We describe a method for projecting single-beam optical traps whose potential energy wells are extended along one-dimensional curves. This technique exploits shape-phase holography in which computer-generated phase-only diffractive optical elements are used to implement complex and amplitude-only holograms. The resulting optical traps can have specified intensity and phase profiles along their lengths and can extend along curves in three dimensions. We demonstrate the extended traps’ operation and characterize their potential energy profiles through digital video microscopy of trapped colloidal spheres.

Created by bringing a beam of light to a sharp focus, an optical tweezer establishes a potential energy well that confines mesoscopic objects in three dimensions. This Letter describes a generalized optical tweezer whose domain of influence extends along a specified curve with specified intensity and phase profiles. Such extended optical traps establish tailored potential energy landscapes along their lengths while rigidly confining trapped objects in transverse directions. These capabilities can be exploited for orienting and assembling anisotropic objects such as nanowires [1, 2], rapidly assessing interparticle forces in colloidal dispersions [3–5], and continuously fractionating fluid-borne objects [6, 7].

The archetypal extended optical trap is a so-called line tweezer, in which an appropriately structured beam of light focuses to a line segment rather than a point. Such extended optical traps have been created in a time-averaged sense by scanning a single conventional optical tweezer rapidly across the field of view [4, 5, 8–10]. Continuously illuminated line tweezers have been implemented interferometrically [11, 12], by modifying conventional optical tweezers with rectangular apertures [13], with cylindrical lenses [1, 14], or with their holographic equivalent [15]. When particular care is taken to avoid introducing astigmatism [14] the resulting lines can trap objects stably in three-dimensions. The generalized phase contrast (GPC) method [16] also can project extended traps with arbitrary intensity profiles; three-dimensional trapping can be achieved with counterpropagating GPC traps [17].

Our method, based on the holographic optical trapping technique [18, 19], uses computer-designed diffractive optical elements (DOEs) to implement the complex-valued holograms encoding extended traps, through an approach that we call shape-phase modulation. This method projects extended optical traps with independent control over the intensity and phase profiles along their lengths. It requires only single-sided optical access, lends itself to adaptive optimization [20], and is easily integrated with multi-mode holographic optical traps [19, 21, 22].

Figure 1 schematically represents a typical holographic optical trapping train that can be used to project extended optical traps. Here, a laser (Coherent Verdi, λ = 532 nm) provides a beam of light that is brought to a diffraction-limited focus by an objective lens (Nikon 100× NA 1.4, oil immersion Plan Apo) mounted in an inverted optical microscope (Nikon TE-2000U). The beam is reflected into the objective’s input pupil by a dichroic mirror that also permits images of trapped objects to pass through to a CCD camera. The addition of a DOE in a plane conjugate to the lens’ input aperture enables the system to project both conventional holographic optical traps as well as extended optical traps. In our system, the DOE is implemented with a computer-addressed spatial light modulator (SLM) (Hamamatsu X8267 PPM), which imprints a phase pattern, \( \varphi(\rho) \), discretized into a 768 × 768 array onto the laser beam’s otherwise feature-
less wavefronts. The resulting field,
\[ \psi(\rho) = u(\rho) \exp(i\varphi(\rho)), \]  
(1)
retains the input beam’s amplitude profile \( u(\rho) \) and polarization. When relayed to the objective lens’ input aperture, the modified beam creates the intended optical traps.

An ideal line tweezer focuses as a conical wedge to a line segment with specified intensity and phase profiles. This can be achieved in principle by inverting the Fraunhofer diffraction integral [23] relating the intended trapping field, \( \Psi(\mathbf{r}) \), to the field at the DOE, \( \psi(\rho) \),
\[ \psi(\rho) = \frac{1}{fL} \int_{\Omega} \Psi(\mathbf{r}) \exp \left( \frac{2\pi}{fL} \mathbf{r} \cdot \rho \right) d^2 \rho. \]  
(2)
Here, \( f \) is the lens’ focal length and \( \Omega \) is the optical train’s aperture. We have omitted irrelevant phase factors in Eq. (2).

For example, the field
\[ \Psi(\mathbf{r}) = \begin{cases} \delta(x), & |y| < \frac{L}{2} \\ 0, & \text{otherwise} \end{cases} \]  
(3)
describes a uniformly bright line tweezer of length \( L \) with uniform phase aligned with the \( \hat{y} \) axis. The associated field in the DOE plane is
\[ \psi(\rho) = \frac{\sin(k\rho_y)}{k\rho_y}, \quad \text{where } k = \frac{\pi}{L}. \]  
(4)
This real-valued function cannot be implemented with a conventional phase-only DOE.

To encode \( \psi(\rho) \) through shape-phase modulation, we separate the desired input field along the line into real-valued amplitude and phase functions,
\[ \psi(\rho) = A(\rho) \exp(i\varphi(\rho)), \]  
(5)
where \( A(\rho) \) is a positive definite amplitude. In the particular case of a uniform line tweezer,
\[ A(\rho) = A_0 \left| \frac{\sin(k\rho_y)}{k\rho_y} \right| \quad \text{and} \]  
(6)
\[ \varphi(\rho) = \begin{cases} \pi, & \sin(k\rho_y) \geq 0 \\ 0, & \sin(k\rho_y) < 0 \end{cases} \]  
(7)
The prefactor \( A_0 \) sets the fraction of the incident light that is to be projected into the line trap. If we assume that the DOE is uniformly illuminated, then \( A(\rho) = A(\rho_y) \) may be interpreted as the fraction of light incident on the DOE at \( \rho_y \) needed to form the trap. In projecting a linear trap with a pixellated DOE, the amplitude function \( A(\rho_y) \) specifies how many pixels at \( \rho_y \) contribute to the hologram, but not which. For example, a uniform line tweezer can be projected with
\[ \varphi_S(\rho) = \varphi(\rho_y) S(\rho), \quad \text{where} \]  
(8)
\[ S(\rho) = \begin{cases} 1, & |\rho_x| < A(\rho_y) \\ 0, & \text{otherwise} \end{cases} \]  
(9)
The shape function \( S(\rho) \) divides the plane of the DOE into assigned \((S = 1)\) and unassigned \((S = 0)\) regions. Light passing through the unassigned region can be diverted [22], diffused, or applied to another task by applying another phase mask, \( \varphi_{1-S}(\rho) \), to the unassigned pixels. Light passing through the assigned region then has both the phase and amplitude structure needed to form the extended optical trap.

Figure 1(b) shows a phase-only hologram that encodes a uniform line tweezer \( L = 15 \, \mu m \) long according to Eq. (8). Light passing through the unassigned pixels is deflected by a blazed grating to form a conventional optical tweezer \( 35 \, \mu m \) away. The calculated intensity pattern, shown in Fig. 1(c), agrees closely with the actual light distribution measured by placing a mirror in the sample plane and collecting the reflected light with the objective lens, Fig. 1(d).

The line tweezer in Fig. 1 suffers from two easily remedied defects. The analytical shape function described by Eq. (9) creates transverse artifacts at the line’s ends. These are eliminated by replacing \( S(\rho) \) with a random distribution that assigns the correct number of pixels in each column. The abrupt intensity gradients called for in Eq. (3) furthermore exceed a practical DOE’s spatial bandwidth, and so cause oscillatory artifacts. This is an example of Gibbs phenomenon, which can be minimized by modifying the trap’s design to reduce gradients, or through standard numerical methods [24]. The results in Fig. 2 show the benefits of these corrections.

When powered by 15 mW of light, each of these line tweezers readily traps micrometer-scale colloidal spheres in three dimensions, while allowing them some freedom of motion along the extended axis. We characterized the extended traps’ potential energy profiles for 1.5 \( \mu m \) diameter polystyrene spheres (Duke Scientific Lot 5238) by placing a single particle on the line and tracking its thermally driven motions at 1/30 sec intervals and 10 nm spatial resolution through digital video microscopy [25]. The local potential \( V(\mathbf{r}) \) can be calculated from the measured probability \( P(\mathbf{r}) \) to find the particle within \( d^2 \mathbf{r} \) of position \( \mathbf{r} \) in equilibrium through
\[ V(\mathbf{r}) - V_0 = k_B T \ln P(\mathbf{r}), \]  
(10)
at absolute temperature \( T \), where \( V_0 \) is an arbitrary reference. A single particle’s trajectory over ten minutes yields the results in Fig. 2(a). The longitudinal potential energy profile closely follows the designed shape and is \( 30 \pm 7 \, k_B T \) deep. The bottom third of the well is plotted in the inset to Fig. 2(a) together with a fit to a parabolic profile. Deviations from the designed shape are smaller than \( 0.8 \, k_B T \). These could be further reduced by adaptive optimization [26]. The transverse profile is broadened by the sphere’s diameter, as expected [6, 7].

Multiple extended optical traps can be projected with the same DOE provided their shape functions \( S_j(\rho) \) are disjoint in the sense that \( \int_{\Omega} S_i(\rho) S_j(\rho) d^2 \rho = 0 \) for \( i \neq j \).
FIG. 2: Imaging photometry of holographic line tweezers’ longitudinal and transverse intensity profiles (circles) with specified intensity profiles (solid) and profiles calculated for each specific hologram (dashed). (a) Gaussian. Insets: Image of projected light and (right) measured potential energy well for a 1.5 \( \mu \text{m} \) diameter polystyrene sphere in water at 15 mW. Dashed curves are fits to parabolic wells. (b) Uniform line. Insets: Projected light and (right) bright-field image of seven spheres trapped on the line. (c) Double-well flat-top profile, with projected light.

The assigned domain then is \( S(\rho) = \sum_j S_j(\rho) \). Other modifications to the phase mask that have been described in other contexts [26–28] can be used to translate the line tweezer along the optical axis, to correct for aberrations in the optical train, and to account for such defects in the optical train as phase scaling errors. Finally, the shape-phase modulation can be generalized for intensity modulation of curved tweezers by applying an appropriate conformal mapping to the phase mask.

This work was supported by the National Science Foundation through grant number DMR-0451589.