], and is built around a 100 × NA 1.4 Plan Apo oil immersion objective lens mounted in a Nikon TE-2000U inverted optical microscope. Laser light at a wavelength of 532 nm provided by a Coherent Verdi laser, is imprinted with computer-generated holograms by a Hamamatsu X8267-16 spatial light modulator (SLM) which acts as a DOE with a 768 × 768 array of phase pixels. The focused optical traps are imaged by placing a mirror in the objective lens’ focal plane and capturing the reflected light with an NEC TI-324AII charge-coupled device (CCD) camera.

Most DOEs, including our SLM, can impose only a limited range of phase delays, which ideally corresponds to one wavelength of light, so that \(\varphi(\rho) \mod 2\pi = \varphi(\rho)\). Introducing the DOE’s phase transfer function, \(f(x)\), such that \(\Phi(\rho) = f(\varphi(\rho))\), and noting that \(\exp(i\varphi)\) is a periodic function of \(\varphi\) with period \(2\pi\), we may expand the DOE’s contribution to the field’s phase factor in a Fourier series

\[
\exp(i\Phi(\rho)) = \sum_{n=-\infty}^{\infty} a_n \exp(i n \varphi(\rho)) \tag{3}
\]

with coefficients

\[
a_n = \frac{1}{2\pi} \int_{0}^{2\pi} \exp(i f(x)) \exp(-i n x) dx. \tag{4}
\]

We accordingly define the generalized \(n\)-th order conjugate fields,

\[
E_n(\rho) = \frac{1}{\lambda f} \int_{\Omega} u_0(\rho) \exp(i \varphi_0(\rho)) \exp(i n \varphi(\rho)) \exp(-i \frac{k \rho^2}{2f^2} z) \exp\left(-i \frac{k r \cdot \rho}{f}\right) d^2\rho. \tag{5}
\]

The projected field is then

\[
E(\rho) = \sum_{n=-\infty}^{\infty} a_n E_n(\rho). \tag{6}
\]

For example, if a DOE created for one wavelength of light, \(\lambda\), is illuminated with another, \(\lambda'\), then \(\Phi(\rho) = \gamma \varphi(\rho)\) with \(\gamma = \lambda/\lambda' \neq 1\) and

\[
a_n = \exp(-i\pi(n - \gamma)) \frac{\sin(\pi(n - \gamma))}{\pi(n - \gamma)}. \tag{7}
\]

Comparable results may be obtained for more general phase transfer functions, including those featuring discrete phase levels. The most stringent test, imperfect binary phase holograms with

\[
f(x) = \begin{cases} 
0 & x \leq a \\
b & a < x \leq 2\pi 
\end{cases}
\]

are described in this way with coefficients

\[
a_n = -\frac{2i}{n\pi} \exp\left(-i\frac{na}{2}\right) \exp\left(i \frac{b}{2}\right) \sin\left(\frac{na}{2}\right) \sin\left(\frac{b}{2}\right). \tag{9}
\]

These terms also fall off with order as \(1/|n|\). Best performance, in this case, is obtained with \(a = b = \pi\).

Equation (6) reveals that the projected image

\[
I(\rho) = \sum_{m,n=-\infty}^{\infty} a_m a_n^* E_m(\rho) E_n^*(\rho) \tag{10}
\]

includes a proportion \(|a_1|^2\) of the intended intensity profile, \(I_1(\rho) = |E_1(\rho)|^2\), even if the DOE imperfectly implements the requisite hologram. This first-order image is a faithful, undistorted realization of the designed pattern whose accuracy is limited only by errors in calculating \(\varphi(\rho)\) and physical imperfections in the optical train. The other terms in Eq. (10) represent artifacts introduced by the DOE’s phase transfer function \(f(x)\).

The term \(|a_0|^2 |E_0(\rho)|^2\) in Eq. (10) describes the undiffracted portion of the input beam, which typically comes to a focus in the center of the plane \(z = -z_0\). Because it receives a fixed proportion of the light, the resulting “central spot” can be brighter than any of the intended traps in \(I_1(\rho)\).

![FIG. 1: Schematic diagram of the geometry of an optimized holographic trapping system.](image)

![FIG. 2: Experimentally realized generalized conjugate images, \(I_0(\rho)\) and \(I_{-1}(\rho)\), to a planar arrangements of optical traps, \(I_1(\rho)\).](image)
Generalized conjugate fields are related by $E_{-n}(r) = E_n^*(r)$ in the plane $z = -z_0$ so that the associated images $I_n(r) = |E_n(r)|^2 = I_{-n}(-r)$, are related by point reflection through the origin, as shown in Fig. 2. Furthermore, $I_n(r) \approx I_0(nr)$ because multiplying $\varphi(p)$ by $n$ proportionately increases the hologram’s spatial frequency. The remaining terms in Eq. (10) thus describe a hierarchy of “ghost” images at locations dictated by integer scale dilations, point inversions, and their superpositions. Ghosts generally act as unintended traps. If they coincide with intended traps, however, the resulting interference can cause large deviations in the traps’ relative intensities.

In conventional holographic optical traps whose DOE is illuminated with collimated light, the entire hierarchy of conjugate fields is focused into the same plane. The central spot, the ghosts, and the undesirable superpositions thus maximally affect the trapping pattern.

The optimized holographic trapping system eliminates most of these defects. Here, the input beam’s curvature is offset by a compensating Fresnel lens function,

$$\varphi_z(p, z_1) = \frac{kq^2z_1}{2f^2},$$

added to the trap-forming hologram and implemented by the DOE [3]. This shifts the trapping pattern a distance $z_1$ back along the optical axis toward the focal plane. Because $\varphi_z(p, z_1)$ also is affected by the DOE’s phase transfer function, it contributes to the hierarchy of conjugate fields in Eq. (5). Noting also that $n \varphi_z(p, z_1) = \varphi_z(p, nz_1)$ shows that the $n$-th order generalized conjugate field $E_n(r)$ comes to best focus in the plane $z \approx -z_0 + nz_1$. Both the central spot and the ghost images therefore are projected away from the intended trapping pattern, and spurious superpositions are strongly suppressed. Artifacts due to practical limitations of the DOE’s phase transfer function therefore should have a minimal influence on the number, configuration, or relative intensity of traps in an optimized holographic trapping system. This is consistent with the observed performance of such systems [3]. The principal ramification of a non-ideal $f(x)$ is a reduction in the overall intensity $|a_1|^2$ of the projected trapping pattern.

Adding $\varphi_z(p, z_1)$ increases the complexity of the projected hologram, which can challenge the capabilities of DOE technologies with limited spatial bandwidths. Systematic metrics for assessing hologram complexity relative to DOE capabilities have yet to be developed. Consequently, the practical limitations of the optimized holographic trapping technique cannot yet be assessed a priori. Nevertheless, complex three-dimensional optimized trapping patterns consisting of hundreds of independent traps have been created with a 768 x 768 array of phase pixels [11].

The final characteristic of holographically projected traps that we will consider is their mode structure. Conventional optical tweezers typically are formed from collimated TEM$_{00}$ modes with planar wavefronts. More exotic traps such as optical vortices [12–14] and Bessel beams [15, 16] derive their interesting and useful properties from the detailed structure of their wavefronts. The structure necessary to create such traps can be imposed on a TEM$_{00}$ beam by a mode-forming hologram. Indeed, the beam-splitting and mode-forming operations can be combined in a single computer-generated hologram to create arrays of multifunctional optical traps [3, 6, 8]. Here again, the phase transfer function, $f(x)$, of the DOE can affect the fidelity with which a particular mode is projected, and thus can influence the associated trap’s functionality.

As a practical example, we consider optical vortices, torque-exerting traps created by focusing helical light beams. Helical modes are characterized by an overall phase factor $\exp(i\ell \theta)$, where $\theta$ is the azimuthal angle about the optical axis. The integer winding number $\ell$ sets the pitch of the helix, and is often referred to as the topological charge [17]. The helical topology suppresses the intensity along the axis of such a beam, not because the amplitude vanishes but rather because of destructive interference due to the coincidence of all phases there. An optical vortex, therefore, focuses to a dark spot surrounded by a bright ring of light. In optical vortices created by imposing a helical phase profile on a Gaussian beam, the ring’s radius scales linearly with topological charge [18–20].

Now we consider what happens to an optical vortex designed to have winding number $\ell$ when projected by a non-ideal DOE. Because the Fourier coefficients in Eq. (6) fall off with index, we approximate the field in the focal plane by the principal terms

$$E(r) \approx a_1 E_1(r) + a_0 E_0(r) + a_{-1} E_{-1}(r).$$

Taking $E_1(r) = u_1(r) \exp(i\ell \theta)$ for an optical vortex centered within a standard holographic optical trapping system, the conjugate field is

$$E_{-1}(r) = E_1^*(-r) = (-1)^\ell u_\ell(r) \exp(-i\ell \theta).$$

The resulting intensity distribution,

$$I(r) = A_0(r)+A_1 \cos(\ell \theta + \theta_1)+A_2 \cos(2\ell \theta + \theta_2),$$

is characterized by azimuthal intensity modulations with both $\ell$-fold and $2\ell$-fold symmetry. Taking $a_n = |a_n| \exp(i\theta_n)$, and defining $a^2 = |a_1|^2 + |a_{-1}|^2$ and $b^2 = 2|a_1|^2|a_{-1}|$ these terms’ relative amplitudes are $A_0(r) = |a_0|^2 u_0^2(r) + a^2 u_\ell^2(r)$, $A_1(r) = 2|a_0| \left[a^2 + (-1)^\ell b^2 \cos(2\beta_0 - \beta_1 - \beta_{-1})\right]^{1/2} u_0(r) u_\ell(r)$, and $A_2(r) = b^2 u_\ell^2(r)$. Their relative phases are given by

$$\tan \theta_1 = \frac{|a_1| \sin(\beta_1 - \beta_0) + (-1)^\ell |a_{-1}| \sin(\beta_0 - \beta_{-1})}{|a_1| \sin(\beta_1 - \beta_0) + (-1)^\ell |a_{-1}| \sin(\beta_0 - \beta_{-1})},$$

(15)
and \( \theta_2 = \ell \pi + \beta_1 - \beta_{-1} \).

In principle, the central spot’s amplitude profile, \( u_0(r) \), is sharply peaked around the optical axis and so should not overlap substantially with the optical vortex’s ring-like profile, \( u_\ell(r) \). The amplitude \( A_1(r) \) of the \( \ell \)-fold intensity corrugation resulting from their interference therefore should be negligible for \( \ell \gg 1 \). Even so, holographically projected optical vortices such as the example in Fig. 3(a) often are surrounded by \( \ell \) radial spokes extending to very large radii. These outer spokes are projected from the hologram’s central region [21], whose features typically are too fine to be reproduced faithfully by a pixellated DOE [18]. The undersampled phase pattern near the optical axis acts as a diffuser and scatters light to larger radii from where it contributes to the visible spokes. Both these and the optical vortex’s higher-order diffraction rings can be eliminated by excising the central region of the mode-forming phase mask [21, 22].

Whereas the \( \ell \)-fold features are due in large part to the DOE’s pixellated structure, the \( 2\ell \)-fold corrugation results from interference between the principal and conjugate fields. This corrugation, which also can be seen in Fig. 3(a), has been described before and significantly affects the dynamics of objects trapped on the circumference of an optical vortex [18]. It can be minimized by displacing the principal vortex away from the center of the field of view. Ideally, this eliminates modulation of the optical vortex’s circumferential intensity profile altogether, as shown in Fig. 3(b). Because optical vortices cover a larger area than conventional optical tweezers, however, some interference with neighboring and ghost traps can occur in more complex configurations [23]. Optimizing the phase transfer function to minimize these interactions thus is more important in creating multifunctional optical traps than in projecting arrays of conventional optical tweezers.

The expansion in generalized conjugate fields introduced in Eqs. (4), (5) and (6) clearly demonstrates that imperfections in a DOE’s phase transfer function only minimally influence the number, distribution, relative intensities and mode structure of optimized holographic traps encoded in a computer-generated hologram. This robustness suggests a strategy for projecting holographic traps in multiple wavelengths simultaneously. Because of the wavelength dependence of \( \varphi_0(\rho) \) in Eq. (2), beams of different wavelengths would focus to different planes in an optimized holographic optical trapping system, even with achromatic optics. Separate holograms can be calculated for each wavelength, each with the appropriate displacement along the optical axis, and the results added to create a multi-wavelength hologram that projects distinct patterns of traps in each color. As in previous approaches to multiwavelength holography [24, 25], all patterns are projected in each wavelength. This is less of a problem for holographic trapping than for data multiplexing [24] or image formation [25] because the unintended patterns in each color are displaced out of the plane of best focus, and typically out of the sample altogether. The result is that only the designed patterns in each color will be projected into the focal volume, as shown in Fig. 4. Suppressing the unintended patterns in each wavelength has been demonstrated for color separation gratings [26]. The same methods could be applied to more demanding multicolor holographic trapping applications.

The ability to project multifunctional optical traps in multiple wavelengths should facilitate simultaneous manipulation and photochemical transformation of light-sensitive systems. This would be useful for non-invasive intracellular surgery and for assembling and photochemically bonding three-dimensional heterostructures [11]. Multicolor arrays also will be useful for sorting objects by their absorptivity or index of refraction, for example in holographically implemented optical fractionation [27]. Combining these functions on a single DOE would sim-
plify the implementation by projecting all wavelengths along a single path.

In summary, we have shown that an optimized holographic optical trapping system’s performance is remarkably insensitive to details of the DOE’s phase transfer function. This accounts for the success of early implementations [4, 7] whose DOEs were not accurately tuned to the wavelength of input light. It also suggests new applications, such as multicolor trapping, and opportunities for simplified implementation of dynamic holographic optical trapping systems.

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