Thermal Stability of the Magnetization in Perpendicularly Magnetized Thin Film Nanomagnets

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We compute the energy barrier and transition state for thermally induced magnetization reversal of thin film nanomagnets with perpendicular magnetic anisotropy (PMA) using the string method. The reversal is generally found to be incoherent; that is, to occur by the nucleation and expansion of a reversed domain. We show that for square elements the energy barrier increases with element size up to a critical size, beyond which the energy barrier is constant. For circular elements the energy barrier continues to increase, albeit more slowly beyond a critical size. In both cases the energy barriers are less than those expected for coherent magnetization reversal. These results have implications for the design of perpendicularly magnetized patterned media and magnetic random access memories.

Thin film elements with perpendicular magnetic anisotropy (PMA) are of great interest in information storage and processing, such in patterned media [1] and spin-transfer MRAM [2]. This is because they can have magnetization directions that are thermally stable at room temperature in nanometer scale elements. A key issue is how their energy barrier and transition state depend on element lateral size. For elements larger than the exchange length (typically ~ 5 nm in transition metal ferromagnets) the assumption of coherent reversal of the magnetization breaks down and the transition state is in generally not uniformly magnetized. Due to the multiscale character of micromagnetism, analytical calculations are complicated and transition states have been calculated only for a handful of physical systems [3–5]. Numerical calculations become necessary for the majority of systems. In this Letter, we use the string method [6] to find the transition states and activation energies in thin film elements with PMA.

We describe the micromagnetic energy of this system by:

\[
E = t \int_\Omega \left[ |A \nabla \mathbf{m}|^2 + \frac{\mu_0}{2} \mathbf{M} \cdot \mathbf{N} \cdot \mathbf{M} - K m_z^2 - \mu_0 H_z M_z \right] \, d\Omega.
\]  

(1)

where \( \mathbf{N} \) is the spatially dependent demagnetizing tensor, and \( \mathbf{m} \) is the normalized magnetization vector. In the macrospin approximation \( A \rightarrow \infty \), this first term becomes a constraint and the energy reduces to:

\[
E_{\text{macrospin}} = t \Omega \left[ \frac{\mu_0 M_s^2}{2} - K m_z^2 - \mu_0 H_z M_z \right].
\]  

(2)

where \( \Omega \) is the inplane cross-sectional area of the film.

The magnetization dynamics is governed by the Landau-Lifshitz-Gilbert equation which we write below as:

\[
\frac{d\mathbf{m}}{dt} = -\gamma' \mathbf{m} \times \nabla_M E - \gamma' \alpha \mathbf{m} \times (\mathbf{m} \times \nabla_M E)
\]  

(3)

where \( \gamma' = \gamma/(1 + \alpha^2) \) is the gyromagnetic ratio and \( \alpha \) is the damping constant. Transition state theory indicates that reversal occurs through the critical points of the magnetization dynamics (i.e. \( d\mathbf{m}/dt = 0 \)) which in this system coincide with the stationary configurations of the energy surface \( \nabla_M E = 0 \). The probability for thermal induced magnetization reversal is expected to follow the Arrhenius law \( e^{-U/k_B T} \) where \( U \) is the energy difference between the transition state and the metastable configuration [7].

We implemented the String Method in conjunction with OOMMF [8] to find the transition states by integrating only the nonconservative term of Eq. 3 (the magnetization evolves parallel to \( -\nabla_M E \)). We use 100 images between the two energy minima and reparametrize the string every 40 ps. Our initial guess path passed through a fully randomized magnetization configuration and was allowed to evolve according to the String Method prescription to its minimal energy path, details are presented elsewhere [9–11].

We studied thin layers of thickness \( t = 1.6 \) nm, with saturation magnetization \( M_s = 713 \times 10^3 \) A/m, exchange constant \( A = 8.3 \times 10^{-12} \) J/m, and anisotropy constant \( K = 403 \times 10^{13} \) J/m\(^3\). A constant field perpendicular to the film \( \mu_0 H_z \) is applied with a maximum magnitude equal to the coercive field \( B_c = (2K - \mu_0 M_s^2)/M_s = 0.245 \) T, this breaks the degeneracy between the two lowest lying magnetization states. We investigate the transition from the metastable state (downward magnetization) to the ground state (upward magnetization) and compare to the macrospin model. A typical result is presented in Fig. 1 where the reversal can be seen to occur by propagation of a Bloch wall across the sample.

The field dependence of the activation energy is shown in Fig. 2 for the large samples the activation energy is clearly lower than the prediction of the macrospin model which from eq. 2 can be derived to be

\[
U_{\text{macrospin}} = t \Omega \left( \frac{\mu_0 M_s H_z^2}{2B_c} + \frac{B_c M_s}{2} - \mu_0 H_z M_s \right)
\]  

(4)

with the transition state at \( m_z = -\mu_0 H_z / B_c \). As expected, the activation energy decreases as the field approaches the coercive field.

Two points needs to be made regarding the way the energy barrier changes with the size of the element in Fig 2. First, the activation energy barrier is independent of the
results indicate that merely increasing the macrospin with different magnetostatic tensors. Our activation energy is compared to two different versions of model. The results are shown in Fig. 3 where the activation energy increases with the size of the element, this clearly be identified. For small samples, the activation energy becomes lower than that predicted by the macrospin model.

Figure 1: Images of the minimum energy path. Images number are at locations 0, 20, 40, 60, 80, 100. Red and blue (light and dark) represent downward and upward magnetization respectively. Reversal occurs by nucleation of a domain on one corner, and propagation of a domain wall across the material. The transition state is located close to image 20.

Figure 2: Field dependence of activation energy for elements with two different sizes. It can be seen that the activation energy is lower than that predicted by the macrospin model.

cell size used in the numerical integration once the cells become smaller than the exchange length; which is the standard requirement of numerical micromagnetics. This is then a real physical effect and not a numerical artifact of changing the relative scales of the system. Second, an effective size reduction can be achieved by increasing the exchange constant to a very large value (10^2 x A). In this limit our result should approach the macrospin model. The results are shown in Fig. 3 where the activation energy is compared to two different versions of the macrospin with different magnetostatic tensor. Our results indicate that merely increasing A is not sufficient to match the macrospin model and the String Method. A further refinement of the magnetostatic energy term is necessary in which the film is not considered an infinite plane. In this case, the demagnetization tensor is calculated using formulae by Newell [12]. This result in a perfect match. Considering the extremely small aspect ratio of the samples this is an important result: it shows that finite size effects are important even for very thin geometries.

Size effects of the activation energy in square geometries are summarized in Fig. 4 where two regimes can clearly be identified. For small samples, the activation energy increases with the size of the element, this regime corresponds to the macrospin model. Beyond a certain size (~ 50 nm) the activation energy becomes a constant. For large elements, the radius of the reversed nucleus is independent of the element size. For all sizes considered, the reverse domain resembles a circular region with domain walls making right angles with the sample’s edge (see second image at Fig. 1). At the given configuration eq. 1 can be approximated as the sum of three dominant terms: a switched domain with area \( \Omega_1 = (\pi R^2/4) \) with energy density per unit area \( \xi_1 \) approximate to \( t \left( \frac{\mu_0 M_s^2}{2} - K - \mu_0 H_z M_s \right) \); an unswitched domain with downward magnetization of area \( \Omega_2 = \Omega - \Omega_1 = \Omega - \pi R^2/4 \) and surface energy density \( \xi_2 \approx t \left( \frac{\mu_0 M_s^2}{2} - K + \mu_0 H_z M_s \right) \); and the domain wall of...
length $s_3 = 2\pi R/4$ with a linear energy density per unit length $\lambda_3 \approx 4\sqrt{AK'}$. Here, $K'$ is an effective crystalline anisotropy constant $K' = (K - \mu_0 M^2 s_2^2)$. The total energy $\xi_1 \Omega_1 + \xi_2 \Omega_2 + s_3 \lambda_3$ is a maximum for $R^* = 38$ nm with an activation energy of $U \approx 16 \times 10^{-20}$J which is a good approximation to the energy plateau presented in figure 4. Our estimate for $R^*$ predicts to good approximation the start of the plateau in activation energy, the samples must be large enough to accommodate the nucleus of size $R^*$ at a distance of at least a few exchange lengths from the corners of the sample.

Fig. 4 presents the computed activation energies for circular samples as a function of in plane diameter. The absence of corners prevents the circular shape of reaching a plateau on the activation energy. As a result circular samples become more thermally stable for increasing size.

In conclusion, using the String Method we have calculated activation energies for thermally induced magnetization reversal and compared the results to a uniform magnetization model. Our results indicate that finite size effects on the macrospin model are important even for very thin samples. In addition, the activation energy for square samples as a function of size present a transition from linear dependence to a constant value. This result mimics previous prediction of activation energies for elongated particles and circular nanorings [5, 14].