Lamb- and Casimir- Energies and Periodic Classical Rays

Martin Schaden
Physics Department of Union College
Science & Engineering Bldg., Schenectady, NY12308

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Many phenomena in atomic physics can be described semi-classically and this approximation is often sufficiently accurate to quantitatively explain the experimental results. Even when this is not the case, the semi-classical description nevertheless can be invaluable for the physical insight it provides. Larry Spruch championed this cause early on and has never been satisfied by a calculation that is not backed up by a physical model (often semiclassical). Lamb-shifts and Casimir energies, quantum effects that involve virtual particles and have no classical limit, have conceptually proven to be among the more difficult to describe semi-classically. Within perturbation theory these effects arise as finite *changes* of otherwise (before renormalization) infinite contributions to the overall energy. It is a challenge to semi-classically *approximate* such a *change* in energy without gross error. Apart from greater insight, the advantage of a semi-classical description is that it usually allows for a generalization to more complex situations in which perturbation theory and other exact methods do not apply or are exceedingly difficult. The following is a short review of the small part of Larry's oevre I had the good fortune to collaborate on. At this point I should perhaps warn any future collaborator that working with Larry is sure to involve many twists and wild turns that almost miraculously lead to new insight.

1 The Bulk Lamb Shift

Although the Lamb shift of an isolated atom is perhaps conceptually somewhat difficult, involving as it does the infinities associated with virtual photons, the physical system itself, especially for a hydrogen atom, is rather simple. Note that it is more or less essential to avoid absolute measurements. For the hydrogen atom one first measured the energy difference between the $2p_{1/2}$ and the $2s_{1/2}$ levels. While a proper relativistic (perturbative) calculation is necessary to reproduce the experimental result quantitatively, a non-relativistic semi-classical approximation is rather good.

In dealing with the Lamb-shift for bulk matter, the physical system itself is rather complicated. Further, there is the additional conceptual complication that virtual photons now give rise to a bulk Lamb shift *and* a Casimir shift. In principle only the sum of the two shifts is physically meaningful, but it is natural to ascribe the contribution proportional to the volume to a bulk Lamb shift and the remainder to a Casimir shift. (The latter is largely a surface effect.) Once again one considers energy differences, and once again the non-relativistic approximation is rather good. A non-relativistic semi-classical approximation becomes exceedingly good if one wishes only to determine the *change* in the Lamb shift when an atom is embedded in a medium.

Milonni[1], using perturbation theory, made the first estimate of a bulk Lamb shift. He obtained the *change* of the Lamb shift of an atom in a medium as compared to the Lamb shift of the free atom perturbatively. The Lamb shift of a dielectric was evaluated non-perturbatively in[2]. Although the result was rather simple – see Eq.(3.17) of[2] – the approach was rather cumbersome and the analysis unnecessarily complicated. A much improved and conceptually more transparent approach was used in[3, 4] to derive this result. All three papers assumed that the permittivity $\varepsilon(\omega)$ of the medium is spatially uniform. This is often though not always the case; it should be possible to eliminate this assumption.

2 Infinity-free Casimir Effects and Classical Periodic Orbits

Larry's interest in Casimir effects at least dates back to his first publications with Kelsey on Rydberg states of He[5]. Mine dates back to a few years ago when Larry first got me interested in the subject. At the time we began working together, I was absolutely convinced that everything that could possibly be said about two parallel plates had been published in the last 50 years. Now I am no longer so sure. Larry had developed[6] and improved on an *almost* classical theory of the effect: essentially considering the interactions that would be induced classically by small electric fields, setting the average energy of such fluctuations with a particular frequency and polarization, the volume integral over $\mathbf{E}^2(\omega)/(8\pi)$ to $\hbar\omega$ "in the very last step"[7]. This explicit semi-classical description of zero-point energies by fluctuating virtual fields has been very successful[8]. It is capable of qualitatively describing many observable Casimir effects and in many instances is found to give exact predictions. This "accuracy" is particularily intriguing since the cause for Casimir effects in this case is more transparent than it is in most other field theoretic derivations.

In the past few years some progress has been made in connecting this (for some) intuitive picture of Casimir effects with the field theoretic one. The better understanding is based on the semi-classical approximation to the spectral density. More precisely, it is the *change* in the spectral density under an adiabatic change of the boundary conditions that is approximated semi-classically. The spectral density $\rho(E)$ of a cavity generally is a well-behaved function even if the zero-point energy of the cavity, proportional to the first moment of the spectral density, generically diverges. It diverges due to the ultraviolet behavior of the spectral density. This behavior does not depend on the global shape nor the boundary conditions of the cavity. In the path integral, the ultraviolet behavior of the spectral density is associated with arbitrarily short periodic paths that at most reflect local properties of the boundary (such as its local curvature).

The approach of Gutzwiller[9] avoids infinite contributions to the energy that do not depend on the boundary by semi-classically computing only the *difference* in the spectral density in the presence and absence of the cavity walls. Semi-classically this difference is associated with *classical* periodic paths of *finite* length that arise due to the boundary. Since the Casimir effect is due to (highly) relativistic virtual photons, Gutzwiller's semi-classical periodic orbit theory for non-relativistic particles had to be adapted to this situation[10]. The periodic paths in the massless case are classical periodic rays. When the electromagnetic field cannot be viewed as equivalent to two independent massless scalars the situation is somewhat more complicated; the transport of the polarization vector along the classical periodic orbit then has to be taken into account. Loosely speaking the shorter the classical periodic path, the smaller its action and the more important its contribution to the Casimir energy. The contribution of a single periodic classical path to the Casimir energy typically is inversely proportional to some power of its length, and this approach therefore lends itself to further approximating Casimir energies of complicated cavities by taking only the shortest periodic rays into account. The relation to geometrical optics leads to a semi-classical interpretation of the Casimir force between paraxial mirrors as due to the focusing of virtual photons[11].

We recently[12] used the periodic orbit approach to obtain the temperature dependence of the Casimir energy for some simple cavities. Semi-classically the temperature dependence is due to classical periodic paths that for $T > 0$ can wind a number of times about a periodic fourth dimension with a periodic length of $\hbar c/kT$. For the relatively simple cases we analyzed, this semi-classical description reproduces the finite temperature corrections found in the literature[13].

From a practical point of view, the main merit of the periodic orbit approach is its relative simplicity and conceptual transparency. The theoretical appeal derives from the implication that periodic classical rays that change as boundary conditions are altered semi-classically lead to a Casimir energy. We have found the approach more involved and considerably less useful when diffraction effects are important. The semi-classical description of diffraction by extremal rays was developed[14] long ago and there is some hope that semi-classical Casimir energies may eventually be related to *extremal* classical rays at a considerable cost in the simplicity of the approach.

The description of the Casimir energy in terms of the change in the sum over the zero-point energies of each oscillator is dual to its description in terms of classical periodic paths in the sense that, at least for simple cavities, the two can be related by Poisson's summation formula: the change in the Casimir energy due to a single mode can also be obtained from a coherent sum of classical paths and vice versa. The Casimir energy thus can often be relatively accurately estimated with only a few (short) classical paths. The method could have the potential of providing a systematic geometrical expansion of the Casimir energy in terms of classical periodic rays.

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