Relativistic *vs* nonrelativistic scattering of slow electrons

An article published in which it was claimed that even at non-relativistic incident kinetic energies one would not use the Schroedinger (S) equation to describe scattering. The argument was based on the fact that the S and D irac (D) equations give different results for a given potential V. The last statement is correct but the claim was wrong.

The flaw in the claim is that for a given problem one should use different potentials in the S and D equations, as pointed out initially by M. Rotenberg. Thus [46], if one reduces the D equation containing a potential to be denoted by V_D to an S equation, the potential V_S appearing in the S equation will differ from V_D. The point is more simply made by considering the relativistic Klein-Gordon (KG) equation with the vector potential taken to be zero, $\left(-\hbar^2 c^2 \nabla^2 + m^2 c^4\right)\Psi_{KG} = \left(E - V_{KG}\right)^2 \Psi_{KG}$.

Setting $E^2 = m^2c^4 + p^2$ this equation can be written as

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + \left(\frac{E V_{KG}}{mc^2} - \frac{V_{KG}^2}{2m^{C2}}\right)\right]\Psi_{KG} = \frac{p^2}{2m}\Psi_{KG}. \quad (1)$$

(There is a typo – the V^2 term appears with a plus sign – in [46].) Eq. (1) can be interpreted as an S equation with an energy dependent potential V_S that differs from V_{KG}. In the non-relativistic domain, we have $E \approx mc^2$, and we arrive at $\left(-\frac{\hbar^2}{2m}\nabla^2 + V_S\right)\Psi_S = \frac{p^2}{2m}\Psi_S$, where $V_S = V_{KG} - \left(V_{KG}^2 / 2mc^2\right)$, with no restriction on the strength of V_{VG}.