Exchange currents, magnetic moments, and isomeric transitions

When a neutron and a proton within a nucleus interact via the exchange of a charged meson, the current J_{ex} carried by the meson contributes to the magnetic moment of the nucleus and to isomeric transitions. Unfortunately, a proper treatment of $\vec{J}_{ex}$ depends upon meson theory, not well understod when the papers to be discussed were written and not that well understood even today. However, one component of $\vec{J}_{ex}$ can be given a solid theoretical basis. One uses a very clever argument – not due to us – to determine the contribution to J_{ex} associated with the space-exchange component $P_{jk}V$ of the full potential. V is a function of the separation $\vec{r}_{jk} = \vec{r}_j - \vec{r}_k$ of the nucleons and can depend upon the spins. P_{jk} is the space exchange operator, defined by $P_{jk}\, f\left(\vec{r}_{jk}\right) = f\left(\vec{r}_{kj}\right)$.

We now give a slight indication of how one can deduce the $\overleftarrow{J}_{jk}$ that arises from $P_{jk}V$. (The actual proof is a bit complicated.) We begin by noting that it follows from a Taylor series expansion that $e^{\vec{a}\cdot\vec{\nabla}} f\left(\vec{x}\right) = e^{(i/\hbar)\vec{a}\cdot\vec{p}} f\left(\vec{x}\right) = f\left(\vec{x}+\vec{a}\right)$, where $\vec{p} = -i\hbar\vec{\nabla}$, for any constant vector $\vec{a}$. It does not follow that with $\vec{r} = \vec{r}_{jk}\ and\ \vec{p}\ = \vec{p}_j - \vec{p}_k, e^{(i/\hbar)2\vec{r}\cdot\vec{p}} f\left(\vec{r}\right) = f\left(-\vec{r}\right) = P_{jk}\, f\left(\vec{r}\right)$, since $(i/\hbar)2\vec{r}$ is not constant, but Eq. (1) does at least suggest that it might be possible to express P_{kj} as a momentum-dependent operator. Wheeler in act showed that that was the case. To go from P_{jk} for the field-free case to $P_{jk}\left(\vec{B}\right)$ for the nucleus in a magnetic field $\vec{B}$, one makes the usual replacement $\vec{p}_j \to \vec{p}_j - e_j\vec{A}\left(\vec{r}_j\right)/e$. For $\vec{B}$ a uniform field, one has $\vec{A}\left(\vec{r}_j\right) = -\frac{1}{2}\left(\vec{r}_j xB\right)$. Since the magnetic moment of a system is defined by its interaction with an arbitrarily weak $\vec{B}$ – field, simplifications are possible, and one arrives at

$$\vec{M}_{ex} = (ie/2\hbar c)\sum_{\pi}\sum_{\nu}\int\left(\vec{r}_{\pi}\, x\, \vec{r}_{0}\right)\left(\Psi, J_{\pi\nu}P_{\pi\nu}\Psi\right)d\vec{r}_1 \dots d\vec{r}_N, \qquad (1)$$

where the spatial integral is over all nucleons the scalar product is the sum of all spin coordinates, and e is the charge of the proton. (There are no $\nu - \nu$ or $\pi - \pi$ terms, for they arise from neutral mesons.) The result for $\vec{M}_{ex}$ was derived by R.G. Sachs.

We obtained rough estimates of $\vec{M}_{ex}$ of Eq. (1) for light nuclei, using simple models (ℓs or jj coupling). The results did not give agreement with the data but were large enough to warrant consideration.[1] We also studied heavy nuclei; we estimated the magnetic moment $\vec{M}_{ex}$ [2], and the interaction contributions of $\vec{J}_{ex}$ to isomeric transitions [5]. In addition, we considered other interaction contributions, with forms limited by invariance and symmetry considerations, which are not velocity dependent and which arise from second-order meson calculations. Most of the calculations were based on the shell model.